

Equations and Inequalities

PA

M11.A.1.3.2

M11.A.3.1.1

M11.D.2.1.3

M11.D.2.1.3

M11.D.2.1.3

M11.D.2.1.1

M11.D.2.1.1

1.1 Apply Properties of Real Numbers

1.2 Evaluate and Simplify Algebraic Expressions

1.3 Solve Linear Equations

1.4 Rewrite Formulas and Equations

1.5 Use Problem Solving Strategies and Models

1.6 Solve Linear Inequalities

1.7 Solve Absolute Value Equations and Inequalities

Before

In previous courses, you learned the following skills, which you'll use in Chapter 1: simplifying numerical expressions, using formulas, and writing algebraic expressions.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **area** of the rectangle is ?.
2. The **perimeter** of the rectangle is ?.
3. The **opposite** of any number a is ?.



SKILLS CHECK

Perform the indicated operation. (Review p. 975 for 1.1, 1.2.)

4. $5 \cdot (-3)$
5. $3 + (-4)$
6. $-28 \div (-7)$
7. $8 - (-15)$

Find the area of the figure. (Review pp. 991–992 for 1.4.)

8. A square with side length 7 ft
9. A circle with radius 3 m

Write an expression to answer the question. (Review p. 984 for 1.5.)

10. How much is a 15% tip on a restaurant bill of x dollars?
11. You have \$15 and buy r raffle tickets for \$.50 each. How much money do you have left?

Reporting
Category E

Data Analysis and Probability

M11.E.1.1.1 Create and/or use appropriate graphical representations of data, including box-and-whisker plots, stem-and-leaf plots or scatter plots.

- 129. (OE)** The following data represents the living area (in hundreds of square feet) of a home and the selling price (in thousands of dollars) of the home. Draw a scatter plot to represent the data. (118, prob. 10–15)

Living Area (hundreds of sq ft)	14	36	23	17	17	13	21	25
Selling price (thousands of dollars)	145	228	150	130	160	114	142	165

M11.E.2.1.1 Calculate or select the appropriate measure of central tendency (mean, mode or median) of a set of data given or represented on a table, line plot or stem-and-leaf plot.

- 130. (OE)** During a week in February the low temperature at Fargo, North Dakota, was recorded each day. The results are presented in the table below. Find both the median and the mean for the set of numbers. (p.747, prob. 3–8)

M	Tu	W	Th	F	Sa	Su
–11	–17	–15	–18	–20	–2	20

M11.E.2.1.3 Describe how outliers affect measures of central tendency.

- 131. (OE)** The data set below gives the number of cars sold at a car dealership over 6 months. (page 748, prob. 26)

5, 6, 4, 6, 23, 4

- A.** Find the mean, median, and mode.
- B.** It is suspected that 23 is an outlier. Calculate the new mean, median, and mode without 23 included.
- C.** How did the outlier affect the mean, median, and mode?



Now

In Chapter 1, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 60. You will also use the key vocabulary listed below.

Big Ideas

- ① Using properties to evaluate and simplify expressions
- ② Using problem solving strategies and verbal models
- ③ Solving linear and absolute value equations and inequalities

KEY VOCABULARY

- reciprocal, p. 4
- power, p. 10
- exponent, p. 10
- base, p. 10
- variable, p. 11
- coefficient, p. 12
- like terms, p. 12
- equivalent expressions, p. 12
- linear equation, p. 18
- equivalent equations, p. 18
- solve for a variable, p. 26
- linear inequality, p. 41
- compound inequality, p. 41
- absolute value, p. 51
- extraneous solution, p. 52

Why?

You can use equations to solve problems about work rates. For example, if two people complete a job at different rates, you can find how long it will take them if they work together.

Animated Algebra

The animation illustrated below for Exercise 76 on page 24 helps you answer this question: If two people paint a community mural at different rates, how long will it take them to complete the mural if they work together?



You want to paint a mural covering a given area. You can work with a friend.

How long will it take to paint 11 square yards of the mural if you work separately? How long will it take if you work together?

☒ You alone
☐ Friend alone
☐ Both you and your friend

Worker	Worker rate
You	
Friend	

Amount of time worked:

Amount of job finished:

Enter your work rates, then find how long it takes working alone or with your friend.

Animated Algebra at classzone.com

Other animations for Chapter 1: pages 5, 11, 20, 27, 34, 42, and 53

1.1 Apply Properties of Real Numbers

PA

M11.A.1.3.2

Compare and/or order any real numbers (rational and irrational may be mixed).

Before

You performed operations with real numbers.

Now

You will study properties of real numbers.

Why?

So you can order elevations, as in Ex. 58.

Key Vocabulary

- opposite
- reciprocal

KEY CONCEPT

For Your Notebook

Subsets of the Real Numbers

The *real numbers* consist of the *rational numbers* and the *irrational numbers*. Two subsets of the rational numbers are the *whole numbers* (0, 1, 2, 3, ...) and the *integers* (... , -3, -2, -1, 0, 1, 2, 3, ...).

REAL NUMBERS

$\frac{3}{4} = 0.75$	Rational Numbers	$-\frac{1}{3} = -0.333...$	Irrational Numbers
-4	Integers	-1 -27	$\sqrt{2} = 1.414213...$
0 5	Whole Numbers	16	$-\sqrt{14} = -3.74165...$
			$\pi = 3.14159...$

Rational Numbers

- can be written as quotients of integers
- can be written as decimals that terminate or repeat

Irrational Numbers

- cannot be written as quotients of integers
- cannot be written as decimals that terminate or repeat

NUMBER LINE Real numbers can be graphed as points on a line called a *real number line*, on which numbers increase from left to right.

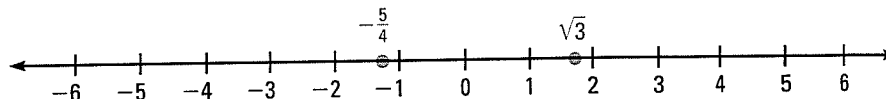
EXAMPLE 1 Graph real numbers on a number line

Graph the real numbers $-\frac{5}{4}$ and $\sqrt{3}$ on a number line.

Solution

Note that $-\frac{5}{4} = -1.25$. Use a calculator to approximate $\sqrt{3}$ to the nearest tenth: $\sqrt{3} \approx 1.7$. (The symbol $\approx$ means *is approximately equal to*.)

So, graph $-\frac{5}{4}$ between -2 and -1, and graph $\sqrt{3}$ between 1 and 2, as shown on the number line below.



**EXAMPLE 2** Standardized Test Practice

The table shows the lowest elevations of six continents. Which list shows the elevations from lowest to highest?

Continent	Africa	Asia	Australia	Europe	North America	South America
Lowest elevation	-156 m	-408 m	-16 m	-28 m	-86 m	-40 m

- (A) -408, -156, -86, -28, -40, -16 (B) -408, -156, -28, -86, -40, -16
 (C) -16, -28, -40, -86, -156, -408 (D) -408, -156, -86, -40, -28, -16

Solution

From lowest to highest, the elevations are -408, -156, -86, -40, -28, and -16.

► The correct answer is D. (A) (B) (C) (D)

ELIMINATE CHOICES

The problem asks for the elevations from lowest to highest, not from highest to lowest. So, you can eliminate choice C.

**GUIDED PRACTICE** for Examples 1 and 2

- Graph the numbers -0.2 , $\frac{7}{10}$, -1 , $\sqrt{2}$, and -4 on a number line.
- Which list shows the numbers in increasing order?

(A) -0.5 , 1.5 , -2 , -0.75 , $\sqrt{7}$ (B) -0.5 , -2 , -0.75 , 1.5 , $\sqrt{7}$
 (C) -2 , -0.75 , -0.5 , 1.5 , $\sqrt{7}$ (D) $\sqrt{7}$, 1.5 , -0.5 , -0.75 , -2

PROPERTIES OF REAL NUMBERS You learned in previous courses that when you add or multiply real numbers, there are several properties you can use.

KEY CONCEPT*For Your Notebook***Properties of Addition and Multiplication**

Let a , b , and c be real numbers.

Property	Addition	Multiplication
Closure	$a + b$ is a real number.	ab is a real number.
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	$a + 0 = a$, $0 + a = a$	$a \cdot 1 = a$, $1 \cdot a = a$
Inverse	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1$, $a \neq 0$

The following property involves both addition and multiplication.

Distributive $a(b + c) = ab + ac$

EXAMPLE 3 Identify properties of real numbers

Identify the property that the statement illustrates.

a. $7 + 4 = 4 + 7$

b. $13 \cdot \frac{1}{13} = 1$

Solution

a. Commutative property of addition

b. Inverse property of multiplication

KEY CONCEPT*For Your Notebook***Defining Subtraction and Division**

Subtraction is defined as *adding the opposite*. The **opposite**, or *additive inverse*, of any number b is $-b$. If b is positive, then $-b$ is negative. If b is negative, then $-b$ is positive.

$$a - b = a + (-b) \quad \text{Definition of subtraction}$$

Division is defined as *multiplying by the reciprocal*. The **reciprocal**, or *multiplicative inverse*, of any nonzero number b is $\frac{1}{b}$.

$$a \div b = a \cdot \frac{1}{b}, b \neq 0 \quad \text{Definition of division}$$

EXAMPLE 4 Use properties and definitions of operations

Use properties and definitions of operations to show that $a + (2 - a) = 2$. Justify each step.

Solution

$$\begin{aligned} a + (2 - a) &= a + [2 + (-a)] && \text{Definition of subtraction} \\ &= a + [(-a) + 2] && \text{Commutative property of addition} \\ &= [a + (-a)] + 2 && \text{Associative property of addition} \\ &= 0 + 2 && \text{Inverse property of addition} \\ &= 2 && \text{Identity property of addition} \end{aligned}$$

**GUIDED PRACTICE** for Examples 3 and 4

Identify the property that the statement illustrates.

3. $(2 \cdot 3) \cdot 9 = 2 \cdot (3 \cdot 9)$

4. $15 + 0 = 15$

5. $4(5 + 25) = 4(5) + 4(25)$

6. $1 \cdot 500 = 500$

Use properties and definitions of operations to show that the statement is true. Justify each step.

7. $b \cdot (4 \div b) = 4$ when $b \neq 0$

8. $3x + (6 + 4x) = 7x + 6$

UNIT ANALYSIS When you use operations in real-life problems, you should use *unit analysis* to check that the units in your calculations make sense.

EXAMPLE 5 Use unit analysis with operations

- You work 4 hours and earn \$36. What is your earning rate?
- You travel for 2.5 hours at 50 miles per hour. How far do you go?
- You drive 45 miles per hour. What is your speed in feet per second?

Solution

- $\frac{36 \text{ dollars}}{4 \text{ hours}} = 9 \text{ dollars per hour}$
- $(2.5 \text{ hours}) \left(\frac{50 \text{ miles}}{1 \text{ hour}} \right) = 125 \text{ miles}$
- $\left(\frac{45 \text{ miles}}{1 \text{ hour}} \right) \left(\frac{1 \text{ hour}}{60 \text{ minutes}} \right) \left(\frac{1 \text{ minute}}{60 \text{ seconds}} \right) \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) = 66 \text{ feet per second}$

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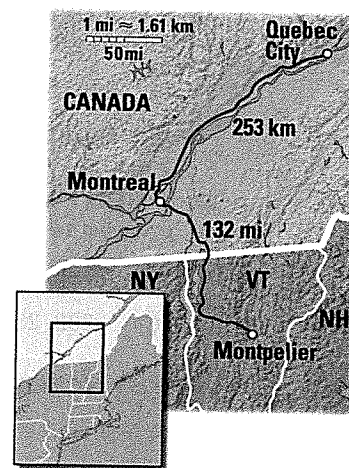
EXAMPLE 6 Use unit analysis with conversions

DRIVING DISTANCE The distance from Montpelier, Vermont, to Montreal, Canada, is about 132 miles. The distance from Montreal to Quebec City is about 253 kilometers.

- Convert the distance from Montpelier to Montreal to kilometers.
- Convert the distance from Montreal to Quebec City to miles.

Solution

- $132 \text{ miles} \cdot \frac{1.61 \text{ kilometers}}{1 \text{ mile}} \approx 213 \text{ kilometers}$
- $253 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}} \approx 157 \text{ miles}$



GUIDED PRACTICE for Examples 5 and 6

Solve the problem. Use unit analysis to check your work.

- You work 6 hours and earn \$69. What is your earning rate?
- How long does it take to travel 180 miles at 40 miles per hour?
- You drive 60 kilometers per hour. What is your speed in miles per hour?

Perform the indicated conversion.

- 150 yards to feet
- 4 gallons to pints
- 16 years to seconds

REVIEW MEASURES

For help with converting units, see the Table of Measures on p. 1025.

1.1 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 21, 31, and 59

★ = STANDARDIZED TEST PRACTICE Exs. 2, 9, 10, 23, 24, 60, and 61

SKILL PRACTICE

EXAMPLE 1

on p. 2
for Exs. 3–8

1. **VOCABULARY** Copy and complete: The ? of any nonzero number b is $\frac{1}{b}$.

2. ★ **WRITING** Express the associative property of addition in words.

GRAPHING NUMBERS Graph the numbers on a number line.

3. $-\frac{3}{4}, 5, \frac{9}{2}, -2, -1$

4. $-3, \frac{5}{2}, 2, -\frac{9}{4}, 4$

5. $1, \sqrt{3}, -\frac{2}{3}, -\frac{5}{4}, 2$

6. $6, -\sqrt{5}, 2.7, -2, \frac{7}{3}$

7. $-0.4, \frac{3}{2}, 0, \sqrt{10}, -1$

8. $-1.7, 5, \frac{9}{2}, -\sqrt{8}, -3$

EXAMPLE 2

on p. 3
for Exs. 9–10

ORDERING NUMBERS In Exercises 9 and 10, use the table of elevations below.

State	Alabama	California	Kentucky	Louisiana	Tennessee
Highest elevation	2407 ft	14,494 ft	4145 ft	535 ft	6643 ft
Lowest elevation	0 ft	-282 ft	257 ft	-8 ft	178 ft

9. ★ **MULTIPLE CHOICE** Which list shows the highest elevations in order from least to greatest?

(A) 2407; 14,494; 4145; 535; 6643

(B) 535; 2407; 4145; 6643; 14,494

(C) 14,494; 2407; 4145; 535; 6643

(D) 14,494; 6643; 4145; 2407; 535

10. ★ **MULTIPLE CHOICE** Which list shows the lowest elevations in order from greatest to least?

(A) 0, -8, 178, 257, -282

(B) -282, -8, 0, 178, 257

(C) -282, 257, 178, -8, 0

(D) 257, 178, 0, -8, -282

Louisiana bayou

EXAMPLE 3

on p. 4
for Exs. 11–16

IDENTIFYING PROPERTIES Identify the property that the statement illustrates.

11. $(4 + 9) + 3 = 4 + (9 + 3)$

12. $15 \cdot 1 = 15$

13. $6 \cdot 4 = 4 \cdot 6$

14. $5 + (-5) = 0$

15. $7(2 + 8) = 7(2) + 7(8)$

16. $(6 \cdot 5) \cdot 7 = 6 \cdot (5 \cdot 7)$

EXAMPLE 4

on p. 4
for Exs. 17–22

USING PROPERTIES Use properties and definitions of operations to show that the statement is true. *Justify* each step.

17. $6 \cdot (a \div 3) = 2a$

18. $15 \cdot (3 \div b) = 45 \div b$

19. $(c - 3) + 3 = c$

20. $(a + b) - c = a + (b - c)$

(21) $7a + (4 + 5a) = 12a + 4$

22. $(12b + 15) - 3b = 15 + 9b$

23. ★ **OPEN-ENDED MATH** Find values of a and b such that a is a whole number, b is a rational number but not an integer, and $a \div b = -8$.

24. ★ **OPEN-ENDED MATH** Write three equations using integers to illustrate the distributive property.

EXAMPLE 5

on p. 5
for Exs. 25–30

OPERATIONS AND UNIT ANALYSIS Solve the problem. Use unit analysis to check your work.

25. You work 10 hours and earn \$85. What is your earning rate?
26. You travel 60 kilometers in 1.5 hours. What is your average speed?
27. You work for 5 hours at \$7.25 per hour. How much do you earn?
28. You buy 6 gallons of juice at \$1.25 per gallon. What is your total cost?
29. You drive for 3 hours at 65 miles per hour. How far do you go?
30. You ride in a train for 175 miles at an average speed of 50 miles per hour. How many hours does the trip take?

EXAMPLE 6

on p. 5
for Exs. 31–40

CONVERSION OF MEASUREMENTS Perform the indicated conversion.

- | | |
|----------------------------|------------------------------|
| 31. 350 feet to yards | 32. 15 meters to millimeters |
| 33. 2.2 kilograms to grams | 34. 5 hours to minutes |
| 35. 7 quarts to gallons | 36. 3.5 tons to pounds |
| 37. 56 ounces to tons | 38. 6800 seconds to hours |

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ERROR ANALYSIS Describe and correct the error in the conversion.

39.

$$25 \text{ dollars} \cdot \frac{1 \text{ dollar}}{0.82 \text{ euro}} \approx 30.5 \text{ euros}$$



40.

$$5 \text{ pints} \cdot \frac{1 \text{ cup}}{2 \text{ pints}} = 2.5 \text{ cups}$$



CONVERSION OF RATES Convert the rate into the given units.

- | | |
|----------------------------------|------------------------------------|
| 41. 20 mi/h to feet per second | 42. 6 ft/sec to miles per hour |
| 43. 50 km/h to miles per hour | 44. 40 mi/h to kilometers per hour |
| 45. 1 gal/h to ounces per second | 46. 6 oz/sec to gallons per hour |

47. **ROCKET SLED** On a track at an Air Force base in New Mexico, a rocket sled travels 3 miles in 6 seconds. What is the average speed in miles per hour?

48. **ELEVATOR SPEED** The elevator in the Washington Monument takes 60 seconds to rise 500 feet. What is the average speed in miles per hour?

REASONING Tell whether the statement is *always*, *sometimes*, or *never* true for real numbers a , b , and c . Explain your answer.

- | | | |
|---|---|---------------------------------|
| 49. $(a + b) + c = a + (b + c)$ | 50. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ | 51. $(a - b) - c = a - (b - c)$ |
| 52. $(a \div b) \div c = a \div (b \div c)$ | 53. $a(b - c) = ab - ac$ | 54. $a(b \div c) = ab \div ac$ |

55. **REASONING** Show that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{c} \div \frac{b}{d}$ for nonzero real numbers a , b , c , and d . Justify each step in your reasoning.

56. **CHALLENGE** Let $\frac{a}{b}$ and $\frac{c}{d}$ be two distinct rational numbers. Find the rational number that lies exactly halfway between $\frac{a}{b}$ and $\frac{c}{d}$ on a number line.

PROBLEM SOLVING

EXAMPLE 2

on p. 3
for Exs. 57–59

57. **MINIATURE GOLF** The table shows the scores of people playing 9 holes of miniature golf.

Lance	+2	+1	0	0	-1	+1	+3	0	0
Darcy	-1	+3	0	-1	+1	0	0	+1	-1
Javier	+1	0	+1	0	0	-1	+1	0	+1
Sandra	-1	-1	0	0	+1	-1	0	0	0

- Find the sum of the scores for each player.
- List the players from best (lowest) to worst (highest) total score.

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58. **VOLCANOES** The following list shows the elevations (in feet) of several volcano summits above or below sea level.

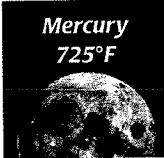
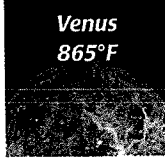
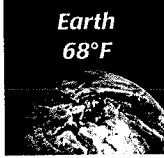
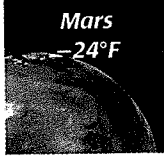
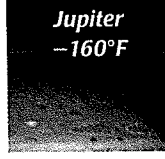
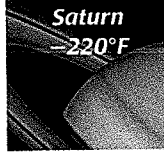
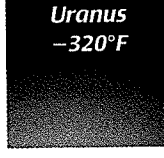
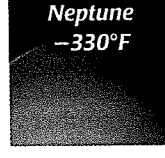
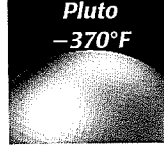
641, 3976, 610, -59, 1718, 1733, -137

Order the elevations from lowest to highest.

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59. **MULTI-STEP PROBLEM** The chart shows the average daytime surface temperatures on the planets in our solar system.

- Sort by Temperature** List the planets in order from least to greatest daytime surface temperature.
- Sort by Distance** List the planets in order from least to greatest distance from the sun.
- Find Patterns** What pattern do you notice between surface temperature and distance from the sun?
- Analyze** Which planet does not follow the general pattern you found in part (c)?

Daytime Surface Temperatures		
Mercury 725°F 	Venus 865°F 	Earth 68°F 
Mars -24°F 	Jupiter -160°F 	Saturn -220°F 
Uranus -320°F 	Neptune -330°F 	Pluto -370°F 

60. **★ EXTENDED RESPONSE** The average weight of the blue whale (the largest mammal) is 120 tons, and the average weight of the bumblebee bat (the smallest mammal) is 0.07 ounce.

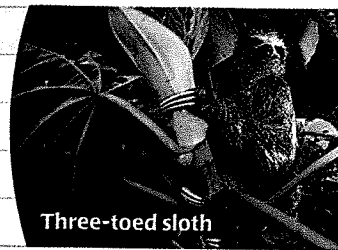
- Convert** Convert the weight of the blue whale from tons to pounds. Convert the weight of the bumblebee bat from ounces to pounds.
- Compare** About how many times as heavy as the bat is the blue whale?
- Find a Method** Besides converting the weights to pounds, what is another method for comparing the weights of the mammals?

EXAMPLES 5 and 6

on p. 5
for Exs. 60–61

61. ★ **SHORT RESPONSE** The table shows the maximum speeds of various animals in miles per hour or feet per second.

Animal	Speed (mi/h)	Speed (ft/s)
Cheetah	70	?
Three-toed sloth	?	0.22
Squirrel	12	?
Grizzly bear	?	44



Three-toed sloth

- a. Copy and complete the table.
- b. *Compare* the speeds of the fastest and slowest animals in the table.
62. **CHALLENGE** A newspaper gives the exchange rates of some currencies with the U.S. dollar, as shown below. Copy and complete the statements.

	1 USD	in USD
Australian dollar	1.31234	0.761998
Canadian dollar	1.1981	0.834655
Hong Kong dollar	7.7718	0.12867
New Zealand dollar	1.43926	0.694801
Singapore dollar	1.6534	0.604814

This row indicates that
\$1 U.S. $\approx$ \$1.31 Australian and
\$1 Australian $\approx$ \$.76 U.S.

- a. 1 Singapore dollar $\approx$? Canadian dollar(s)
- b. 1 Hong Kong dollar $\approx$? New Zealand dollar(s)

PA

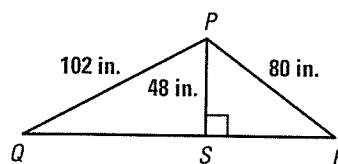
PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

63. Susan purchased a television on sale for \$315. The original price of the television was \$370. Which expression can be used to determine the percent of the original price that Susan saved on the purchase of this television?

- (A) $\frac{315}{370} \times 100$
- (B) $\frac{370}{315} \times 100$
- (C) $\frac{370 - 315}{315 \times 100}$
- (D) $\frac{370 - 315}{370} \times 100$

64. In the figure, what is the length of $\overline{QR}$ in inches?



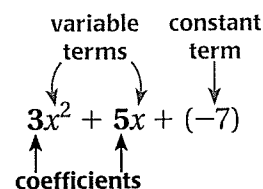
- (A) 86 in. (B) 90 in. (C) 122 in. (D) 154 in.

Terms and Coefficients

In an expression that can be written as a sum, the parts added together are called **terms**.

A term that has a variable part is called a **variable term**. A term that has no variable part is called a **constant term**.

When a term is a product of a number and a power of a variable, the number is called the **coefficient** of the power.



SIMPLIFYING An expression is simplified if it contains no grouping symbols and all *like terms* are combined. **Like terms** are terms that have the same variable parts. (Constant terms are also considered like terms.) The distributive property allows you to *combine like terms* by adding coefficients.

EXAMPLE 4 Simplify by combining like terms

a. $8x + 3x = (8 + 3)x$ **Distributive property**
 $= 11x$ **Add coefficients.**

b. $5p^2 + p - 2p^2 = (5p^2 - 2p^2) + p$ **Group like terms.**
 $= 3p^2 + p$ **Combine like terms.**

c. $3(y + 2) - 4(y - 7) = 3y + 6 - 4y + 28$ **Distributive property**
 $= (3y - 4y) + (6 + 28)$ **Group like terms.**
 $= -y + 34$ **Combine like terms.**

d. $2x - 3y - 9x + y = (2x - 9x) + (-3y + y)$ **Group like terms.**
 $= -7x - 2y$ **Combine like terms.**

AVOID ERRORS

The terms $3p^2$ and p are not like terms. They use the same variable but different exponents, so the terms cannot be combined.

IDENTITIES Two algebraic expressions are **equivalent expressions** if they have the same value for all values of their variable(s). For instance, in part (a) of Example 4, the expressions $8x + 3x$ and $11x$ are equivalent. A statement such as $8x + 3x = 11x$ that equates two equivalent expressions is called an **identity**.

**GUIDED PRACTICE** for Example 4

8. Identify the terms, coefficients, like terms, and constant terms in the expression $2 + 5x - 6x^2 + 7x - 3$. Then simplify the expression.

Simplify the expression.

9. $15m - 9m$

10. $2n - 1 + 6n + 5$

11. $3p^3 + 5p^2 - p^3$

12. $2q^2 + q - 7q - 5q^2$

13. $8(x - 3) - 2(x + 6)$

14. $-4y - x + 10x + y$

EXAMPLE 5 Simplify a mathematical model

DIGITAL PHOTO PRINTING You send 15 digital images to a printing service that charges \$.80 per print in large format and \$.20 per print in small format. Write and simplify an expression that represents the total cost if n of the 15 prints are in large format. Then find the total cost if 5 of the 15 prints are in large format.



Solution

Write a verbal model. Then write an algebraic expression.

Price of large print (dollars/print)	•	Number of large prints (prints)	+	Price of small print (dollars/print)	•	Number of small prints (prints)
↓		↓		↓		↓
0.8	•	n	+	0.2	•	$(15 - n)$

An expression for the total cost is $0.8n + 0.2(15 - n)$.

$$\begin{aligned}
 0.8n + 0.2(15 - n) &= 0.8n + 3 - 0.2n && \text{Distributive property} \\
 &= (0.8n - 0.2n) + 3 && \text{Group like terms.} \\
 &= 0.6n + 3 && \text{Combine like terms.}
 \end{aligned}$$

► When $n = 5$, the total cost is $0.6(5) + 3 = 3 + 3 = \$6$.

INTERPRET EXPRESSIONS

The total number of prints is 15, so if n are in large format, then $15 - n$ are in small format.



GUIDED PRACTICE for Example 5

15. **WHAT IF?** In Example 5, write and simplify an expression for the total cost if the price of a large print is \$.75 and the price of a small print is \$.25.

1.2 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 21, 29, and 59
 ★ = STANDARDIZED TEST PRACTICE Exs. 2, 24, 33, 51, and 59
 ◆ = MULTIPLE REPRESENTATIONS Ex. 61

SKILL PRACTICE

- VOCABULARY** Copy 12^7 and label the base and the exponent.
- ★ WRITING** Explain what it means for terms to be like terms.
- ERROR ANALYSIS** Describe and correct the error in evaluating the power shown at the right.

$$-3^4 = 81 \quad \times$$

EXAMPLE 1
on p. 10
for Exs. 4–15

EVALUATING POWERS Evaluate the power.

- | | | | |
|--------------|--------------|--------------|--------------|
| 4. 2^3 | 5. 3^4 | 6. 4^3 | 7. 7^2 |
| 8. -5^2 | 9. -2^5 | 10. -8^3 | 11. -10^4 |
| 12. $(-3)^2$ | 13. $(-4)^3$ | 14. $(-2)^8$ | 15. $(-8)^2$ |

EXAMPLE 2

on p. 11
for Exs. 16–24

ORDER OF OPERATIONS Evaluate the expression for the given value of the variable.

16. $5d - 6$ when $d = 7$

17. $-10f + 15$ when $f = 2$

18. $6h \div 2 + h$ when $h = 4$

19. $5j - 3j \cdot 5$ when $j = 10$

20. $(k + 2)^2 - 6k$ when $k = 5$

21. $8m + (2m - 9)^3$ when $m = 6$

22. $n^3 - 4n + 10$ when $n = -3$

23. $2x^4 - 4x^3$ when $x = -1$

Animated Algebra at classzone.com

24. ★ **MULTIPLE CHOICE** What is the value of $2x^2 - 6x + 15$ when $x = -2$?

(A) 11

(B) 19

(C) 35

(D) 43

EXAMPLE 4

on p. 12
for Exs. 25–33

SIMPLIFYING EXPRESSIONS Simplify the expression.

25. $9x - 4x + 5$

26. $y^2 + 2y + 3y^2$

27. $5z^2 - 2z + 8z^2 + 10$

28. $10w^2 - 4w + 3w^2 + 18w$

29. $7(m - 3) + 4(m + 5)$

30. $10(n^2 + n) - 6(n^2 - 2)$

31. $4p^2 - 12p - 9p^2 + 3(4p + 7)$

32. $6(q - 2) - 2(q^2 + 6q)$

33. ★ **MULTIPLE CHOICE** Which terms are like terms?

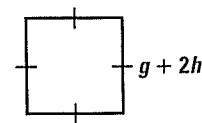
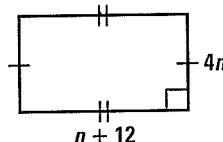
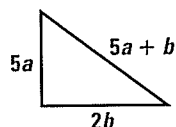
(A) $2x, 2y$ (B) $3x^2, 4x$ (C) x^2, y^2 (D) $10x^3, 2x^3$

GEOMETRY Write a simplified expression for the perimeter of the figure. Then evaluate the expression for the given value(s) of the variable(s).

34. $a = 3, b = 10$

35. $n = 2$

36. $g = 5, h = 4$



EVALUATING EXPRESSIONS Evaluate the expression for the given values of x and y .

37. $5x + 6y$ when $x = 16$ and $y = -9$

38. $16x + 11y$ when $x = -2$ and $y = -3$

39. $x^3 + 5y$ when $x = 4$ and $y = -3$

40. $(3x)^2 - y^3$ when $x = 4$ and $y = 5$

41. $\frac{x - y}{x + y}$ when $x = 10$ and $y = 8$

42. $\frac{x + 2y}{4x - y}$ when $x = -3$ and $y = 4$

HINT

Fraction bars
are grouping
symbols.

SIMPLIFYING EXPRESSIONS Simplify the expression.

43. $16c - 10d + 3d - 5c$

44. $9j + 4k - 2j - 7k$

45. $2m^2 - 5n^2 + 6n^2 - 8m$

46. $p^3 + 3q^2 - q + 3p^3$

47. $10m^2 + 3n - 8 + 3m^2 - 3n + 3$

48. $3y^2 + 5x - 12x + 9y^2 - 5$

49. $8(s - t) + 16(t - s)$

50. $3(x^2 - y) + 9(x^2 + 2y)$

51. ★ **OPEN-ENDED MATH** Write an algebraic expression that includes three coefficients, two like terms, and one constant term. Then simplify the expression.

GROUPING SYMBOLS Add parentheses to make a true statement.

52. $9 + 12 \div 3 - 1 = 15$

53. $4 + 3 \cdot 5 - 2 = 21$

54. $8 + 5^2 - 6 \div 3 = 9$

55. $3 \cdot 4^2 - 2^3 + 3^2 = 23$

56. **CHALLENGE** Under what conditions are the expressions $(x + y)^2$ and $x^2 + y^2$ equal? Are the expressions equivalent? *Explain.*

PROBLEM SOLVING

EXAMPLE 3

on p. 11
for Exs. 57–59

57. **MOVIE COSTS** In the United States, the average movie ticket price (in dollars) since 1974 can be modeled by $0.131x + 1.89$ where x is the number of years since 1974. What values of x should you use to find the ticket prices in 1974, 1984, 1994, and 2004? Find the ticket prices for those years.

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58. **MILEAGE** You start driving a used car when the odometer reads 96,882. After a typical month of driving, the reading is 97,057. Write an expression for the reading on the odometer after m months, assuming the amount you drive each month is the same. Predict the reading after 12 months.

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59. **★ SHORT RESPONSE** A student has a debit card with a prepaid amount of \$270 to use for school lunches. The cafeteria charges \$4.50 per lunch. Write an expression for the balance on the card after buying x lunches. Does your expression make sense for all positive integer values of x ? *Explain.*

EXAMPLE 5

on p. 13
for Exs. 60–62

60. **CROSS-TRAINING** You exercise for 60 minutes, spending w minutes walking and the rest of the time running. Use the information in the diagram below to write and simplify an expression for the number of calories burned. Find the calories burned if you spend 20 minutes walking.

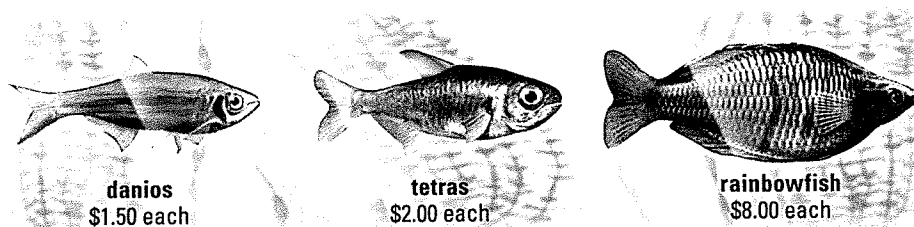


61. **◆ MULTIPLE REPRESENTATIONS** A theater has 30 rows of seats with 20 seats in each row. Tickets for the seats in the n rows closest to the stage cost \$45 and tickets for the other rows cost \$35.

- Visual Thinking** Make a sketch of the theater seating.
- Modeling** Write a verbal model for the income if all seats are sold.
- Simplifying** Write and simplify an expression for the income.
- Making a Table** Make a table for the income when $n = 5, 10$, and 15 .

62. **COMPUTERS** A company offers each of its 80 workers either a desktop computer that costs \$900 or a laptop that costs \$1550. Write and simplify an expression for the cost of all the computers when n workers choose desktop computers. Find the cost if 65 workers choose desktop computers.

63. **CHALLENGE** You want to buy 25 fish for an aquarium. You decide to buy danios, tetras, and rainbowfish.



Write and simplify an expression for the total cost of x danios, y tetras, and the rest rainbowfish. You buy 8 danios, 10 tetras, and the rest rainbowfish. What is the total cost?

PA

PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

64. A roadside fruit stand sells three apples for a total of \$0.79. The total cost, c , of purchasing n apples can be found by—
- (A) multiplying n by c
 - (B) multiplying n by the cost of 1 apple
 - (C) dividing n by c
 - (D) dividing c by the cost of 1 apple
65. A rectangle has a length of 6 feet and a perimeter of 22 feet. What is the perimeter of a similar rectangle with a width of 20 feet?
- (A) 52 ft
 - (B) 82 ft
 - (C) 88 ft
 - (D) 100 ft

QUIZ for Lessons 1.1–1.2

Graph the numbers on a number line. (p. 2)

1. $-5, \frac{7}{2}, 1, -\frac{4}{3}$

2. $-6.2, 5.4, \sqrt{5}, -2.5$

3. $0, -7.3, -\frac{2}{5}, 2\sqrt{3}$

Identify the property that the statement illustrates. (p. 2)

4. $6(4 + 9) = 6(4) + 6(9)$

5. $-5 \cdot 8 = 8 \cdot (-5)$

6. $17 + (-17) = 0$

Evaluate the expression for the given value of the variable. (p. 10)

7. $10m + 32$ when $m = -5$

8. $12 + (8 - n)^3$ when $n = 5$

9. $p^3 - 3p^2$ when $p = -2$

Simplify the expression. (p. 10)

10. $8x + 6x^2 - 9x^2 - 4x$

11. $5(x + 9) - 2(4 - x)$

12. $24x - 6y + 15y - 18x$

13. **CD COSTS** CDs are on sale for \$8 each and you have a gift card worth \$100. Write an expression for the amount of money left on the gift card after purchasing n CDs. Evaluate the expression to find the amount of money left after purchasing 6 CDs. (p. 10)



1.2 Evaluate Expressions

QUESTION How can you use a calculator to evaluate expressions?

You can use a scientific calculator or a graphing calculator to evaluate expressions. Keystrokes for evaluating several expressions are shown below.

Note that to enter a negative number, you use the $\pm$ key on a scientific calculator or the $(-)$ key (not the $-$ key) on a graphing calculator.

EXAMPLE Evaluate expressions

EXPRESSION	CALCULATOR	KEYSTROKES	RESULT
a. $-4^2 + 6$	Scientific	4 x^2 $\pm$ 6 $=$	-10
$-4^2 + 6$	Graphing	$(-)$ 4 x^2 + 6 ENTER	-10
b. $(-4)^2 + 6$	Scientific	4 $\pm$ x^2 + 6 $=$	22
$(-4)^2 + 6$	Graphing	($(-)$ 4) x^2 + 6 ENTER	22
c. $(39 \div 3)^3$	Scientific	(39 $\div$ 3) y^x 3 $=$	2197
$(39 \div 3)^3$	Graphing	(39 $\div$ 3) $\wedge$ 3 ENTER	2197
d. $\frac{64 - 5 \cdot 8}{4}$	Scientific	(64 - 5 $\times$ 8) $\div$ 4 $=$	6
$\frac{64 - 5 \cdot 8}{4}$	Graphing	(64 - 5 $\times$ 8) $\div$ 4 ENTER	6

PRACTICE

Use a calculator to evaluate the expression.

- $50.2 - 15 \div 3$
- $-11(20) - 66$
- $21(-8) + 51$
- $(-4)^4$
- $7(44.5 - 8^2)$
- $\frac{9.2 - 15.9}{-19 + 14}$

Use a calculator to evaluate the expression when $x = -3$, $y = 5$, and $z = -6$.

- $7z + y$
- x^6
- $6y - z^3$
- $\frac{10x}{2z - 3}$
- $(x + y)^2 + 3z$
- $(-4x + 9) \div (y + 2)$

13. **ERROR ANALYSIS** A student evaluated the expression $7 + (-4)^3$ on a graphing calculator by pressing 7 + ($-$ 4) $\wedge$ 3 ENTER. The calculator displayed an error message. Describe and correct the error.

1.3 Solve Linear Equations



PA M11.D.2.1.3 Write, solve and/or apply a linear equation (including problem situations).

Before You simplified algebraic expressions.

Now You will solve linear equations.

Why? So you can solve problems about earnings, as in Example 2.

Key Vocabulary

- equation
- linear equation
- solution
- equivalent equations

An **equation** is a statement that two expressions are equal. A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$ where a and b are constants and $a \neq 0$.

A number is a **solution** of an equation in one variable if substituting the number for the variable results in a true statement. Two equations are **equivalent equations** if they have the same solution(s).

KEY CONCEPT

For Your Notebook

Transformations That Produce Equivalent Equations

Addition Property of Equality	Add the same number to each side.	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	Subtract the same number from each side.	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	Multiply each side by the same nonzero number.	If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.
Division Property of Equality	Divide each side by the same nonzero number.	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

EXAMPLE 1 Solve an equation with a variable on one side

ANOTHER WAY

You can also solve the equation in Example 1 by multiplying each side by 5 first.

$$\begin{aligned} 5\left(\frac{4}{5}x + 8\right) &= 5(20) \\ 4x + 40 &= 100 \\ 4x &= 60 \\ x &= 15 \end{aligned}$$

Solve $\frac{4}{5}x + 8 = 20$.

$$\frac{4}{5}x + 8 = 20 \quad \text{Write original equation.}$$

$$\frac{4}{5}x = 12 \quad \text{Subtract 8 from each side.}$$

$$x = \frac{5}{4}(12) \quad \text{Multiply each side by } \frac{5}{4}, \text{ the reciprocal of } \frac{4}{5}.$$

$$x = 15 \quad \text{Simplify.}$$

► The solution is 15.

CHECK Check $x = 15$ in the original equation.

$$\frac{4}{5}x + 8 = \frac{4}{5}(15) + 8 = 12 + 8 = 20 \quad \checkmark$$

EXAMPLE 2 Write and use a linear equation

RESTAURANT During one shift, a waiter earns wages of \$30 and gets an additional 15% in tips on customers' food bills. The waiter earns \$105. What is the total of the customers' food bills?

Solution

Write a verbal model. Then write an equation. Write 15% as a decimal.

Income (dollars)	=	Wages (dollars)	+	Percent for tips	·	Food bills (dollars)
↓		↓		↓		↓
105	=	30	+	0.15	·	x

$$105 = 30 + 0.15x$$

Write equation.

$$75 = 0.15x$$

Subtract 30 from each side.

$$500 = x$$

Divide each side by 0.15.

► The total of the customers' food bills is \$500.

**GUIDED PRACTICE** for Examples 1 and 2

Solve the equation. Check your solution.

1. $4x + 9 = 21$

2. $7x - 41 = -13$

3. $-\frac{3}{5}x + 1 = 4$

4. **REAL ESTATE** A real estate agent's base salary is \$22,000 per year. The agent earns a 4% commission on total sales. How much must the agent sell to earn \$60,000 in one year?

**EXAMPLE 3** Standardized Test Practice

What is the solution of $7p + 13 = 9p - 5$?

(A) -9

(B) -4

(C) 4

(D) 9

Solution

$$7p + 13 = 9p - 5 \quad \text{Write original equation.}$$

$$13 = 2p - 5 \quad \text{Subtract } 7p \text{ from each side.}$$

$$18 = 2p \quad \text{Add 5 to each side.}$$

$$9 = p \quad \text{Divide each side by 2.}$$

► The correct answer is D. (A) (B) (C) (D)

CHECK $7p + 13 = 9p - 5$ Write original equation.

$$7(9) + 13 \stackrel{?}{=} 9(9) - 5 \quad \text{Substitute 9 for } p.$$

$$63 + 13 \stackrel{?}{=} 81 - 5 \quad \text{Multiply.}$$

$$76 = 76 \checkmark \quad \text{Solution checks.}$$

EXAMPLE 4 Solve an equation using the distributive propertySolve $3(5x - 8) = -2(-x + 7) - 12x$.

$$3(5x - 8) = -2(-x + 7) - 12x \quad \text{Write original equation.}$$

$$15x - 24 = 2x - 14 - 12x \quad \text{Distributive property}$$

$$15x - 24 = -10x - 14 \quad \text{Combine like terms.}$$

$$25x - 24 = -14 \quad \text{Add } 10x \text{ to each side.}$$

$$25x = 10 \quad \text{Add 24 to each side.}$$

$$x = \frac{2}{5} \quad \text{Divide each side by 25 and simplify.}$$

► The solution is $\frac{2}{5}$.

$$\text{CHECK} \quad 3\left(5 \cdot \frac{2}{5} - 8\right) \stackrel{?}{=} -2\left(-\frac{2}{5} + 7\right) - 12 \cdot \frac{2}{5} \quad \text{Substitute } \frac{2}{5} \text{ for } x.$$

$$3(-6) \stackrel{?}{=} \frac{4}{5} - 14 - \frac{24}{5} \quad \text{Simplify.}$$

$$-18 = -18 \checkmark \quad \text{Solution checks.}$$

EXAMPLE 5 Solve a work problem

CAR WASH It takes you 8 minutes to wash a car and it takes a friend 6 minutes to wash a car. How long does it take the two of you to wash 7 cars if you work together?

Solution**STEP 1** Write a verbal model. Then write an equation.

Your rate (cars/minute)	•	Time (minutes)	+	Friend's rate (cars/minute)	•	Time (minutes)	=	Cars washed (cars)
↓		↓		↓		↓		↓
$\frac{1 \text{ car}}{8 \text{ min}}$	•	$t \text{ min}$	+	$\frac{1 \text{ car}}{6 \text{ min}}$	•	$t \text{ min}$	=	7 cars

STEP 2 Solve the equation for t .

$$\frac{1}{8}t + \frac{1}{6}t = 7 \quad \text{Write equation.}$$

$$24\left(\frac{1}{8}t + \frac{1}{6}t\right) = 24(7) \quad \text{Multiply each side by the LCD, 24.}$$

$$3t + 4t = 168 \quad \text{Distributive property}$$

$$7t = 168 \quad \text{Combine like terms.}$$

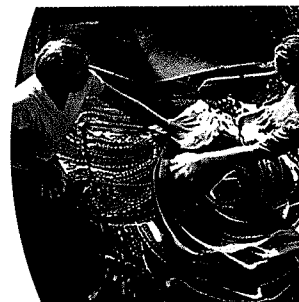
$$t = 24 \quad \text{Divide each side by 7.}$$

► It will take 24 minutes to wash 7 cars if you work together.

CHECK You wash $\frac{1}{8} \cdot 24 = 3$ cars and your friend washes $\frac{1}{6} \cdot 24 = 4$ cars in 24 minutes. Together, you wash 7 cars. ✓

AVOID ERRORS

Be sure to multiply *both* sides of the equation by the LCD, not just one side.



**GUIDED PRACTICE** for Examples 3, 4, and 5

Solve the equation. Check your solution.

5. $-2x + 9 = 2x - 7$

6. $10 - x = -6x + 15$

7. $3(x + 2) = 5(x + 4)$

8. $-4(2x + 5) = 2(-x - 9) - 4x$

9. $\frac{1}{4}x + \frac{2}{5}x = 39$

10. $\frac{2}{3}x + \frac{5}{6} = x - \frac{1}{2}$

11. **WHAT IF?** In Example 5, suppose it takes you 9 minutes to wash a car and it takes your friend 12 minutes to wash a car. How long does it take the two of you to wash 7 cars if you work together?

1.3 EXERCISES**HOMEWORK KEY**

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 23, 43, and 71
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 19, 32, 72, and 77
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 74

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If a number is substituted for a variable in an equation and the resulting statement is true, the number is called a(n) ? of the equation.

2. ★ **WRITING** Give an example of two equivalent equations. How do you know they are equivalent?

EXAMPLE 1
on p. 18
for Exs. 3–19

VARIABLE ON ONE SIDE Solve the equation. Check your solution.

3. $x + 8 = 11$

4. $y - 4 = 7$

5. $z - 13 = -1$

6. $-3 = w + 5$

7. $5d = 30$

8. $4 = \frac{2}{5}g$

9. $\frac{9}{2}h = -1$

10. $-16k = -8$

11. $6m - 3 = 21$

12. $4n - 10 = 12$

13. $3 = 2p + 5$

14. $-3q + 4 = 13$

15. $1 = \frac{1}{3}a - 5$

16. $\frac{3}{11}b + 5 = 5$

17. $7 - \frac{5}{3}c = 22$

18. $3 + \frac{8}{7}d = -1$

19. ★ **MULTIPLE CHOICE** What is the solution of $4x - 7 = -15$?

(A) -12

(B) -2

(C) 2

(D) $\frac{11}{2}$

EXAMPLE 3
on p. 19
for Exs. 20–32

VARIABLE ON BOTH SIDES Solve the equation. Check your solution.

20. $3a + 4 = 2a + 15$

21. $5w + 2 = 2w + 5$

22. $6x + 7 = 2x + 59$

23. $5b - 4 = 2b + 8$

24. $3y + 7 = y - 3$

25. $2z - 3 = 6z + 25$

26. $4n - 7 = 5 - 2n$

27. $2c + 14 = 6 - 4c$

28. $5m - 2 = -m - 2$

29. $p + 5 = 25 - 4p$

30. $6 - 5q = q + 9$

31. $17 - 6r = 25 - 3r$

32. ★ **MULTIPLE CHOICE** What is the solution of $7t - 5 = 3t + 11$?

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) $\frac{8}{5}$

(D) 4

EXAMPLE 4

on p. 20
for Exs. 33–40

THE DISTRIBUTIVE PROPERTY Solve the equation. Check your solution.

33. $2(b + 3) = 4b - 2$

34. $5d + 17 = 4(d + 3)$

35. $3(m - 5) = 6(m + 1)$

36. $-4(n + 2) = 3(n - 4)$

37. $12(r + 3) = 2(r + 5) - 3r$

38. $7(t - 3) = 2(t - 9) + 2t$

39. $10(w - 4) = 4(w + 4) + 4w$

40. $3(2x - 5) - x = -7(x + 3)$

ERROR ANALYSIS Describe and correct the error in solving the equation.

41.

$$\frac{3}{7}x + 2 = 17$$

$$\frac{3}{7}x = 15$$

$$x = 15 - \frac{3}{7}$$

$$x = 14\frac{4}{7}$$



42.

$$\frac{1}{5}x + \frac{1}{2} = 1$$

$$10\left(\frac{1}{5}x + \frac{1}{2}\right) = 1$$

$$2x + 5 = 1$$

$$x = -2$$

**EXAMPLE 5**

on p. 20
for Exs. 43–50

EQUATIONS WITH FRACTIONS Solve the equation. Check your solution.

43. $\frac{1}{2}t + \frac{1}{3}t = 10$

44. $\frac{1}{5}d + \frac{1}{8}d = 2$

45. $\frac{2}{3}m - \frac{3}{5}m = 4$

46. $\frac{4}{7}z + \frac{2}{3}z = 13$

47. $\frac{3}{7}w - \frac{2}{9} = \frac{4}{9}w + \frac{1}{7}$

48. $\frac{1}{2}x + 4 = -\frac{2}{3}x + \frac{1}{2}$

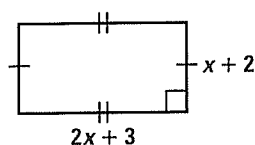
49. $\frac{2}{5}k + \frac{1}{6} = \frac{3}{10}k + \frac{1}{3}$

50. $\frac{2}{3}q - \frac{1}{12} = q + \frac{1}{8}$

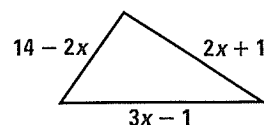
 **Animated Algebra** at classzone.com

 **GEOMETRY** Solve for x . Then find the length of each side of the figure.

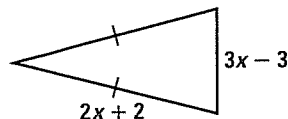
51. Perimeter = 46



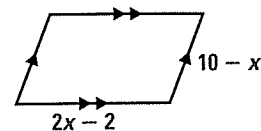
52. Perimeter = 26



53. Perimeter = 15



54. Perimeter = 26



EQUATIONS WITH DECIMALS Solve the equation. Check your solution.

55. $0.6g + 0.5 = 2.9$

56. $1.1h + 1.3 = 6.8$

57. $0.4k - 0.6 = 1.3k + 1.2$

58. $6.5m + 1.5 = 4.3m - 0.7$

59. $3.8w + 3.2 = 2.3(w + 4)$

60. $1.7(x + 5) = 2.1x + 9.7$

61. $2.25b + 3.81 = 1.75b + 5.26$

62. $18.13 - 5.18c = 6.32c - 8.32$

SPECIAL EQUATIONS Solve the equation. If there is no solution, write *no solution*. If the equation is always true, write *all real numbers*.

63. $5(x - 4) = 5x + 12$

64. $3(x + 5) = 3x + 15$

65. $5(2 - x) = 3 - 2x + 7 - 3x$

66. $-2(4 - 3x) + 7 = 6(x + 1)$

67. **CHALLENGE** Solve the equation $ax + b = cx + d$ for x in terms of a , b , c , and d . Under what conditions is there no solution? Under what conditions are all real numbers solutions?

PROBLEM SOLVING

EXAMPLE 2
on p. 19
for Exs. 68–71

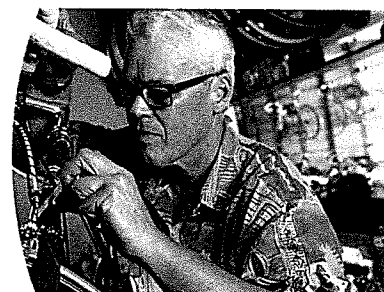
68. **CATALOG PURCHASE** You are ordering T-shirts from a catalog. Each T-shirt costs \$15. The cost of shipping is \$6 no matter how many you order. The total cost is \$111. How many T-shirts did you order?

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69. **BICYCLE REPAIR** The bill for the repair of your bicycle was \$180. The cost of parts was \$105. The cost of labor was \$25 per hour. How many hours did the repair work take?

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70. **CAR SALES** A salesperson at a car dealership has a base salary of \$25,000 per year and earns a 5% commission on total sales. How much must the salesperson sell to earn \$50,000 in one year?



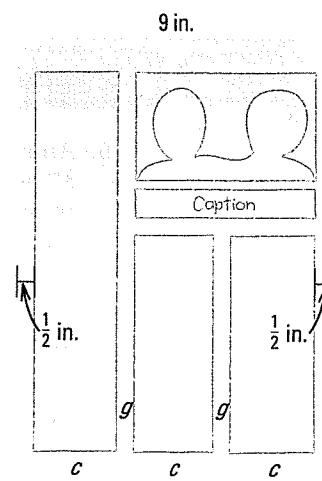
71. **SUMMER JOBS** You have two summer jobs. In the first job, you work 25 hours per week and earn \$7.75 per hour. In the second job, you earn \$6.25 per hour and can work as many hours as you want. You want to earn \$250 per week. How many hours must you work at the second job?

72. **★ SHORT RESPONSE** Your friend bought a total of 10 CDs and DVDs as gifts for \$199. The price per CD was \$15 and the price per DVD was \$22. Write and solve an equation to find how many CDs and how many DVDs your friend bought. How would your answer change if the total cost of the CDs and DVDs was \$185? *Explain.*

73. **MULTI-STEP PROBLEM** You are working on the layout of a yearbook. The page is 9 inches wide, has $\frac{1}{2}$ inch margins, and has three columns of equal width.

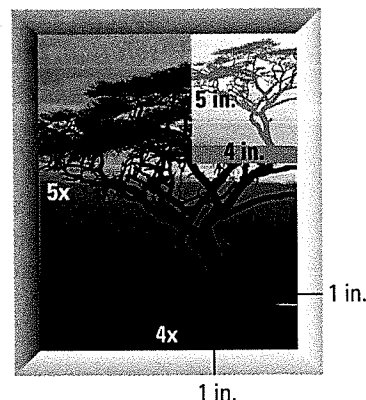
- Write and simplify an equation that relates the column width c and the gap g between columns to the total width of the page.
- Copy and complete the table by substituting the given value into your equation from part (a) and solving to find the unknown value.

Gap, g (in.)	$\frac{5}{8}$	?	$\frac{3}{8}$	?
Column width, c (in.)	?	$2\frac{1}{3}$	?	$2\frac{1}{2}$



74. **◆ MULTIPLE REPRESENTATIONS** You want to enlarge a 4 inch by 5 inch photo to fit into a 1 inch wide frame that has an outer perimeter of 53 inches.

- Using a Diagram** Write an expression for the outer perimeter of the picture frame.
- Making a Table** Evaluate the perimeter expression when $x = 1.5, 2, 2.5, 3$, and 3.5 . Make a table of your results. For what value of x is the perimeter 53 inches?
- Using an Equation** Write and solve an equation to find x . *Explain* what the value of x tells you about how much you should enlarge the original photo.



EXAMPLE 5
on p. 20
for Exs. 75–77

75. **RAKING LEAVES** It takes you 30 minutes to rake the leaves in your yard and it takes your brother 45 minutes. How long does it take the two of you to rake the leaves when working together?

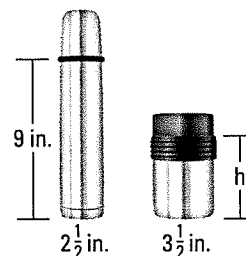
76. **MURAL PAINTING** You paint 2 square yards of a community mural in 3 hours and a friend paints 4 square yards in 5 hours. How long does it take the two of you to paint 11 square yards when working together?

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77. **★ MULTIPLE CHOICE** Three students use calligraphy pens to write the names of graduating seniors on their diplomas. One writes 7 names in 6 minutes, another writes 17 names in 10 minutes, and the third writes 23 names in 15 minutes. How long, to the nearest minute, will the students take to write names on 440 diplomas if they work together?

(A) 97 minutes (B) 100 minutes (C) 103 minutes (D) 290 minutes

78. **CHALLENGE** A cylindrical thermos with an inside diameter of $2\frac{1}{2}$ inches is filled with liquid to a height of 9 inches. If the liquid is poured into a cylindrical travel mug with an inside diameter of $3\frac{1}{2}$ inches, what will be the height h of the liquid?



PA

PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

79. Andy is saving money for a digital music player that costs \$350. He makes \$7 per hour as a lifeguard. How many hours must he work to earn enough money to buy the digital music player if he uses a coupon for 20% off?
- (A) 10 h (B) 25 h (C) 40 h (D) 43 h
80. Two runners are running at constant speeds in the same direction around a track. The faster runner travels 8 miles per hour and completes 4 laps each time the slower runner completes 3 laps. What is the slower runner's speed?
- (A) 2 mph (B) 4 mph (C) 6 mph (D) 7 mph

1.3 Use Tables to Solve Equations

QUESTION How can you use tables to solve linear equations?

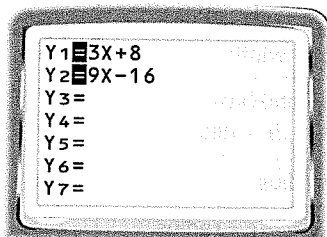
You can use the *table* feature of a graphing calculator to solve linear equations.

EXAMPLE Solve a linear equation

Use the *table* feature of a graphing calculator to solve the equation $3x + 8 = 9x - 16$.

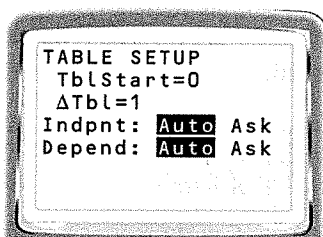
STEP 1 Enter expressions

Press **Y=**. Enter the left side of the equation as $y_1 = 3x + 8$. Enter the right side of the equation as $y_2 = 9x - 16$.



STEP 2 Make a table

Press **2nd** [TBLSET]. Set the starting x -value TblStart to 0 and the step value ΔTbl (the value by which the x -values increase) to 1.



STEP 3 Identify solution

Press **2nd** [TABLE] to display the table. Scroll through the table until you find an x -value for which both sides of the equation have the same value.

X	Y1	Y2
0	8	-16
1	11	-7
2	14	2
3	17	11
4	20	20
5	23	29

X=4

Both sides of the equation have a value of 20 when $x = 4$. So, the solution of $3x + 8 = 9x - 16$ is 4.

PRACTICE

Use the *table* feature of a graphing calculator to solve the equation.

- $7x - 3 = -x + 13$
- $-6x + 8 = 12 - 5x$
- $-2x - 13 = -3x - 5$
- $22 + 15x = -9x - 2$
- $4x + 27 = -8 + 11x$
- $7 - 8x = -9 - 10x$

7. **REASONING** Consider the equation $4x + 18 = 9x - 9$.

- Attempt to solve the equation using the *table* feature of a graphing calculator with step value $\Delta Tbl = 1$. Between what two integers does the solution lie? How do you know?
- Use a smaller value of ΔTbl to find the exact solution.

8. **WRITING** Solve the equation $3x + 8 = 9x - 16$ by writing it in the form $ax + b = 0$, entering $y_1 = ax + b$ on a graphing calculator, and using a table to find the x -value for which $y_1 = 0$. What are the advantages and disadvantages of this method compared to the method shown above?

1.4 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS1 for Exs. 3, 9, and 35

★ = STANDARDIZED TEST PRACTICE
Exs. 2, 6, 15, 27, 36, and 38

SKILL PRACTICE

EXAMPLES 1 and 2

on pp. 26–27
for Exs. 3–6

1. **VOCABULARY** Copy and complete: A(n) ? is an equation that relates two or more quantities.

2. ★ **WRITING** What does it mean to solve for a variable in an equation?

REWRITING FORMULAS Solve the formula for the indicated variable. Then use the given information to find the value of the variable.

3. ○ Solve $A = lw$ for l . Then find the length of a rectangle with a width of 50 millimeters and an area of 250 square millimeters.

4. Solve $A = \frac{1}{2}bh$ for b . Then find the base of a triangle with a height of 6 inches and an area of 24 square inches.

5. Solve $A = \frac{1}{2}(b_1 + b_2)h$ for h . Then find the height of a trapezoid with bases of lengths 10 centimeters and 15 centimeters and an area of 75 square centimeters.

6. ★ **MULTIPLE CHOICE** What equation do you obtain when you solve the formula $A = \frac{1}{2}(b_1 + b_2)h$ for b_1 ?

(A) $b_1 = \frac{2A}{h} - b_2$

(B) $b_1 = \frac{A}{2h} - b_2$

(C) $b_1 = 2A - b_2h$

(D) $b_1 = \frac{2A}{h - b_2}$

EXAMPLE 3

on p. 28
for Exs. 7–17

REWRITING EQUATIONS Solve the equation for y . Then find the value of y for the given value of x .

7. $3x + y = 26$; $x = 7$

8. $4y + x = 24$; $x = 8$

9. ○ $6x + 5y = 31$; $x = -4$

10. $15x + 4y = 9$; $x = -3$

11. $9x - 6y = 63$; $x = 5$

12. $10x - 18y = 84$; $x = 6$

13. $8y - 14x = -22$; $x = 5$

14. $9y - 4x = -30$; $x = 8$

15. ★ **MULTIPLE CHOICE** What equation do you obtain when you solve the equation $4x - 5y = 20$ for y ?

(A) $x = \frac{5}{4}y + 5$

(B) $y = -\frac{4}{5}x + 4$

(C) $y = \frac{4}{5}x - 4$

(D) $y = \frac{4}{5}x - 20$

ERROR ANALYSIS Describe and correct the error in solving the equation for y .

16.

$$-7x + 5y = 2$$

$$5y = 7x + 2$$

$$y = \frac{7}{5}x + 2$$



17.

$$4y - xy = 9$$

$$4y = 9 + xy$$

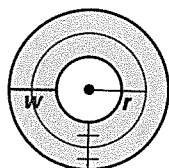
$$y = \frac{9 + xy}{4}$$



GEOMETRY Solve the formula for the variable in red. Then use the given information to find the value of the variable. Round to the nearest tenth.

18. Area of a circular ring

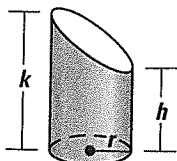
$$A = 2\pi r w$$



Find r if $w = 4$ ft and $A = 120$ ft².

19. Lateral surface area of a truncated cylinder

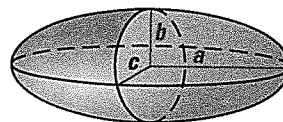
$$S = \pi r(h + k)$$



Find h if $r = 2$ cm, $k = 3$ cm, and $S = 50$ cm².

20. Volume of an ellipsoid

$$V = \frac{4}{3}\pi abc$$



Find c if $a = 4$ in., $b = 3$ in., and $V = 60$ in.³

EXAMPLE 4

on p. 28
for Exs. 21–26

REWRITING EQUATIONS Solve the equation for y . Then find the value of y for the given value of x .

21. $xy - 3x = 40$; $x = 5$

22. $7x - xy = -18$; $x = -4$

23. $3xy - 28 = 16x$; $x = 4$

24. $9y + 6xy = 30$; $x = -6$

25. $y - 2xy = 15$; $x = -1$

26. $4x + 7y + 5xy = 0$; $x = 1$

27. ★ **SHORT RESPONSE** Consider the equation $15x - 9y = 27$. To find the value of y when $x = 2$, you can use two methods.

Method 1 Solve the original equation for y and then substitute 2 for x .

Method 2 Substitute 2 for x and then solve the resulting equation for y .

Show the steps of the two methods. Which method is more efficient if you need to find the value of y for several values of x ? *Explain.*

REASONING Solve for the indicated variable.

28. Solve $xy = x + y$ for y .

29. Solve $xyz = x + y + z$ for z .

30. Solve $\frac{1}{x} + \frac{1}{y} = 1$ for y .

31. Solve $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ for z .

32. **CHALLENGE** Write a formula giving the area of a circle in terms of its circumference.

PROBLEM SOLVING

EXAMPLE 5

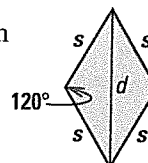
on p. 29
for Exs. 33–38

33. **TREE DIAMETER** You can estimate the diameter of a tree without boring through it by measuring its circumference. Solve the formula $C = \pi d$ for d . Then find the diameter of an oak that has a circumference of 113 inches.

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34. **DESIGN** The fabric panels on a kite are rhombuses. For the panel shown, a formula for the length of the long diagonal d is $d = s\sqrt{3}$ where s is the length of a side. Solve the formula for s . Then find the value of s when $d = 15$ inches.

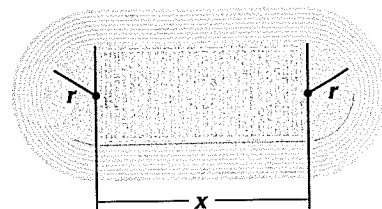
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35. **TEMPERATURE** The formula for converting temperatures from degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Solve the formula for C . Then find the temperature in degrees Celsius that corresponds to 50°F .

36. **★ EXTENDED RESPONSE** A quarter mile running track is shaped as shown. The formula for the inside perimeter is $P = 2\pi r + 2x$.

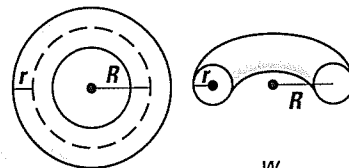
- Solve the perimeter formula for r .
- For a quarter mile track, $P = 440$ yards. Find r when $x = 75$ yards, 100 yards, 120 yards, and 150 yards.
- What are the greatest and least possible values of r if $P = 440$ yards? *Explain* how you found the values, and sketch the track corresponding to each extreme value.



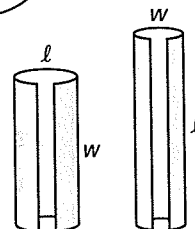
37. **MULTI-STEP PROBLEM** A tuxedo shop rents classic tuxedos for \$80 and designer tuxedos for \$150. Write an equation that represents the shop's revenue. Solve the equation for the variable representing the number of designer tuxedos rented. The shop owner wants \$60,000 in revenue during prom season. How many designer tuxedos must be rented if the number of classic tuxedos rented is 600? 450? 300?

38. **★ OPEN-ENDED MATH** The volume of a donut-like shape called a *torus* is given by the formula $V = 2\pi^2 r^2 R$ where r and R are the radii shown and $r \leq R$.

- Solve the formula for R .
- A metal ring in the shape of a torus has a volume of 100 cubic centimeters. Choose three possible values of r , and find the corresponding values of R .



39. **CHALLENGE** A rectangular piece of paper with length ℓ and width w can be rolled to form the lateral surface of a cylinder in two ways, assuming no overlapping. Write a formula for the volume of each cylinder in terms of ℓ and w .



PA

PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

- Jill is mailing a gift in a rectangular box that is 14 inches by 10 inches by 8 inches. She wants to mail this box in a larger box that is 18 inches by 15 inches by 10 inches. How many cubic inches of packing material does she need to surround the gift?
 (A) 1120 in.³ (B) 1580 in.³ (C) 2700 in.³ (D) 3820 in.³
- If $\angle A$ and $\angle B$ are supplementary angles and $m\angle A$ is 56° , what is $m\angle B$?
 (A) 34° (B) 112° (C) 124° (D) 306°
- What is the solution of the equation $3(r - 1) = -2(r + 7) + 1$?
 (A) -3 (B) -2 (C) 2 (D) 3



Lessons 1.1–1.4

1. **CAR RENTALS** There is a \$50 fee to join an urban car rental service. Using the car costs \$8.50 per hour. What is the cost to join and drive for 20 hours?

A \$119
B \$135
C \$170
D \$220

2. **MUSEUM COSTS** You visit a museum. You have \$50 to spend. Admission to the museum is \$15. Admission to each special exhibit inside the museum is \$10. What is the maximum number of special exhibits you can include in your visit?

A 2
B 3
C 6
D 7

3. **HOCKEY STATISTICS** In hockey, each player has a statistic called plus/minus, which is the difference between the number of goals scored by the player's team and the number of goals scored by the other team when the player is on the ice. Which list shows the players in order from least to greatest plus/minus?

Player	Plus/Minus
Vincent Lecavalier	23
Dave Andreychuk	-9
Ruslan Fedotenko	14
Martin St. Louis	35
Cory Sarich	5
Tim Taylor	-5

- A Andreychuk, Taylor, Sarich, Fedotenko, Lecavalier, St. Louis
B St. Louis, Lecavalier, Fedotenko, Andreychuk, Sarich, Taylor
C Taylor, Andreychuk, Sarich, Fedotenko, Lecavalier, St. Louis
D St. Louis, Lecavalier, Fedotenko, Sarich, Taylor, Andreychuk

4. **PRINTING MONEY** In one year, the Bureau of Engraving and Printing printed \$10 and \$20 bills with a total value of \$66,368,000. The total number of \$10 and \$20 bills printed was 3,577,600. What was the number of \$20 bills printed?

A 172,800
B 518,400
C 3,059,200
D 3,404,800

5. **SCHOOL PICNIC SUPPLIES** You are in charge of buying food for a school picnic. You have \$45 to spend on ground beef and chicken. Ground beef costs \$1.80 per pound and chicken costs \$1.00 per pound. You want to buy equal amounts of ground beef and chicken. About how many total pounds of meat can you buy?

A 16.07 pounds
B 25 pounds
C 32.14 pounds
D 112.5 pounds

6. **SUMMER JOBS** You have two summer jobs. You mow lawns for \$20 per hour. You also work at a restaurant for \$7.50 per hour. In one week, you earn \$825 working a total of 50 hours. Which equation can be used to find the number of hours you mowed lawns?

A $825 = 20x + 7.5x$
B $825 = 20x + 7.5(50 - x)$
C $825 = 20x + 7.5(x - 50)$
D $825 = 20(x + 50) + 7.5x$

7. **OPEN-ENDED** You work 45 hours per week for a construction company during the summer. You earn \$8 per hour for office work and \$9 per hour for outside work. You earn \$399 one week.

	Hours	Earnings
office work	x	$8x$
outside work	?	?
Total	45	\$399

- A. Find the missing values in the table in terms of x .
B. Write and solve an equation to find the number of hours you worked in the office and the number of hours you worked outside.
C. Compare the number of hours you worked in the office to the number hours you worked outside.

1.5 Use Problem Solving Strategies and Models



PA

M11.D.2.1.3

Write, solve and/or apply a linear equation (including problem situations).

Before

You wrote and solved equations.

Now

You will solve problems using verbal models.

Why?

So you can solve constant rate problems, as in Ex. 26.

Key Vocabulary

- verbal model

As you have seen in this chapter, it is helpful when solving real-life problems to write an equation in words *before* you write it in mathematical symbols. This word equation is called a **verbal model**.

Sometimes problem solving strategies can be used to write a verbal or algebraic model. Examples of such strategies are *use a formula*, *look for a pattern*, and *draw a diagram*.

EXAMPLE 1 Use a formula

HIGH-SPEED TRAIN The Acela train travels between Boston and Washington, a distance of 457 miles. The trip takes 6.5 hours. What is the average speed?

Solution

You can use the formula for distance traveled as a verbal model.

$$\begin{array}{ccccc} \text{Distance} & = & \text{Rate} & \cdot & \text{Time} \\ \text{(miles)} & & \text{(miles/hour)} & & \text{(hours)} \\ \downarrow & & \downarrow & & \downarrow \\ 457 & = & r & \cdot & 6.5 \end{array}$$

An equation for this situation is $457 = 6.5r$. Solve for r .

$$457 = 6.5r \quad \text{Write equation.}$$

$$70.3 \approx r \quad \text{Divide each side by 6.5.}$$

► The average speed of the train is about 70.3 miles per hour.

CHECK You can use unit analysis to check your answer.

$$457 \text{ miles} \approx \frac{70.3 \text{ miles}}{1 \text{ hour}} \cdot 6.5 \text{ hours}$$

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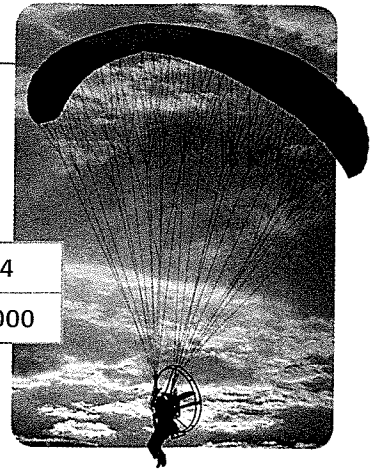
GUIDED PRACTICE for Example 1

1. **AVIATION** A jet flies at an average speed of 540 miles per hour. How long will it take to fly from New York to Tokyo, a distance of 6760 miles?

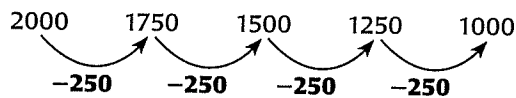
EXAMPLE 2 Look for a pattern

PARAMOTORING A paramotor is a parachute propelled by a fan-like motor. The table shows the height h of a paramotorist t minutes after beginning a descent. Find the height of the paramotorist after 7 minutes.

Time (min), t	0	1	2	3	4
Height (ft), h	2000	1750	1500	1250	1000

**Solution**

The height decreases by 250 feet per minute.



You can use this pattern to write a verbal model for the height.

$$\begin{array}{ccccccc}
 \text{Height} & = & \text{Initial height} & - & \text{Rate of descent} & \cdot & \text{Time} \\
 \text{(feet)} & & \text{(feet)} & & \text{(feet/minute)} & & \text{(minutes)} \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 h & = & 2000 & - & 250 & \cdot & t
 \end{array}$$

An equation for the height is $h = 2000 - 250t$.

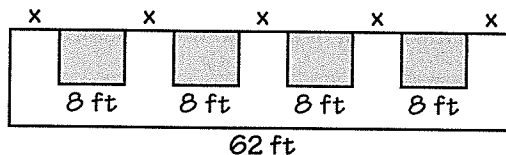
► So, the height after 7 minutes is $h = 2000 - 250(7) = 250$ feet.

EXAMPLE 3 Draw a diagram

BANNERS You are hanging four championship banners on a wall in your school's gym. The banners are 8 feet wide. The wall is 62 feet long. There should be an equal amount of space between the ends of the wall and the banners, and between each pair of banners. How far apart should the banners be placed?

Solution

Begin by drawing and labeling a diagram, as shown below.



From the diagram, you can write and solve an equation to find x .

$$x + 8 + x + 8 + x + 8 + x + 8 + x = 62 \quad \text{Write equation.}$$

$$5x + 32 = 62 \quad \text{Combine like terms.}$$

$$5x = 30 \quad \text{Subtract 32 from each side.}$$

$$x = 6 \quad \text{Divide each side by 5.}$$

► The banners should be placed 6 feet apart.

REVIEW STRATEGIES

For help with other problem solving strategies, see p. 998.

**EXAMPLE 4****Standardized Test Practice**

A car used 16 gallons of gasoline and traveled a total distance of 460 miles. The car's fuel efficiency is 30 miles per gallon on the highway and 25 miles per gallon in the city. How many gallons of gasoline were used on the highway?

- (A) 8 gallons (B) 12 gallons (C) $15\frac{1}{3}$ gallons (D) 16 gallons

Solution

STEP 1 Write a verbal model. Then write an equation.

		highway miles				city miles		
Total distance (miles)	=	Fuel efficiency (miles/gallon)	•	Gas used (gallons)	+	Fuel efficiency (miles/gallon)	•	Gas used (gallons)
↓		↓		↓		↓		↓
460	=	30	•	g	+	25	•	$(16 - g)$

An equation for the situation is $460 = 30g + 25(16 - g)$.

STEP 2 Solve for g to find the number of gallons used on the highway.

$$460 = 30g + 25(16 - g) \quad \text{Write equation.}$$

$$460 = 30g + 400 - 25g \quad \text{Distributive property}$$

$$460 = 5g + 400 \quad \text{Combine like terms.}$$

$$60 = 5g \quad \text{Subtract 400 from each side.}$$

$$12 = g \quad \text{Divide each side by 5.}$$

The car used 12 gallons on the highway.

► The correct answer is B. (A) (B) (C) (D)

CHECK $30 \cdot 12 + 25(16 - 12) = 360 + 100 = 460 \checkmark$

**GUIDED PRACTICE** for Examples 2, 3, and 4

- 2. PARAMOTORING** The table shows the height h of a paramotorist after t minutes. Find the height of the paramotorist after 8 minutes.

Time (min), t	0	1	2	3	4
Height (ft), h	2400	2190	1980	1770	1560

- 3. WHAT IF?** In Example 3, how would your answer change if there were only three championship banners?
- 4. FUEL EFFICIENCY** A truck used 28 gallons of gasoline and traveled a total distance of 428 miles. The truck's fuel efficiency is 16 miles per gallon on the highway and 12 miles per gallon in the city. How many gallons of gasoline were used in the city?

1.5 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS
on p. WS2 for Exs. 3, 11, and 27
- ★ = STANDARDIZED TEST PRACTICE
Exs. 2, 15, 16, 21, and 27
- ◆ = MULTIPLE REPRESENTATIONS
Ex. 28

SKILL PRACTICE

- VOCABULARY** Copy and complete: A word equation that represents a real-life problem is called a(n) ?.
- ★ WRITING** Give an example of how a problem solving strategy can help you write an equation that models a real-life problem.

EXAMPLE 1

on p. 34
for Exs. 3–10

USING A FORMULA Use the formula $d = rt$ for distance traveled to solve for the missing variable.

3. $d = 20$ mi, $r = 40$ mi/h, $t = ?$
4. $d = 300$ mi, $r = ?$, $t = 4$ h
5. $d = ?$, $r = 30$ mi/h, $t = 3$ h
6. $d = 250$ mi, $r = 50$ mi/h, $t = ?$

GEOMETRY Use the formula $P = 2\ell + 2w$ for the perimeter of a rectangle to solve for the missing variable.

7. $P = ?$, $\ell = 15$ ft, $w = 12$ ft
8. $P = 46$ in., $\ell = ?$, $w = 4$ in.
9. $P = 100$ m, $\ell = 30$ m, $w = ?$
10. $P = 25$ cm, $w = 5$ cm, $\ell = ?$

EXAMPLE 2

on p. 35
for Exs. 11–15

USING PATTERNS Look for a pattern in the table. Then write an equation that represents the table.

11.

x	0	1	2	3
y	11	15	19	23

12.

x	0	1	2	3
y	60	45	30	15

13.

x	0	1	2	3
y	46	36	26	16

14.

x	0	1	2	3
y	57	107	157	207

15. **★ MULTIPLE CHOICE** Which equation represents the table at the right?

x	0	1	2	3
y	12	19	26	33

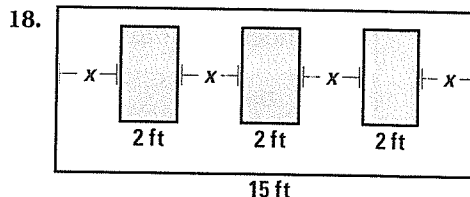
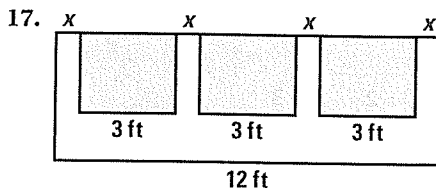
- (A) $y = 5x + 7$ (B) $y = 7x + 5$
(C) $y = 12x - 5$ (D) $y = 7x + 12$

16. **★ SHORT RESPONSE** The first story of a building is 24 feet high, and each additional story is 18 feet high. Write an expression for the height to the top of the n th story. *Explain* the meaning of each term in the expression.

EXAMPLE 3

on p. 35
for Exs. 17–18

USING DIAGRAMS Write and solve an equation to find x .



ERROR ANALYSIS Describe and correct the error in writing the equation.

19.

x	0	1	2	3
y	75	65	55	45

An equation that represents the table is $y = 75x - 10$.



20.

x	0	5	10	15
y	7	17	27	37

An equation that represents the table is $y = 7 + 10x$.



21. **★ MULTIPLE CHOICE** A car used 15 gallons of gasoline and traveled a total distance of 350 miles. The car's fuel efficiency is 25 miles per gallon on the highway and 20 miles per gallon in the city. Which equation can you solve to find h , the number of gallons that were used on the highway?

(A) $350 = 25(15 - h) + 20h$

(B) $25h + 20(15 - h) = 350$

(C) $350 = \left(\frac{25 + 20}{2}\right)h$

(D) $15 = \frac{350}{25h} + \frac{350}{20h}$

CHALLENGE Write an equation that represents the table.

22.

x	0	3	6	9
y	12	30	48	66

23.

x	4	5	6	7
y	12	19	26	33

PROBLEM SOLVING

EXAMPLE 1

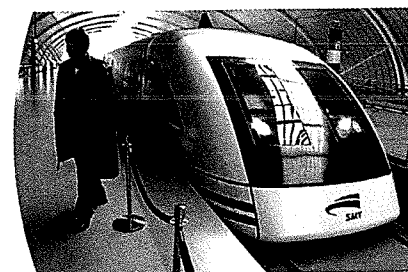
on p. 34
for Exs. 24–26

24. **DAYTONA 500** A recent Daytona 500 race was won by Dale Earnhardt, Jr. He completed the 500 mile race in 3.2 hours. What was his average racing speed?

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25. **MAGLEV TRAIN** A magnetic levitation (maglev) train travels between the city center of Shanghai, China, and Pudong International Airport. The trip covers 30 kilometers in just 8 minutes. What is the average speed of the train?

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26. **SCUBA DIVING** A scuba diver is returning to the surface from a depth of 165 feet. The safe ascent rate for a diver is 30 feet per minute. How many minutes will it take for the diver to return to the surface?

EXAMPLE 2

on p. 35
for Exs. 27–28

27. **★ SHORT RESPONSE** The table shows the height of a bamboo shoot during a period of fast growth. Use the table to write an equation modeling the growth. Do you think it is reasonable to assume the pattern in the table continues indefinitely? *Explain.*

Day	0	1	2	3	4
Bamboo height (ft)	15	16.5	18	19.5	21

EXAMPLE 3
on p. 35
for Exs. 29–30

EXAMPLE 4
on p. 36
for Exs. 31–32

28. **MULTIPLE REPRESENTATIONS** Your cell phone plan costs \$40 per month plus \$.10 per text message. You receive a bill for \$53.80.

- a. **Making a Table** Copy and complete the table below. Use the table to estimate how many text messages you sent.

Text messages	0	50	100	150	200
Monthly bill	\$40	?	?	?	?

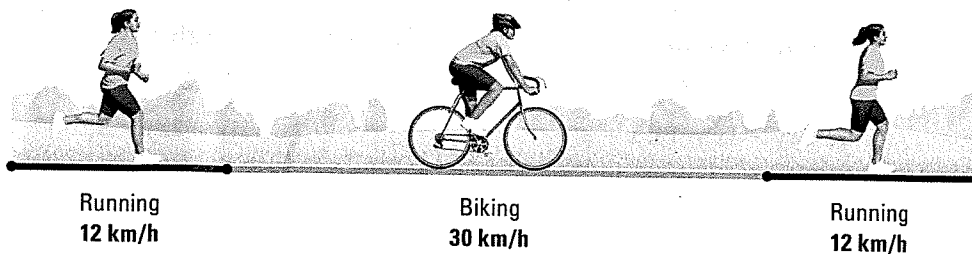
- b. **Writing a Model** Write an equation for the situation. Solve it to find exactly how many text messages you sent.
- c. **Comparing Answers** Is your estimate from part (a) compatible with the exact answer from part (b)? *Explain.*

29. **WOOD SHOP** You have a piece of wood that is 72 inches long. You cut the wood into three pieces. The second piece is 6 inches longer than the first piece. The third piece is 6 inches longer than the second piece. Draw a diagram and then write and solve an equation to find the lengths of the three pieces.

30. **POSTERS** You want to tape five posters on a wall so that the spaces between posters are the same. You also want the spaces at the left and right of the group of posters to be three times the space between any two adjacent posters. The wall is 15 feet wide and the posters are 1.5 feet wide. Draw a diagram and then write and solve an equation to find how to position the posters.

31. **PACKING WEIGHT** A moving company weighs 20 boxes you have packed that contain either books or clothes and says the total weight is 404 pounds. You know that a box of books weighs 40 pounds and a box of clothes weighs 7 pounds. Write and solve an equation to find how many boxes of books and how many boxes of clothes you packed.

32. **MULTI-STEP PROBLEM** A duathlon consists of a run, a bike ride, and a second run. Use the information below about the average rates of one participant who completed a 55 kilometer duathlon in 2 hours 35 minutes.



- a. **Model** Write a verbal model that shows the race distance as the sum of the total running distance and the biking distance.
- b. **Translate** Write an equation based on the verbal model.
- c. **Solve** Solve the equation to find how much time the participant spent running and how much time the participant spent biking.
- d. **Check** Find the total running distance and the biking distance, and verify that their sum is 55 kilometers.
33. **CHALLENGE** You are hanging fliers around a cylindrical kiosk that has a diameter of 5 feet. You want to hang 15 fliers that are 8.5 inches wide so they are evenly spaced. How far apart should the fliers be placed?

34. Curtis takes a bag of trail mix on a camping trip. On the first day, he eats one fourth of the trail mix. On the second day, he eats half of the remaining trail mix. On the third day, he eats one third of the remaining trail mix. When Curtis goes home, he has one-half pound of trail mix. How many pounds of trail mix did Curtis take on the camping trip?
- (A) 2 lb (B) 4 lb (C) 8 lb (D) 12 lb
35. The number of students participating in extracurricular activities at Alexander High School this year is 25% higher than the previous year's participation of 740 students. What percent of this year's participation is last year's participation?
- (A) 20% (B) 57% (C) 75% (D) 80%
36. How many yards of rope are needed to rope off a rectangular region having a width of 9 yards and a diagonal of 15 yards?
- (A) 24 yd (B) 33 yd (C) 36 yd (D) 42 yd

QUIZ for Lessons 1.3–1.5

Solve the equation. Check your solution. (p. 18)

1. $5b - 2 = 8$

2. $2d - 3 = 8d + 15$

3. $2(m - 4) = m + 2$

4. $\frac{2}{3}k + \frac{2}{7} = \frac{3}{7}k + \frac{1}{2}$

Solve the equation for y . Then find the value of y for the given value of x . (p. 26)

5. $4x + y = 12$; $x = 4$

6. $3x - 2y = 14$; $x = 6$

7. $3xy - 4x = 19$; $x = 2$

8. $11y + 2xy = 9$; $x = -5$

Look for a pattern in the table. Then write an equation that represents the table. (p. 34)

9.

x	0	1	2	3
y	0	13	26	39

10.

x	0	1	2	3
y	-5	-2	1	4

11. **TUTORING FEE** A chess tutor charges a fee for the first lesson that is 1.5 times the fee for later lessons. You spend \$315 for 10 lessons. How much does the first lesson cost? How much does a later lesson cost? (p. 34)
12. **FLOWER PRICES** You buy some calla lilies and peonies at a flower store. Calla lilies cost \$3.50 each and peonies cost \$5.50 each. The total cost of 12 flowers is \$52. How many calla lilies and how many peonies did you buy? (p. 34)

1.6 Solve Linear Inequalities



PA M11.D.2.1.1 Solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).

Before

You solved linear equations.

Now

You will solve linear inequalities.

Why?

So you can describe temperature ranges, as in Ex. 54.

Key Vocabulary

- linear inequality
- compound inequality
- equivalent inequalities

A **linear inequality** in one variable can be written in one of the following forms, where a and b are real numbers and $a \neq 0$:

$$ax + b < 0 \quad ax + b > 0 \quad ax + b \leq 0 \quad ax + b \geq 0$$

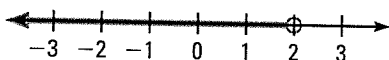
A **solution** of an inequality in one variable is a value that, when substituted for the variable, results in a true statement. The **graph** of an inequality in one variable consists of all points on a number line that represent solutions.

EXAMPLE 1 Graph simple inequalities

a. Graph $x < 2$.

The solutions are all real numbers less than 2.

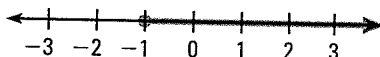
An open dot is used in the graph to indicate 2 is *not* a solution.



b. Graph $x \geq -1$.

The solutions are all real numbers greater than or equal to -1 .

A solid dot is used in the graph to indicate -1 *is* a solution.



COMPOUND INEQUALITIES A compound inequality consists of two simple inequalities joined by “and” or “or.”

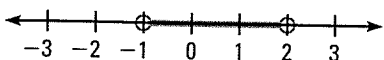
EXAMPLE 2 Graph compound inequalities

READ INEQUALITIES

The compound inequality $-1 < x < 2$ is another way of writing “ $x > -1$ and $x < 2$.”

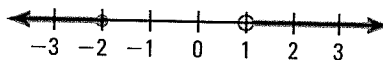
a. Graph $-1 < x < 2$.

The solutions are all real numbers that are greater than -1 and less than 2.



b. Graph $x \leq -2$ or $x > 1$.

The solutions are all real numbers that are less than or equal to -2 or greater than 1.



GUIDED PRACTICE for Examples 1 and 2

Graph the inequality.

1. $x > -5$

2. $x \leq 3$

3. $-3 \leq x < 1$

4. $x < 1$ or $x \geq 2$

SOLVING INEQUALITIES To solve a linear inequality in one variable, you isolate the variable using transformations that produce **equivalent inequalities**, which are inequalities that have the same solutions as the original inequality.

KEY CONCEPT

For Your Notebook

Transformations That Produce Equivalent Inequalities

Transformation applied to inequality	Original inequality	Equivalent inequality
Add the same number to each side.	$x - 7 < 4$	$x < 11$
Subtract the same number from each side.	$x + 3 \geq -1$	$x \geq -4$
Multiply each side by the same <i>positive</i> number.	$\frac{1}{2}x > 10$	$x > 20$
Divide each side by the same <i>positive</i> number.	$5x \leq 15$	$x \leq 3$
Multiply each side by the same <i>negative</i> number and <i>reverse</i> the inequality.	$-x < 17$	$x > -17$
Divide each side by the same <i>negative</i> number and <i>reverse</i> the inequality.	$-9x \geq 45$	$x \leq -5$

EXAMPLE 3 Solve an inequality with a variable on one side

FAIR You have \$50 to spend at a county fair. You spend \$20 for admission. You want to play a game that costs \$1.50. Describe the possible numbers of times you can play the game.



ANOTHER WAY

For alternative methods for solving the problem in Example 3, turn to page 48 for the **Problem Solving Workshop**.

Solution

STEP 1 Write a verbal model. Then write an inequality.

Admission fee (dollars)	+	Cost per game (dollars/game)	•	Number of games (games)	≤	Amount you can spend (dollars)
20	+	1.50	•	g	≤	50

An inequality is $20 + 1.5g \leq 50$.

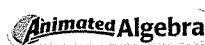
STEP 2 Solve the inequality.

$$20 + 1.5g \leq 50 \quad \text{Write inequality.}$$

$$1.5g \leq 30 \quad \text{Subtract 20 from each side.}$$

$$g \leq 20 \quad \text{Divide each side by 1.5.}$$

► You can play the game 20 times or fewer.

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EXAMPLE 4 Solve an inequality with a variable on both sides

Solve $5x + 2 > 7x - 4$. Then graph the solution.

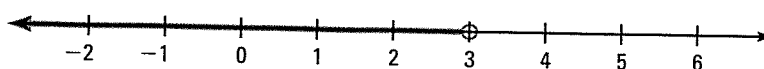
$$5x + 2 > 7x - 4 \quad \text{Write original inequality.}$$

$$-2x + 2 > -4 \quad \text{Subtract } 7x \text{ from each side.}$$

$$-2x > -6 \quad \text{Subtract 2 from each side.}$$

$$x < 3 \quad \text{Divide each side by } -2 \text{ and reverse the inequality.}$$

► The solutions are all real numbers less than 3. The graph is shown below.

**AVOID ERRORS**

Don't forget to reverse the inequality symbol if you multiply or divide each side of an inequality by a negative number.

**GUIDED PRACTICE** for Examples 3 and 4

Solve the inequality. Then graph the solution.

5. $4x + 9 < 25$

6. $1 - 3x \geq -14$

7. $5x - 7 \leq 6x$

8. $3 - x > x - 9$

EXAMPLE 5 Solve an "and" compound inequality

Solve $-4 < 6x - 10 \leq 14$. Then graph the solution.

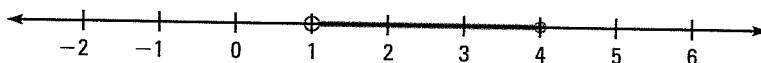
$$-4 < 6x - 10 \leq 14 \quad \text{Write original inequality.}$$

$$-4 + 10 < 6x - 10 + 10 \leq 14 + 10 \quad \text{Add 10 to each expression.}$$

$$6 < 6x \leq 24 \quad \text{Simplify.}$$

$$1 < x \leq 4 \quad \text{Divide each expression by 6.}$$

► The solutions are all real numbers greater than 1 and less than or equal to 4. The graph is shown below.

**EXAMPLE 6** Solve an "or" compound inequality

Solve $3x + 5 \leq 11$ or $5x - 7 \geq 23$. Then graph the solution.

Solution

A solution of this compound inequality is a solution of *either* of its parts.

First Inequality

$$3x + 5 \leq 11 \quad \text{Write first inequality.}$$

$$3x \leq 6 \quad \text{Subtract 5 from each side.}$$

$$x \leq 2 \quad \text{Divide each side by 3.}$$

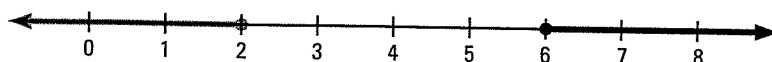
Second Inequality

$$5x - 7 \geq 23 \quad \text{Write second inequality.}$$

$$5x \geq 30 \quad \text{Add 7 to each side.}$$

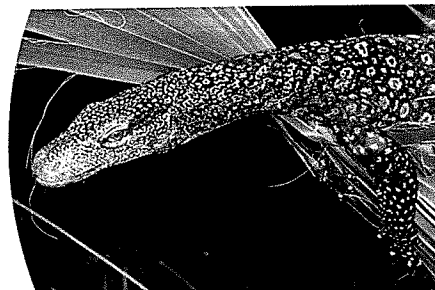
$$x \geq 6 \quad \text{Divide each side by 5.}$$

► The graph is shown below. The solutions are all real numbers less than or equal to 2 or greater than or equal to 6.



EXAMPLE 7 Write and use a compound inequality

BIOLOGY A monitor lizard has a temperature that ranges from 18°C to 34°C . Write the range of temperatures as a compound inequality. Then write an inequality giving the temperature range in degrees Fahrenheit.



Monitor lizard

Solution

The range of temperatures C can be represented by the inequality $18 \leq C \leq 34$. Let F represent the temperature in degrees Fahrenheit.

$$18 \leq C \leq 34$$

Write inequality.

$$18 \leq \frac{5}{9}(F - 32) \leq 34$$

Substitute $\frac{5}{9}(F - 32)$ for C .

$$32.4 \leq F - 32 \leq 61.2$$

Multiply each expression by $\frac{9}{5}$, the reciprocal of $\frac{5}{9}$.

$$64.4 \leq F \leq 93.2$$

Add 32 to each expression.

► The temperature of the monitor lizard ranges from 64.4°F to 93.2°F .

USE A FORMULA

In Example 7, use the temperature formula

$$C = \frac{5}{9}(F - 32).$$

**GUIDED PRACTICE** for Examples 5, 6, and 7

Solve the inequality. Then graph the solution.

9. $-1 < 2x + 7 < 19$

10. $-8 \leq -x - 5 \leq 6$

11. $x + 4 \leq 9$ or $x - 3 \geq 7$

12. $3x - 1 < -1$ or $2x + 5 \geq 11$

13. **WHAT IF?** In Example 7, write a compound inequality for a lizard whose temperature ranges from 15°C to 30°C . Then write an inequality giving the temperature range in degrees Fahrenheit.

1.6 EXERCISES**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS**
on p. WS2 for Exs. 13, 25, and 55

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 15, 36, 56, and 59

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The set of all points on a number line that represent solutions of an inequality is called the ? of the inequality.

2. ★ **WRITING** The first transformation on page 42 can be written as follows:

If a , b , and c are real numbers and $a > b$, then $a + c > b + c$.

Write similar statements for the other transformations listed on page 42.

EXAMPLE 1

on p. 41
for Exs. 3–10

GRAPHING INEQUALITIES Graph the inequality.

3. $x > 4$

4. $x < -1$

5. $x \leq -5$

6. $x \geq 3$

7. $6 \geq x$

8. $-2 < x$

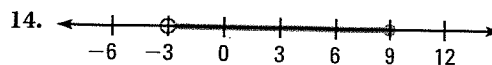
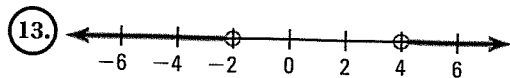
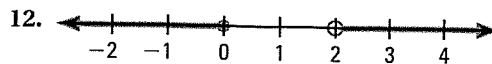
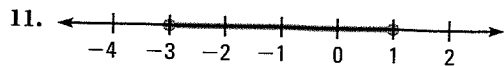
9. $x \geq -3.5$

10. $x < 2.5$

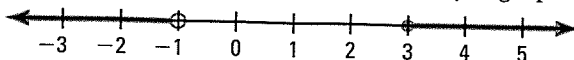
EXAMPLE 2

on p. 41
for Exs. 11–21

WRITING COMPOUND INEQUALITIES Write the compound inequality that is represented by the graph.



15. ★ **MULTIPLE CHOICE** What compound inequality is graphed below?



(A) $-1 < x < 3$

(B) $x \leq -1$ or $x > 3$

(C) $x < -1$ or $x \geq 3$

(D) $x > -1$ or $x \leq 3$

GRAPHING COMPOUND INEQUALITIES Graph the compound inequality.

16. $2 \leq x \leq 5$

17. $-3 < x < 4$

18. $5 \leq x < 10$

19. $x < 0$ or $x > 2$

20. $x \leq -1$ or $x > 1$

21. $x > -2$ or $x < -5$

EXAMPLES 3 and 4

on pp. 42–43
for Exs. 22–35

SOLVING INEQUALITIES Solve the inequality. Then graph the solution.

22. $x + 4 > 10$

23. $x - 3 \leq -5$

24. $4x - 8 \geq -4$

25. $15 - 3x > 3$

26. $11 + 8x \geq 7$

27. $4 + \frac{3}{2}x \leq 13$

28. $2x - 6 > 3 - x$

29. $4x + 14 < 3x + 6$

30. $5 - 8x \leq 19 - 10x$

31. $21x + 7 < 3x + 16$

32. $18 + 2x \leq 9x + 4$

33. $2(x - 4) > 4x + 6$

ERROR ANALYSIS Describe and correct the error in solving the inequality.

34.

$$2x + 8 \leq 6x - 4$$

$$-4x \leq -12$$

$$x \leq 3$$



35.

$$10 + 3x > 5x$$

$$10 < 2x$$

$$5 < x$$



36. ★ **OPEN-ENDED MATH** Write two different inequalities of the form $ax + b > c$ that have a solution of $x > 5$.

EXAMPLE 5

on p. 43
for Exs. 37–42

“AND” COMPOUND INEQUALITIES Solve the inequality. Then graph the solution.

37. $-5 < x + 1 < 4$

38. $2 \leq x - 3 \leq 6$

39. $-3 < 4 - x \leq 3$

40. $2 < 3x - 1 \leq 6$

41. $-4 \leq 2 + 4x < 0$

42. $0 \leq \frac{3}{4}x + 3 \leq 4$

EXAMPLE 6

on p. 43
for Exs. 43–48

“OR” COMPOUND INEQUALITIES Solve the inequality. Then graph the solution.

43. $x + 1 < -3$ or $x - 2 > 0$

44. $x - 4 \leq -6$ or $x + 2 > 5$

45. $2x - 3 \leq -4$ or $3x + 1 \geq 4$

46. $2 + 3x < -13$ or $4 + 2x > 7$

47. $0.3x - 0.5 < -1.7$ or $0.4x \geq 2.4$

48. $-x - 4 \geq 1$ or $2 - 5x \leq -8$

CHALLENGE Solve the inequality. If there is no solution, write *no solution*. If the inequality is always true, write *all real numbers*.

49. $2(x - 4) > 2x + 1$

50. $4x - 5 \leq 4(x + 2)$

51. $2(3x - 1) > 3(2x + 3)$

PROBLEM SOLVING

EXAMPLE 3
on p. 42
for Exs. 52–53

- 52. SWIMMING** You have budgeted \$100 to improve your swimming over the summer. At your local pool, it costs \$50 to join the swim association and \$5 for each swim class. Write and solve an inequality to find the possible numbers of swim classes you can attend within your budget.

@HomeTutor for problem solving help at classzone.com

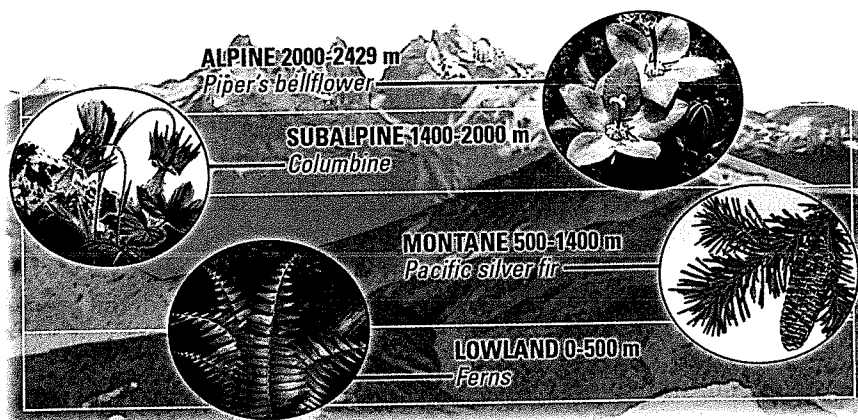
- 53. VIDEO CONTEST** You and some friends have raised \$250 to help make a video for a contest. You need \$35 to buy videotapes. It costs \$45 per day to rent the video camera. Write and solve an inequality to find the possible numbers of days you can rent the video camera.

@HomeTutor for problem solving help at classzone.com

- 54. WAKEBOARDING** What you wear when you wakeboard depends on the air temperature. Copy and complete the table by writing an inequality for each temperature range. Assume each range includes the lower temperature but not the higher temperature. (The first inequality has been written for you.)

Temperature	Gear	Inequality
60°F to 65°F	Full wetsuit	$60 \leq T < 65$
65°F to 72°F	Full leg wetsuit	?
72°F to 80°F	Wetsuit trunks	?
80°F or warmer	No special gear	?

- 55. BOTANY** In Olympic National Park in Washington, different plants grow depending on the elevation, as shown in the diagram. Assume each range includes the lower elevation but not the higher elevation.



- Write an inequality for elevations in the lowland zone.
 - Write an inequality for elevations in the alpine and subalpine zones combined.
 - Write an inequality for elevations *not* in the montane zone.
- 56. ★ MULTIPLE CHOICE** Canoe rental costs \$18 for the first two hours and \$3 per hour after that. You want to canoe for more than 2 hours but can spend no more than \$30. Which inequality represents the situation, where t is the total number of hours you can canoe?
- (A) $18 + t \leq 30$ (B) $18 + 3t \leq 30$
 (C) $18 + 3(t + 2) \leq 30$ (D) $18 + 3(t - 2) \leq 30$

EXAMPLE 7

on p. 44
for Exs. 57–58

- 57. LAPTOP COMPUTERS** A computer manufacturer states that its laptop computer can operate within a temperature range of 50°F to 95°F. Write a compound inequality for the temperature range. Then rewrite the inequality in degrees Celsius.
- 58. MULTI-STEP PROBLEM** On a certain highway, there is a minimum speed of 45 miles per hour and a maximum speed of 70 miles per hour.
- Write a compound inequality for the legal speeds on the highway.
 - Write a compound inequality for the illegal speeds on the highway.
 - Write each compound inequality from parts (a) and (b) so that it expresses the speeds in kilometers per hour. (1 mi $\approx$ 1.61 km)
- 59. ★ EXTENDED RESPONSE** A math teacher announces that grades will be calculated by adding 65% of a student's homework score, 15% of the student's quiz score, and 20% of the student's final exam score. All scores range from 0 to 100 points.
- Write Inequalities** Write an inequality for each student that can be used to find the possible final exam scores that result in a grade of 85 or better.
 - Solve** Solve the inequalities from part (a).
 - Interpret** For which students is a grade of 85 or better possible? *Explain.*
- | Name | Homework | Quiz | Exam |
|-------|----------|------|------|
| Amy | 84 | 80 | w |
| Brian | 80 | 100 | x |
| Clara | 75 | 95 | y |
| Dan | 80 | 90 | z |
- 60. CHALLENGE** You are shopping for single-use cameras to hand out at a party. The daylight cameras cost \$2.75 and the flash cameras cost \$4.25. You must buy exactly 20 cameras and you want to spend between \$65 and \$75, inclusive. Write and solve a compound inequality for this situation. Then list all the solutions that involve whole numbers of cameras.

PA**PENNSYLVANIA MIXED REVIEW****TEST PRACTICE** at classzone.com

- 61.** Steve has 6 fewer trading cards than Kevin. Thomas has twice as many trading cards as Steve. The three students have a total of 22 trading cards. Which equation can be used to find the number of trading cards that Kevin has?
- (A) $x - 6x + \frac{1}{2}x = 22$
- (B) $x + (x - 6) + 2x = 22$
- (C) $x + (x - 6) + 2(x - 6) = 22$
- (D) $2x + (x - 6) + (x - 6) = 22$
- 62.** The radius and height of a cylindrical can are doubled. How does the surface area of the new cylindrical can compare with the surface area of the original cylindrical can?
- (A) The new surface area is two times the original surface area.
- (B) The new surface area is four times the original surface area.
- (C) The new surface area is six times the original surface area.
- (D) The new surface area is eight times the original surface area.



**PROBLEM SOLVING
WORKSHOP**
LESSON 1.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 42



MULTIPLE REPRESENTATIONS Example 3 of Lesson 1.6 involved solving an inequality using algebra. You can also solve an inequality using a table or a graphing calculator's *test* feature, which tells when an inequality is true or false.

PROBLEM

FAIR You have \$50 to spend at a county fair. You spend \$20 for admission. You want to play a game that costs \$1.50. Describe the possible numbers of times you can play the game.

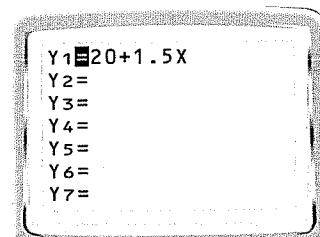
METHOD 1

Using a Table One alternative approach is to make a table of values.

STEP 1 Write an expression for the total cost of admission and playing x games.

Admission fee	+	Cost per game	·	Number of games
▼		▼		▼
20	+	1.50	·	x

STEP 2 Enter the equation $y = 20 + 1.5x$ into a graphing calculator.



STEP 3 Make a table of values for the equation.

Use TblStart = 0 and $\Delta Tbl = 1$ to see these values.

X	Y1
0	20
1	21.5
2	23
3	24.5
4	26
X=0	

STEP 4 Scroll through the table of values to find when the total cost is \$50. You can see that $y = 50$ when $x = 20$.

X	Y1
18	47
19	48.5
20	50
21	51.5
22	53
X=20	

► The table suggests that $20 + 1.5x \leq 50$ when $x \leq 20$. So, you can play the game at the fair 20 times or fewer.

METHOD 2

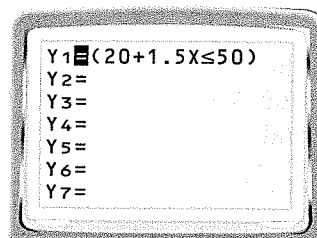
Using a Graph Another approach is to use a graph.

If your graphing calculator has a *test* feature, you can enter the inequality and evaluate its truth for various values of x .

- When the inequality is *true*, the calculator returns a 1.
- When the inequality is *false*, the calculator returns a 0.

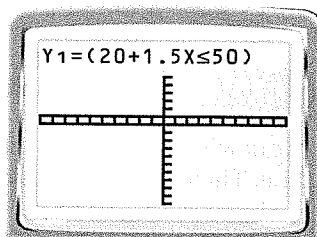
STEP 1 Enter $y = (20 + 1.5x \leq 50)$ into a graphing calculator.

Press **2nd** [TEST] **6** to enter the $\leq$ symbol.



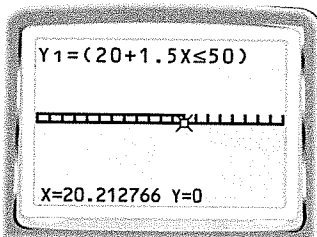
STEP 2 Graph the result.

The y-value is 1 for all x-values that make the inequality true.



STEP 3 Find the point where the inequality changes from true to false by using the *trace* feature.

- The graph suggests that the inequality is true when $x \leq 20$. So, you can play the game at the fair 20 times or fewer.



PRACTICE

1. **REASONING** Determine the equation that gives the table below. For what x -values is $y < -500$?

X	Y1
0	200
1	165
2	130
3	95
4	60
X=0	

2. **GIFT** You have \$16.50 to spend for a friend's birthday. You spend \$3 on a card and want to buy some chocolates that cost \$.75 each. What are the numbers of chocolates you can buy? Solve using a table and using a graph.

3. **SALESPERSON** A salesperson has a weekly salary of \$1550 and gets a 5% commission on sales. What are the amounts the salesperson can sell to earn at least \$1900 per week? Solve using a table and using a graph.

4. **WRITING** Explain how to use a table like the one below to solve $0.5x - 1.5 \leq 3 - 0.4x$.

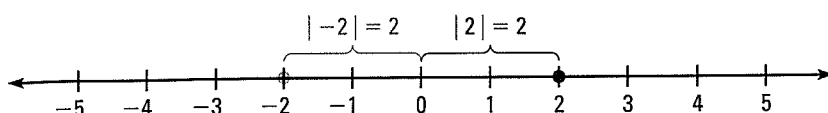
X	Y1	Y2
0	-1.5	3
1	-1	2.6
2	-.5	2.2
3	0	1.8
4	.5	1.4
X=0		

1.7 Absolute Value Equations and Inequalities

MATERIALS • 13 index cards numbered with the integers from -6 to 6

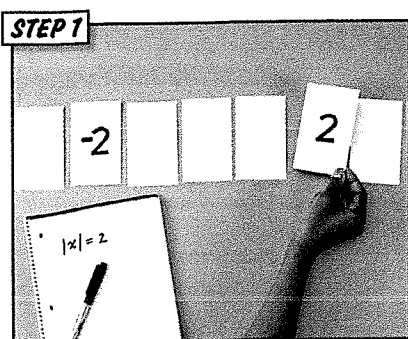
QUESTION What does the solution of an absolute value equation or inequality look like on a number line?

The *absolute value* of a number x , written $|x|$, is the distance the number is from 0 on a number line. Because 2 and -2 are both 2 units from 0, $|2| = 2$ and $|-2| = 2$. The absolute value of a number is never negative.



EXPLORE Find solutions of absolute value equations and inequalities

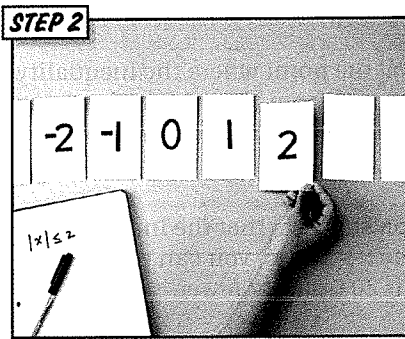
Work with a partner. Place the numbered index cards in a row to form a number line. Then turn all the cards face down.



Solve equations

Turn over cards to reveal numbers that are solutions of the equations below.

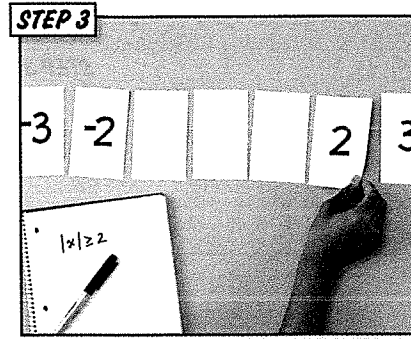
- a. $|x| = 2$
- b. $|x - 2| = 1$
- c. $|x + 1| = 3$



Solve inequalities with $\leq$

Turn over cards to reveal numbers that are solutions of the inequalities below.

- d. $|x| \leq 2$
- e. $|x - 2| \leq 1$
- f. $|x + 1| \leq 3$



Solve inequalities with $\geq$

Turn over cards to reveal numbers that are solutions of the inequalities below.

- g. $|x| \geq 2$
- h. $|x - 2| \geq 1$
- i. $|x + 1| \geq 3$

DRAW CONCLUSIONS Use your observations to complete these exercises

- Describe the solutions of the absolute value equations in Step 1. Will all absolute value equations have the same number of solutions? *Explain.*
- Compare the solutions of the absolute value inequalities in Steps 2 and 3. How does the inequality symbol ($\leq$ or $\geq$) affect the pattern of the solutions?

1.7 Solve Absolute Value Equations and Inequalities

PA

M11.D.2.1.1

Solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).

Before

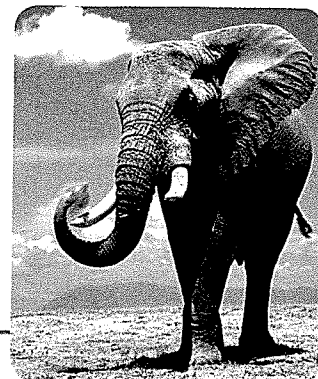
You solved linear equations and inequalities.

Now

You will solve absolute value equations and inequalities.

Why?

So you can describe hearing ranges of animals, as in Ex. 81.



Key Vocabulary

- absolute value
- extraneous solution

Recall that the **absolute value** of a number x , written $|x|$, is the distance the number is from 0 on a number line. This understanding of absolute value can be extended to apply to simple absolute value equations.

$$|x| = \begin{cases} x, & \text{if } x \text{ is positive} \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \text{ is negative} \end{cases}$$

KEY CONCEPT

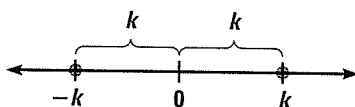
For Your Notebook

Interpreting Absolute Value Equations

Equation $|x| = |x - 0| = k$

Meaning The distance between x and 0 is k .

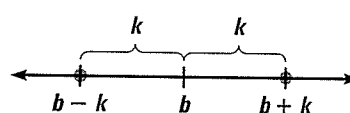
Graph



Solutions $x - 0 = -k$ or $x - 0 = k$
 $x = -k$ or $x = k$

$|x - b| = k$

The distance between x and b is k .



$x - b = -k$ or $x - b = k$
 $x = b - k$ or $x = b + k$

EXAMPLE 1

Solve a simple absolute value equation

Solve $|x - 5| = 7$. Graph the solution.

Solution

$|x - 5| = 7$

Write original equation.

$x - 5 = -7$ or $x - 5 = 7$

Write equivalent equations.

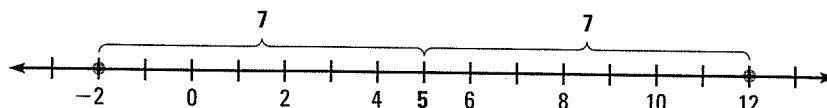
$x = 5 - 7$ or $x = 5 + 7$

Solve for x .

$x = -2$ or $x = 12$

Simplify.

► The solutions are -2 and 12 . These are the values of x that are 7 units away from 5 on a number line. The graph is shown below.



Solving an Absolute Value Equation

Use these steps to solve an absolute value equation $|ax + b| = c$ where $c > 0$.

STEP 1 Write two equations: $ax + b = c$ or $ax + b = -c$.

STEP 2 Solve each equation.

STEP 3 Check each solution in the original absolute value equation.

EXAMPLE 2 Solve an absolute value equation

Solve $|5x - 10| = 45$.

$$|5x - 10| = 45$$

Write original equation.

$$5x - 10 = 45 \quad \text{or} \quad 5x - 10 = -45$$

Expression can equal 45 or -45.

$$5x = 55 \quad \text{or} \quad 5x = -35$$

Add 10 to each side.

$$x = 11 \quad \text{or} \quad x = -7$$

Divide each side by 5.

► The solutions are 11 and -7. Check these in the original equation.

CHECK $5x - 10 = 45$ $5(11) - 10 \stackrel{?}{=} 45$ $45 \stackrel{?}{=} 45$ $45 = 45 \checkmark$	$5x - 10 = 45$ $5(-7) - 10 \stackrel{?}{=} 45$ $-45 \stackrel{?}{=} 45$ $45 = 45 \checkmark$
--	--

EXTRANEIOUS SOLUTIONS When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

EXAMPLE 3 Check for extraneous solutions

Solve $|2x + 12| = 4x$. Check for extraneous solutions.

$$|2x + 12| = 4x$$

Write original equation.

$$2x + 12 = 4x \quad \text{or} \quad 2x + 12 = -4x$$

Expression can equal $4x$ or $-4x$.

$$12 = 2x \quad \text{or} \quad 12 = -6x$$

Subtract $2x$ from each side.

$$6 = x \quad \text{or} \quad -2 = x$$

Solve for x .

Check the apparent solutions to see if either is extraneous.

CHECK $2x + 12 = 4x$ $2(6) + 12 \stackrel{?}{=} 4(6)$ $24 \stackrel{?}{=} 24$ $24 = 24 \checkmark$	$2x + 12 = 4x$ $2(-2) + 12 \stackrel{?}{=} 4(-2)$ $8 \stackrel{?}{=} -8$ $8 \neq -8$
---	--

► The solution is 6. Reject -2 because it is an extraneous solution.

AVOID ERRORS

Always check your solutions in the original equation to make sure that they are not extraneous.

**GUIDED PRACTICE** for Examples 1, 2, and 3

Solve the equation. Check for extraneous solutions.

1. $|x| = 5$

2. $|x - 3| = 10$

3. $|x + 2| = 7$

4. $|3x - 2| = 13$

5. $|2x + 5| = 3x$

6. $|4x - 1| = 2x + 9$

INEQUALITIES You can solve an absolute value inequality by rewriting it as a compound inequality and then solving each part.**KEY CONCEPT***For Your Notebook***Absolute Value Inequalities**

Inequality	Equivalent form	Graph of solution
$ ax + b < c$	$-c < ax + b < c$	
$ ax + b \leq c$	$-c \leq ax + b \leq c$	
$ ax + b > c$	$ax + b < -c$ or $ax + b > c$	
$ ax + b \geq c$	$ax + b \leq -c$ or $ax + b \geq c$	

**EXAMPLE 4** Solve an inequality of the form $|ax + b| > c$ Solve $|4x + 5| > 13$. Then graph the solution.**Solution**The absolute value inequality is equivalent to $4x + 5 < -13$ or $4x + 5 > 13$.**First Inequality**

$4x + 5 < -13$

$4x < -18$

$x < -\frac{9}{2}$

Write inequalities.

Subtract 5 from each side.

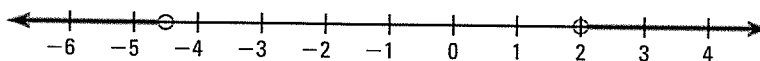
Divide each side by 4.

Second Inequality

$4x + 5 > 13$

$4x > 8$

$x > 2$

► The solutions are all real numbers less than $-\frac{9}{2}$ or greater than 2. The graph is shown below.

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**GUIDED PRACTICE** for Example 4

Solve the inequality. Then graph the solution.

7. $|x + 4| \geq 6$

8. $|2x - 7| > 1$

9. $|3x + 5| \geq 10$

READING

Tolerance is the maximum acceptable deviation of an item from some ideal or mean measurement.

EXAMPLE 5 Solve an inequality of the form $|ax + b| \leq c$

BASEBALL A professional baseball should weigh 5.125 ounces, with a *tolerance* of 0.125 ounce. Write and solve an absolute value inequality that describes the acceptable weights for a baseball.

Solution

STEP 1 Write a verbal model. Then write an inequality.

Actual weight (ounces)	–	Ideal weight (ounces)	≤	Tolerance (ounces)
↓		↓		↓
w	–	5.125	≤	0.125

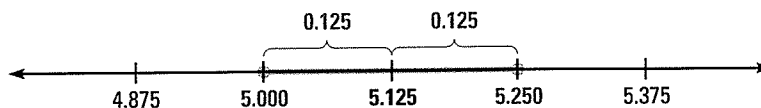
STEP 2 Solve the inequality.

$$|w - 5.125| \leq 0.125 \quad \text{Write inequality.}$$

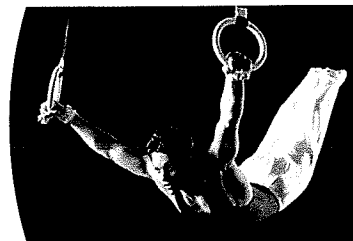
$$-0.125 \leq w - 5.125 \leq 0.125 \quad \text{Write equivalent compound inequality.}$$

$$5 \leq w \leq 5.25 \quad \text{Add 5.125 to each expression.}$$

► So, a baseball should weigh between 5 ounces and 5.25 ounces, inclusive. The graph is shown below.

**EXAMPLE 6** Write a range as an absolute value inequality

GYMNASTICS The thickness of the mats used in the rings, parallel bars, and vault events must be between 7.5 inches and 8.25 inches, inclusive. Write an absolute value inequality describing the acceptable mat thicknesses.

**Solution**

STEP 1 Calculate the mean of the extreme mat thicknesses.

$$\text{Mean of extremes} = \frac{7.5 + 8.25}{2} = 7.875$$

STEP 2 Find the tolerance by subtracting the mean from the upper extreme.

$$\text{Tolerance} = 8.25 - 7.875 = 0.375$$

STEP 3 Write a verbal model. Then write an inequality.

Actual thickness (inches)	–	Mean of extremes (inches)	≤	Tolerance (inches)
↓		↓		↓
t	–	7.875	≤	0.375

► A mat is acceptable if its thickness t satisfies $|t - 7.875| \leq 0.375$.

REVIEW MEAN

For help with finding a mean, see p. 1005.

**GUIDED PRACTICE** for Examples 5 and 6

Solve the inequality. Then graph the solution.

10. $|x + 2| < 6$

11. $|2x + 1| \leq 9$

12. $|7 - x| \leq 4$

13. **GYMNASTICS** For Example 6, write an absolute value inequality describing the *unacceptable* mat thicknesses.

1.7 EXERCISES**HOMEWORK KEY**

- = **WORKED-OUT SOLUTIONS**
on p. WS2 for Exs. 21, 47, and 77
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 33, 40, 63, and 64
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 78

SKILL PRACTICE

1. **VOCABULARY** What is an extraneous solution of an equation?

2. ★ **WRITING** The absolute value of a number cannot be negative. How, then, can the absolute value of x be $-x$ for certain values of x ?

CHECKING SOLUTIONS Decide whether the given number is a solution of the equation.

3. $|b - 1| = 14$; -13

4. $|d + 6| = 10$; -4

5. $|32 - 6f| = 20$; -2

6. $|2m + 6| = 10$; -8

7. $|3n - 7| = 4$; 1

8. $|17 - 8r| = 15$; 4

EXAMPLE 1

on p. 51
for Exs. 9–20

SOLVING EQUATIONS Solve the equation. Graph the solution.

9. $|x| = 9$

10. $|y| = -5$

11. $|z| = 0$

12. $|f - 5| = 3$

13. $|g - 2| = 7$

14. $|h - 4| = 4$

15. $|k + 3| = 6$

16. $|m + 5| = 1$

17. $|n + 9| = 10$

18. $|6 - p| = 4$

19. $|5 - q| = 7$

20. $|-4 - r| = 4$

EXAMPLE 2

on p. 52
for Exs. 21–32

SOLVING EQUATIONS Solve the equation.

21. $|2d - 5| = 13$

22. $|3g + 14| = 7$

23. $|7h - 10| = 4$

24. $|3p - 6| = 21$

25. $|2q + 3| = 11$

26. $|4r + 7| = 43$

27. $|5 + 2j| = 9$

28. $|6 - 3k| = 21$

29. $|20 - 9m| = 7$

30. $|\frac{1}{4}x - 3| = 10$

31. $|\frac{1}{2}y + 4| = 6$

32. $|\frac{2}{3}z - 6| = 12$

33. ★ **SHORT RESPONSE** The equation $|5x - 10| = 45$ in Example 2 has two solutions. Does the equation $|5x - 10| = -45$ also have two solutions? Explain.

EXAMPLE 3

on p. 52
for Exs. 34–42

EXTRANEIOUS SOLUTIONS Solve the equation. Check for extraneous solutions.

34. $|3x - 4| = x$

35. $|x + 24| = -7x$

36. $|8x - 1| = 6x$

37. $|4x + 5| = 2x + 4$

38. $|9 - 2x| = 10 + 3x$

39. $|8 + 5x| = 7 - x$

40. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of $|3x + 7| = 5x$?

(A) $-4, -\frac{2}{3}$

(B) $-\frac{7}{8}, \frac{7}{2}$

(C) $\frac{7}{8}, \frac{7}{2}$

(D) $\frac{7}{2}$

ERROR ANALYSIS Describe and correct the error in solving the equation.

41.

$$|5x - 9| = x + 3$$

$$5x - 9 = x + 3 \text{ or } 5x - 9 = -x + 3$$

$$4x - 9 = 3 \text{ or } 6x - 9 = 3$$

$$4x = 12 \text{ or } 6x = 12$$

$$x = 3 \text{ or } x = 2$$

The solutions are 3 and 2.

42.

$$|n - 7| = 3n - 1$$

$$n - 7 = 3n - 1 \text{ or } n - 7 = -3n + 1$$

$$-7 = 2n - 1 \text{ or } 4n - 7 = 1$$

$$-6 = 2n \text{ or } 4n = 8$$

$$-3 = n \text{ or } n = 2$$

The solutions are -3 and 2.

EXAMPLES

4 and 5

on pp. 53–54
for Exs. 43–63

SOLVING INEQUALITIES Solve the inequality. Then graph the solution.

43. $|j| \leq 5$

44. $|k| > 4$

45. $|m - 2| < 7$

46. $|n - 11| \geq 1$

47. $|d + 4| \geq 3$

48. $|f + 6| < 2$

49. $|g - 1| > 0$

50. $|h + 10| \leq 10$

51. $|3w - 15| < 30$

52. $|2x + 6| \geq 10$

53. $|4y - 9| \leq 7$

54. $|5z + 1| > 14$

55. $|16 - p| > 3$

56. $|24 - q| \leq 11$

57. $|7 - 2r| < 19$

58. $|19 - 5t| > 7$

59. $|\frac{1}{2}x - 10| \leq 4$

60. $|\frac{1}{3}m - 15| < 6$

61. $|\frac{1}{7}y + 2| - 5 > 3$

62. $|\frac{2}{5}n - 8| + 4 \geq 12$

Animated Algebra at classzone.com

63. ★ **MULTIPLE CHOICE** What is the solution of $|6x - 9| \geq 33$?

(A) $-4 \leq x \leq 7$

(B) $-7 \leq x \leq 4$

(C) $x \leq -4 \text{ or } x \geq 7$

(D) $x \leq -7 \text{ or } x \geq 4$

64. ★ **MULTIPLE CHOICE** Which absolute value inequality represents the graph shown below?



(A) $-1 < |x| < 5$

(B) $|x + 2| < 3$

(C) $|x - 2| < 3$

(D) $|x - 2| < 5$

65. **REASONING** For the equation $|ax + b| = c$ (where a , b , and c are real numbers and $a \neq 0$), describe the value(s) of c that yield two solutions, one solution, and no solution.

SOLVING INEQUALITIES Solve the inequality. Then graph the solution.

66. $|x + 1| \geq -16$

67. $|2x - 1| < -25$

68. $|7x + 3| \leq 0$

69. $|x - 9| > 0$

CHALLENGE Solve the inequality for x in terms of a , b , and c . Assume a , b , and c are real numbers and $c > 0$.

70. $|ax + b| < c$ where $a > 0$

71. $|ax + b| \geq c$ where $a > 0$

72. $|ax + b| \leq c$ where $a < 0$

73. $|ax + b| > c$ where $a < 0$

PROBLEM SOLVING

EXAMPLE 5

on p. 54
for Exs. 74–78

74. **GYMNASTICS** The horizontal bar used in gymnastics events should be placed 110.25 inches above the ground, with a tolerance of 0.4 inch. Write an absolute value inequality for the acceptable bar heights.

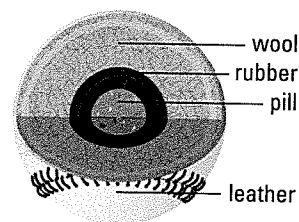
@HomeTutor for problem solving help at classzone.com

75. **SOIL PH LEVELS** Cucumbers grow in soil having a pH level of 6.5, with a tolerance of 1 point on the pH scale. Write an absolute value inequality that describes the pH levels of soil in which cucumbers can grow.

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76. **MULTI-STEP PROBLEM** A baseball has a cushioned cork center called the *pill*. The pill must weigh 0.85 ounce, with a tolerance of 0.05 ounce.

- Write an absolute value inequality that describes the acceptable weights for the pill of a baseball.
- Solve the inequality to find the acceptable weights for the pill.
- Look back at Example 5 on page 54. Find the minimum and maximum percentages of a baseball's total weight that the pill can make up.



77. **MANUFACTURING** A regulation basketball should weigh 21 ounces, with a tolerance of 1 ounce. Write an absolute value inequality describing the weights of basketballs that should be *rejected*.

78. **MULTIPLE REPRESENTATIONS** The strength of eyeglass lenses is measured in units called *diopters*. The diopter number x is negative for nearsighted vision and positive for farsighted vision.

Nearsightedness (focus is in front of retina)				Farsightedness (focus is behind retina)	
Mild	$ x + 1.5 < 1.5$			Mild	$ x - 1 < 1$
Moderate	$ x + 4.5 < 1.5$			Moderate	$ x - 3 < 1$
Severe	$ x + 7.5 < 1.5$			Severe	$ x - 5 < 1$

- Writing Inequalities** Write an equivalent compound inequality for each vision category shown above. Solve the inequalities.
- Making a Graph** Illustrate the six vision categories by graphing their ranges of diopter numbers on the same number line. Label each range with the corresponding category name.

EXAMPLE 6

on p. 54
for Exs. 79–81

79. **SLEEPING BAGS** A manufacturer of sleeping bags suggests that one model is best suited for temperatures between 30°F and 60°F, inclusive. Write an absolute value inequality for this temperature range.

80. **TEMPERATURE** The recommended oven setting for cooking a pizza in a professional brick-lined oven is between 550°F and 650°F, inclusive. Write an absolute value inequality for this temperature range.

81. AUDIBLE FREQUENCIES An elephant can hear sounds with frequencies from 16 hertz to 12,000 hertz. A mouse can hear sounds with frequencies from 1000 hertz to 91,000 hertz. Write an absolute value inequality for the hearing range of each animal.

82. CHALLENGE The depth finder on a fishing boat gives readings that are within 5% of the actual water depth. When the depth finder reading is 250 feet, the actual water depth x lies within a range given by the following inequality:

$$|x - 250| \leq 0.05x$$

- Write the absolute value inequality as a compound inequality.
- Solve each part of the compound inequality for x . What are the possible actual water depths if the depth finder's reading is 250 feet?

PA

PENNSYLVANIA MIXED REVIEW

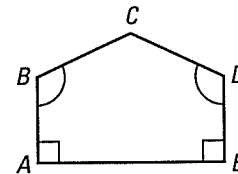
TEST PRACTICE at classzone.com

83. A car dealership hires Anne to wash cars. She is paid \$28 per day plus \$6 for every car she washes. Anne shares the money equally with a friend who assists her. After five days, Anne's share of the pay is \$130. How many cars did Anne and her friend wash?

- (A) 17 (B) 20 (C) 32 (D) 39

84. Pentagon $ABCDE$ is the outline of the front of a cabin. The measure of $\angle ABC$ is 115° . What is the measure of $\angle BCD$?

- (A) 90° (B) 115°
(C) 130° (D) 155°



QUIZ for Lessons 1.6–1.7

Solve the inequality. Then graph the solution. (p. 41)

- $4k - 17 < 27$
- $14n - 8 \geq 90$
- $-9p + 15 \leq 96$
- $-8r - 11 > 45$
- $3(x - 7) < 6(10 - x)$
- $-25 - 4z > 66 - 17z$

Solve the equation or inequality. (p. 51)

- $|x - 6| = 9$
- $|3y + 3| = 12$
- $|2z + 5| = -9z$
- $|p + 7| > 2$
- $|2q - 3| \leq 3$
- $|5 - r| \geq 4$

13. TEST SCORES Your final grade in a course is 80% of your current grade, plus 20% of your final exam score. Your current grade is 83 and your goal is to get a final grade of 85 or better. Write and solve an inequality to find the final exam scores that will meet your goal. (p. 41)

14. GROCERY WEIGHTS A container of potato salad from your grocer's deli is supposed to weigh 1.5 pounds, with a tolerance of 0.025 pound. Write and solve an absolute value inequality that describes the acceptable weights for the container of potato salad. (p. 51)



Lessons 1.5–1.7

1. **HYBRID CAR** A hybrid car gets about 60 miles per gallon of gas in the city and about 51 miles per gallon on the highway. During one week, the hybrid uses 12 gallons of gas and travels 675 miles. How much gas was used on the highway?
- A 4 gallons
B 5 gallons
C 7 gallons
D 8 gallons
2. **POPCORN** A popcorn manufacturer's ideal weight for a bag of microwave popcorn is 3.5 ounces, with a tolerance of 0.25 ounce. What is the range of acceptable weights w (in ounces) of a bag of popcorn?
- A $w \geq 3.75$
B $w \leq 3.25$
C $-0.25 \leq w \leq 0.25$
D $3.25 \leq w \leq 3.75$
3. **LIQUID OXYGEN** Oxygen exists as a liquid between -369°F and -297°F , inclusive. Which compound inequality gives the range of temperatures T for liquid oxygen?
- A $-369 \leq T \leq -297$
B $-369 < T < -297$
C $-297 \leq T \leq -369$
D $-297 < T < -369$
4. **FOOTBALL** A football kicker scores 1 point for each extra point and 3 points for each field goal. One season, a kicker made 34 extra points and scored a total of 112 points. How many field goals did the kicker make?
- A 13
B 26
C 48
D 78
5. **VIDEO RENTAL** A video store rents movies for \$2.95 each. Recently, the store began a special allowing an unlimited number of rentals for \$15.95 per month. How many movies must you rent in a month in order to save money by using the special?
- A 5 or less
B exactly 6
C 6 or more
D 13 or more
6. **SWIMMING POOL** You are draining a swimming pool. The table shows the depth of the water at different times. How long will it take the pool to empty?
- | | | | | |
|------------|----|------|---|-----|
| Time (h) | 0 | 1 | 2 | 3 |
| Depth (ft) | 12 | 10.5 | 9 | 7.5 |
- A 4.8 hours
B 6.7 hours
C 8 hours
D 24 hours
7. **TRIANGLE INEQUALITY** The triangle inequality relationship from geometry states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. If the lengths of the three sides of a triangle are x , $2x$, and 9, which of the following is a possible value of x ?
- A 2
B 3
C 5
D 10
8. **OPEN-ENDED** For a rope trick, a magician cuts a 72 inch piece of rope into three pieces of different lengths. The length of one piece must be the mean of the lengths of the other two pieces, as shown below.
-
- A. Find the length of the second-longest piece.
B. Write an equation that relates a and b . Explain what this equation expresses about the lengths of the ropes.
C. Give a pair of possible lengths for the shortest and longest pieces of rope.

1.6 Solve Linear Inequalities

pp. 41–47

EXAMPLESolve $25 - 3x \leq 10$. Then graph the solution.

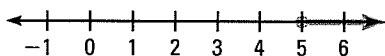
$$25 - 3x \leq 10$$

Write original inequality.

$$-3x \leq -15$$

Subtract 25 from each side.

$$x \geq 5$$

Divide each side by -3 and reverse the inequality.

Graph the solution.

EXERCISES

Solve the inequality. Then graph the solution.

34. $2x - 3 < -1$

35. $7 - 3x \geq -11$

36. $15x + 8 > 9x - 22$

37. $13x + 24 \leq 16 - 3x$

38. $-5 < 10 - x < 5$

39. $-8 \leq 3x + 1 \leq 10$

40. **GEOMETRY** A triangle has sides of lengths 10, $2x$, and $3x$. The sum of the lengths of any two sides is greater than the length of the third side. Write and solve three inequalities to find the possible values of x .

EXAMPLES

1, 2, 3, and 4
on pp. 41–43
for Exs. 34–40

1.7 Solve Absolute Value Equations and Inequalities

pp. 51–58

EXAMPLESolve $|3x - 7| > 2$. Then graph the solution.

$$|3x - 7| > 2$$

Write original inequality.

$$3x - 7 < -2 \quad \text{or} \quad 3x - 7 > 2$$

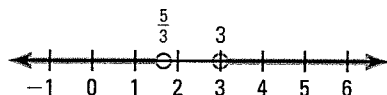
Write equivalent compound inequality.

$$3x < 5 \quad \text{or} \quad 3x > 9$$

Add 7 to each side.

$$x < \frac{5}{3} \quad \text{or} \quad x > 3$$

Divide each side by 3.



Graph the solution.

EXERCISES

Solve the equation. Check for extraneous solutions.

41. $|3p + 2| = 7$

42. $|9q - 5| = 2q$

43. $|8r + 1| = 3r$

Solve the inequality. Then graph the solution.

44. $|x - 5| \geq 1$

45. $|5 - 2y| > 7$

46. $|6z + 5| \leq 25$

47. **VOLLEYBALL** The circumference of a volleyball should be 26 inches, with a tolerance of 0.5 inch. Write and solve an absolute value inequality that describes the acceptable circumferences of a volleyball.

EXAMPLES

2, 3, 4, and 5
on pp. 52–54
for Exs. 41–47

Graph the numbers on a number line.

1. $-2, -\frac{7}{4}, 6.5, \sqrt{30}, \frac{1}{3}$

2. $\frac{9}{2}, 0.8, -5.5, -\sqrt{10}, -\frac{3}{4}$

Use properties and definitions of operations to show that the statement is true. *Justify* each step.

3. $5 + (x - 5) = x$

4. $(3d + 7) - d + 5 = 2d + 12$

Evaluate the expression for the given values of x and y .

5. $4x - 6y$ when $x = 5$ and $y = -3$

6. $3x^2 - 9y$ when $x = 2$ and $y = 4$

Simplify the expression.

7. $5n + 10 - 8n + 6$

8. $10m - 4(3m + 7) + 6m$

9. $11 + q - 3q^2 + 18q^2 - 2$

10. $9t^2 + 14 - 17t + 6t - 8t^2$

11. $5(x - 3y) + 2(4y - x)$

12. $5(2u + 3w) - 2(5u - 7w)$

Solve the equation. Check your solution.

13. $5n + 11 = -9$

14. $6k + 7 = 4 + 12k$

15. $-t - 2 = 9(t - 8)$

Solve the equation for y . Then find the value of y for the given value of x .

16. $12x - 28y = 40; x = 6$

17. $x + 4y = 12; x = 2$

18. $15y + 2xy = -30; x = 5$

Solve the inequality. Then graph the solution.

19. $-5x - 6 < 19$

20. $x + 22 \geq -3x - 10$

21. $5 < 2x + 3 \leq 11$

Solve the equation. Check for extraneous solutions.

22. $|3d - 4| = 14$

23. $|f + 3| = 2f + 4$

24. $|10 - 7g| = 2g$

Solve the inequality. Then graph the solution.

25. $|x - 5| \leq 30$

26. $|3y + 4| > 2$

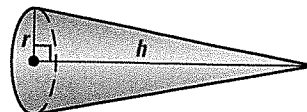
27. $|\frac{2}{3}z - 5| < 5$

28. **WIRELESS NETWORK** To set up a wireless network for Internet access at home, you buy a network router for \$75. The fee for DSL service is \$18 per month. Write an expression for the amount of money you spend in n months. How much money do you spend in 12 months?

29. **CAR REPAIR** The bill for the repair of a car was \$420. The cost of parts was \$240. The cost of labor was \$45 per hour. How many hours did the repair take?

30. **HOUSEHOLD CHORES** You can wash one window in 15 minutes and your sister can wash one window in 20 minutes. How many minutes will it take to wash 12 windows if you work together?

31. **GEOMETRY** The formula $V = \frac{1}{3}\pi r^2 h$ gives the volume V of a cone with height h and base radius r . Solve the formula for h . Then find h when $r = 2$ inches and $V = 45$ cubic inches.



Scoring Rubric

Full Credit

- solution is complete and correct

Partial Credit

- solution is complete but has errors, or
- solution is without error but incomplete

No Credit

- no solution is given, or
- solution makes no sense

OPEN-ENDED QUESTIONS

PROBLEM

A national bank offers a checking account for a fee of \$3.90 per month. The first 10 transactions per month are free, but every additional transaction costs \$.15. A local bank offers a checking account with no monthly fee, but every transaction costs \$.36.

A. When is it less expensive to use the national bank?

B. Explain how you found your answer.

Below are sample solutions to the problem. Read each solution and the comments on the left to see why the sample represents *full credit*, *partial credit*, or *no credit*.

SAMPLE 1: Full credit solution

If you make 10 transactions or fewer per month, the local bank is less expensive because it costs \$3.60 or less, compared to \$3.90 for the national bank. If the number of transactions is more than 10, you can use the following model, where x is the number of transactions per month.

National bank			Local bank	
Cost per transaction over 10	•	Number of transactions over 10	+	Fee
0.15	•	$(x - 10)$	+	3.90

Solve the inequality to find when it is less expensive to use the national bank.

$$0.15(x - 10) + 3.90 < 0.36x$$

$$0.15x - 1.5 + 3.90 < 0.36x$$

$$2.4 < 0.21x$$

$$11.4 < x$$

A noninteger answer does not make sense, so round up. The national bank is less expensive if you make 12 or more transactions per month.

The verbal model explains how the inequality is obtained.

The inequality is solved correctly, step by step.

The answer is correct. An integer makes sense in this context.

The inequality is correct when $x > 10$. The case when $x \leq 10$ is not considered.

The student made an error in the last step. The answer is incorrect.

SAMPLE 2: Partial credit solution

$$0.15(x - 10) + 3.90 < 0.36x$$

$$0.15x - 1.5 + 3.90 < 0.36x$$

$$2.4 < 0.21x$$

$$2.19 < x$$

The national bank is less expensive if you make 3 or more transactions.



SAMPLE 3: Partial credit solution

.....→
Calculations are shown
for 12 transactions only.

Find the cost for each bank when there are 12 transactions.

National bank: $\$3.90 + 2(\$0.15) = \$4.20$ **Local bank:** $12(\$0.36) = \4.32

.....→
The answer is correct,
but it is not justified.

The national bank is less expensive when you make 12 or more transactions.

SAMPLE 4: No credit solution

.....→
The inequality is
incorrect. The national
bank does not charge
\$.15 for every transaction.

$$0.15x + 3.90 < 0.36x$$

$$3.9 < 0.21x$$

$$18.6 < x$$

.....→
The answer is incorrect.

The national bank is less expensive when you make 19 or more transactions.

PRACTICE Apply the Scoring Rubric

Use the rubric on page 66 to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. Explain your reasoning.

PROBLEM You plant a 1.5 foot tall sawtooth oak that grows 3.5 feet per year and a 5 foot tall chestnut oak that grows 2 feet per year.

- When will the sawtooth oak be taller than the chestnut oak?
- Explain your reasoning.

- The sawtooth oak will be taller than the chestnut oak in about 2.33 years.

- Let x be the number of years.

Sawtooth initial height	+	Sawtooth growth	>	Chestnut initial height	+	Chestnut growth
1.5	+	$3.5x$	>	5	+	$2x$

Solve the inequality for x .

$$1.5 + 3.5x > 5 + 2x$$

$$1.5x > 3.5$$

$$x > 2.33$$

The sawtooth oak will be taller than the chestnut oak after about 2.33 years.

- $1.5x + 3.5 > 5x + 2$

$$1.5 > 3.5x$$

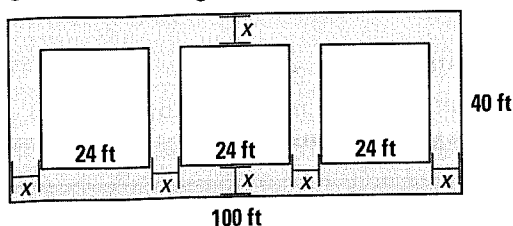
$0.43 > x$, so after 0.43 year, the sawtooth oak will be taller.

OPEN-ENDED

1. A teacher is buying rulers. At an online site, rulers cost \$.89 each plus \$5 for shipping for the entire order. At a store, each ruler costs \$.95.
 - A. Under what conditions is the store less expensive?
 - B. If there is free shipping for online orders over \$50, how does your answer change?

2. Consider the equation $|x + h| = a$.
 - A. For what values of h and a are the solutions of the equation 8 and -12 ?
 - B. Describe the method you used to solve the problem.

3. Consider the diagram below.



- A. Write and solve an equation to find x .
 - B. Then find the area of the shaded region. Explain your reasoning.
4. The table shows the number of goals g and assists a for four players on a girls' varsity soccer team. Each player is assigned a point total given by $p = 2g + a$.

Player	Goals, g	Assists, a
Sandra	2	6
Kim	3	5
Jen	4	1
Melanie	5	2

- A. Which player earned the most points?
 - B. Is it possible for two players to earn the same number of points but have different numbers of goals and assists? Justify your answer.

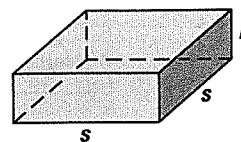
5. A community is having a Taste of the Town event featuring the area's best restaurants. The admission is \$25 in advance and \$35 at the door.

- A. If 220 people pay in advance and the total amount collected is \$7495, how many people pay at the door?
 - B. How much more money is collected in advance than at the door?
 - C. Explain your solution.

6. Jared and 7 of his friends have a bowling party. Jared's parents pay for bowling, shoe rental, and snacks for all 8 children. The prices of these items are shown in the table. Jared's parents want the total cost to be at most \$80.

Bowling	\$2.70 per person, per game
Shoe rental	\$1.25 per pair
Snacks	\$2.80 per person

- A. If each person bowls the same number of games, what are the possible numbers of games each can bowl?
 - B. How does your answer change if Jared's parents decide to pay at most \$100? Explain.
7. The volume V of a rectangular prism with a square base is given by $V = s^2h$ where h is the height and s is the length of one side of the base.



Suppose such a prism has a volume of 1000 cubic centimeters.

- A. Choose three possible values of s and find the corresponding values of h .
 - B. Is there a maximum value that s can have?
 - C. Explain your answer.



OPEN-ENDED

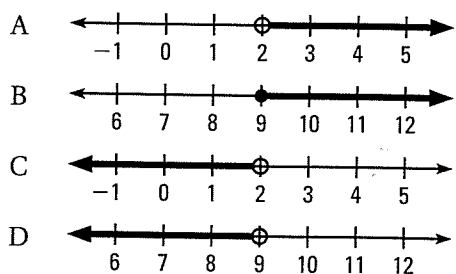
8. The melting points and boiling points of lithium, carbon, nitrogen, oxygen, and magnesium are shown, to the nearest degree.

	Li	C	N	O	Mg
Melting point (°C)	?	3500	?	-218	?
Melting point (°F)	357	?	-346	?	1202
Boiling point (°C)	1347	?	-196	?	1107
Boiling point (°F)	?	8721	?	-297	?

- A. Copy the table. Use the formula $F = \frac{9}{5}C + 32$ to convert the Celsius temperatures in the table to Fahrenheit temperatures. Record the results.
- B. Rewrite the formula so that it gives the Celsius temperature in terms of the Fahrenheit temperature. *Justify* each step.
- C. Use the rewritten formula to convert the Fahrenheit temperatures in the table to Celsius temperatures. Record the results.
9. A baseball pitcher's earned run average (ERA) can be calculated using this formula: $\text{ERA} = 9 \cdot \text{earned runs} \div \text{innings pitched}$.
- A. During one season, Johan Santana gave up 66 earned runs in 228 innings pitched. To the nearest hundredth, what was his ERA?
- B. After pitching 2296 innings, Pedro Martinez had a career ERA of 2.71. Write and solve an equation to find the number of earned runs he allowed in those innings. *Explain* why there are two possible answers.
- C. A pitcher who expects to pitch 200 innings in a season wants his ERA to be less than 4.00. Write and solve an inequality to find the possible numbers of earned runs he can allow. *Explain* how you need to round your answer.

MULTIPLE CHOICE

10. Which graph represents the solution of the inequality $2x - 7 < 11$?



11. Which equation has -5 as a solution?

- A $-3x - 6 = 10$ B $1.5 + 3x = -14.5$
C $5 - x = 10$ D $-9x = -45$

12. What is the solution of the equation $-t + 12 = 5t + 3$?

- A $\frac{4}{9}$ B $\frac{2}{3}$
C $\frac{3}{2}$ D $\frac{9}{4}$

13. What is a solution of the absolute value equation $|x + 5| = 15 - 3x$?

- A -5 B 2.5
C $2.5, 10$ D $-5, -2.5$

14. What is the greatest value of x for which $|2x - 5| \leq 7$?

- A -6 B -1
C 1 D 6

2 Linear Equations and Functions

PA

M11.D.1.1.2

M11.D.3.2.1

M11.D.2.1.2

M11.D.3.2.2

M11.A.2.1.2

M11.E.4.2.1

M11.D.2.1.2

2.1 Represent Relations and Functions

2.2 Find Slope and Rate of Change

2.3 Graph Equations of Lines

2.4 Write Equations of Lines

2.5 Model Direct Variation

2.6 Draw Scatter Plots and Best-Fitting Lines

2.7 Use Absolute Value Functions and Transformations

2.8 Graph Linear Inequalities in Two Variables

Before

In Chapter 1, you learned the following skills, which you'll use in Chapter 2: evaluating algebraic expressions, solving linear equations, and rewriting equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. A **linear equation** in one variable is an equation that can be written in the form $ax + b = c$ where a and b are constants and $a \neq 0$.
2. The **absolute value** of a real number is the distance the number is from 0 on a number line.

SKILLS CHECK

Evaluate the expression for the given value of x . (Review p. 10 for 2.1.)

- | | |
|-------------------------------|---------------------------------|
| 3. $-2(x + 1)$ when $x = -5$ | 4. $11x - 14$ when $x = -3$ |
| 5. $x^2 + x + 1$ when $x = 4$ | 6. $-x^2 - 3x + 7$ when $x = 1$ |

Solve the equation. Check your solution. (Review p. 18 for 2.3.)

- | | | |
|-----------------|--------------------|-----------------------|
| 7. $5x - 2 = 8$ | 8. $-6x - 10 = 20$ | 9. $-x + 9 = 2x - 27$ |
|-----------------|--------------------|-----------------------|

Solve the equation for y . (Review p. 26 for 2.4.)

- | | | |
|-------------------|-------------------|-------------------|
| 10. $2x + 3y = 6$ | 11. $-x - y = 10$ | 12. $x + 4y = -5$ |
|-------------------|-------------------|-------------------|