2 Linear Equations and Functions

Before

In Chapter 1, you learned the following skills, which you'll use in Chapter 2: evaluating algebraic expressions, solving linear equations, and rewriting equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. A **linear equation** in one variable is an equation that can be written in the form _?_ where a and b are constants and $a \neq 0$.
2. The **absolute value** of a real number is the distance the number is from _?_ on a number line.

SKILLS CHECK

Evaluate the expression for the given value of x. ***(Review p. 10 for 2.1.)***

3. $-2(x + 1)$ when $x = -5$
4. $11x - 14$ when $x = -3$
5. $x^2 + x + 1$ when $x = 4$
6. $-x^2 - 3x + 7$ when $x = 1$

Solve the equation. Check your solution. ***(Review p. 18 for 2.3.)***

7. $5x - 2 = 8$
8. $-6x - 10 = 20$
9. $-x + 9 = 2x - 27$

Solve the equation for y. ***(Review p. 26 for 2.4.)***

10. $2x + 3y = 6$
11. $-x - y = 10$
12. $x + 4y = -5$

OPEN-ENDED

8. The melting points and boiling points of lithium, carbon, nitrogen, oxygen, and magnesium are shown, to the nearest degree.

	Li	C	N	O	Mg
Melting point (°C)	?	3500	?	−218	?
Melting point (°F)	357	?	−346	?	1202
Boiling point (°C)	1347	?	−196	?	1107
Boiling point (°F)	?	8721	?	−297	?

A. Copy the table. Use the formula $F = \frac{9}{5}C + 32$ to convert the Celsius temperatures in the table to Fahrenheit temperatures. Record the results.

B. Rewrite the formula so that it gives the Celsius temperature in terms of the Fahrenheit temperature. *Justify* each step.

C. Use the rewritten formula to convert the Fahrenheit temperatures in the table to Celsius temperatures. Record the results.

9. A baseball pitcher's earned run average (ERA) can be calculated using this formula: ERA = 9 • earned runs ÷ innings pitched.

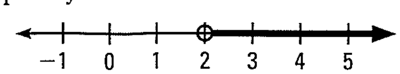
A. During one season, Johan Santana gave up 66 earned runs in 228 innings pitched. To the nearest hundredth, what was his ERA?

B. After pitching 2296 innings, Pedro Martinez had a career ERA of 2.71. Write and solve an equation to find the number of earned runs he allowed in those innings. *Explain* why there are two possible answers.

C. A pitcher who expects to pitch 200 innings in a season wants his ERA to be less than 4.00. Write and solve an inequality to find the possible numbers of earned runs he can allow. *Explain* how you need to round your answer.

MULTIPLE CHOICE

10. Which graph represents the solution of the inequality $2x - 7 < 11$?

A 

B 

C 

D 

11. Which equation has −5 as a solution?

A $-3x - 6 = 10$ B $1.5 + 3x = -14.5$

C $5 - x = 10$ D $-9x = -45$

12. What is the solution of the equation $-t + 12 = 5t + 3$?

A $\frac{4}{9}$ B $\frac{2}{3}$

C $\frac{3}{2}$ D $\frac{9}{4}$

13. What is a solution of the absolute value equation $|x + 5| = 15 - 3x$?

A −5 B 2.5

C 2.5, 10 D −5, −2.5

14. What is the greatest value of x for which $|2x - 5| \le 7$?

A −6 B −1

C 1 D 6

Now

In Chapter 2, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 140. You will also use the key vocabulary listed below.

Big Ideas

1. **Representing relations and functions**
2. **Graphing linear equations and inequalities in two variables**
3. **Writing linear equations and inequalities in two variables**

KEY VOCABULARY

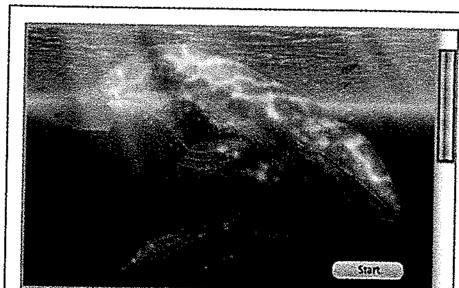
- domain, range, *p. 72*
- function, *p. 73*
- linear function, *p. 75*
- slope, *p. 82*
- rate of change, *p. 85*
- parent function, *p. 89*
- *y*-intercept, *p. 89*
- slope-intercept form, *p. 90*
- *x*-intercept, *p. 91*
- point-slope form, *p. 98*
- direct variation, *p. 107*
- correlation coefficient, *p. 114*
- best-fitting line, *p. 114*
- absolute value function, *p. 123*
- transformation, *p. 123*
- linear inequality in two variables, *p. 132*

Why?

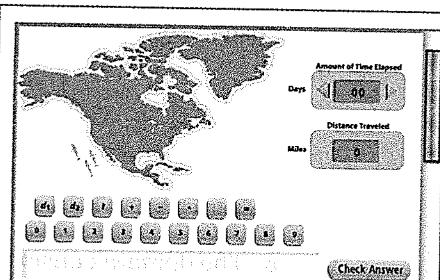
You can use rates of change to find linear models. For example, you can use an average rate of change to model distance traveled as a function of time.

Animated Algebra

The animation illustrated below for Exercise 44 on page 111 helps you answer this question: If a whale migrates at a given rate, how far will it travel in different periods of time?



Gray whales migrate from Mexico's Baja Peninsula to waters near Alaska.



Change the time elapsed to find how far the whales have traveled.

Animated Algebra at classzone.com

Other animations for Chapter 2: pages 73, 86, 90, 95, 98, 102, 107, 115, 133, and 140

2.1 Represent Relations and Functions

PA M11.D.1.1.2 Determine if a relation is a function given a set of points or a graph.

Before You solved linear equations.

Now You will represent relations and graph linear functions.

Why? So you can model changes in elevation, as in Ex. 48.

Key Vocabulary
- relation
- domain
- range
- function
- equation in two variables
- linear function

A **relation** is a *mapping*, or pairing, of input values with output values. The set of input values is the **domain**, and the set of output values is the **range**.

KEY CONCEPT — *For Your Notebook*

Representing Relations

A relation can be represented in the following ways.

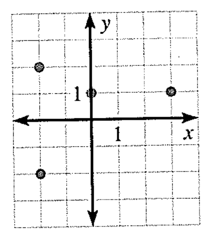
Ordered Pairs

(−2, 2)
(−2, −2)
(0, 1)
(3, 1)

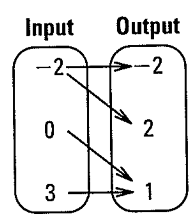
Table

x	y
−2	2
−2	−2
0	1
3	1

Graph



Mapping Diagram



EXAMPLE 1 Represent relations

Consider the relation given by the ordered pairs (−2, −3), (−1, 1), (1, 3), (2, −2), and (3, 1).

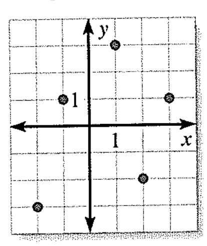
a. Identify the domain and range.

b. Represent the relation using a graph and a mapping diagram.

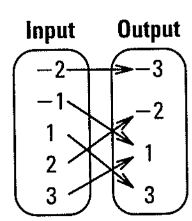
Solution

a. The domain consists of all the x-coordinates: −2, −1, 1, 2, and 3. The range consists of all the y-coordinates: −3, −2, 1, and 3.

b. Graph



Mapping Diagram



REVIEW GRAPHING
For help with plotting points in a coordinate plane, see p. 987.

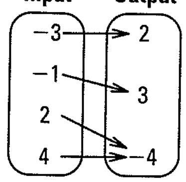
FUNCTIONS A **function** is a relation for which each input has exactly one output. If any input of a relation has more than one output, the relation is *not* a function.

EXAMPLE 2 Identify functions

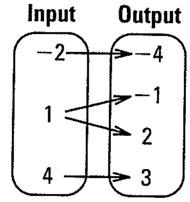
AVOID ERRORS
A relation can map more than one input onto the same output and still be a function.

Tell whether the relation is a function. Explain.

a. Input Output



b. Input Output



Solution

a. The relation *is* a function because each input is mapped onto exactly one output.

b. The relation *is not* a function because the input 1 is mapped onto both −1 and 2.

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. Consider the relation given by the ordered pairs (−4, 3), (−2, 1), (0, 3), (1, −2), and (−2, −4).

a. Identify the domain and range.

b. Represent the relation using a table and a mapping diagram.

2. Tell whether the relation is a function. *Explain.*

x	−2	−1	0	1	3
y	−4	−4	−4	−4	−4

VERTICAL LINE TEST You can use the graph of a relation to determine whether it is a function by applying the *vertical line test.*

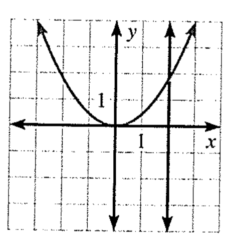
KEY CONCEPT *For Your Notebook*

REVIEW LOGICAL STATEMENTS
For help with "if and only if" statements, see p. 1002.

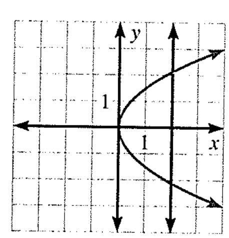
Vertical Line Test

A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

Function

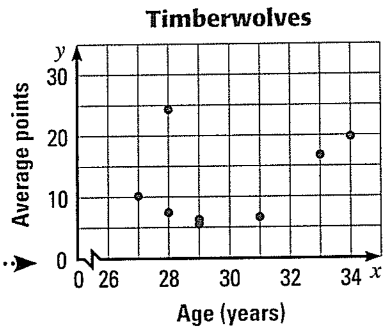


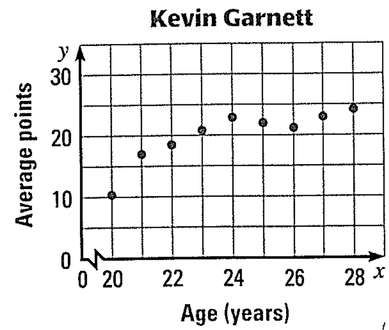
Not a function

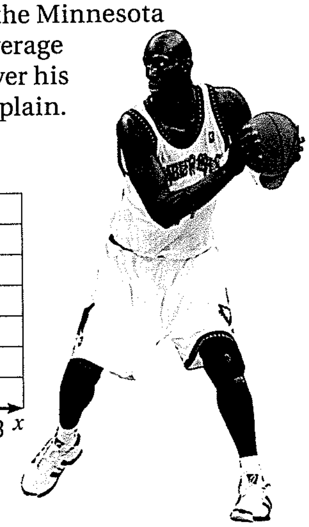


EXAMPLE 3 Use the vertical line test

BASKETBALL The first graph below plots average points per game versus age at the end of the 2003–2004 NBA regular season for the 8 members of the Minnesota Timberwolves with the highest averages. The second graph plots average points per game versus age for one team member, Kevin Garnett, over his first 9 seasons. Are the relations shown by the graphs functions? Explain.







READING GRAPHS
The zigzag symbol on the horizontal axis of each graph indicates that values of x were skipped.

Solution

The team graph *does not* represent a function because vertical lines at $x = 28$ and $x = 29$ each intersect the graph at more than one point. The graph for Kevin Garnett *does* represent a function because no vertical line intersects the graph at more than one point.

✓ GUIDED PRACTICE for Example 3

3. **WHAT IF?** In Example 3, suppose that Kevin Garnett averages 24.2 points per game in his tenth season as he did in his ninth. If the relation given by the second graph is revised to include the tenth season, is the relation still a function? *Explain.*

EQUATIONS IN TWO VARIABLES Many functions can be described by an **equation in two variables**, such as $y = 3x - 5$. The input variable (in this case, x) is called the **independent variable**. The output variable (in this case, y) is called the **dependent variable** because its value *depends* on the value of the input variable.

An ordered pair (x, y) is a **solution** of an equation in two variables if substituting x and y in the equation produces a true statement. For example, $(2, 1)$ is a solution of $y = 3x - 5$ because $1 = 3(2) - 5$ is true. The **graph** of an equation in two variables is the set of all points (x, y) that represent solutions of the equation.

KEY CONCEPT *For Your Notebook*

Graphing Equations in Two Variables

To graph an equation in two variables, follow these steps:

STEP 1 **Construct** a table of values.

STEP 2 **Plot** enough points from the table to recognize a pattern.

STEP 3 **Connect** the points with a line or a curve.

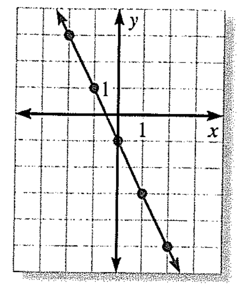
EXAMPLE 4 Graph an equation in two variables

Graph the equation $y = -2x - 1$.

Solution

STEP 1 **Construct** a table of values.

x	-2	-1	0	1	2
y	3	1	-1	-3	-5



STEP 2 **Plot** the points. Notice that they all lie on a line.

STEP 3 **Connect** the points with a line.

READING
The parentheses in $f(x)$ do not indicate multiplication. The symbol $f(x)$ does not mean "f times x."

LINEAR FUNCTIONS The function $y = -2x - 1$ in Example 4 is a **linear function** because it can be written in the form $y = mx + b$ where m and b are constants. The graph of a linear function is a line. By renaming y as $f(x)$, you can write $y = mx + b$ using **function notation.**

$y = mx + b$ **Linear function in *x*-*y* notation**

$f(x) = mx + b$ **Linear function in function notation**

The notation $f(x)$ is read "the value of f at x," or simply "f of x," and identifies x as the independent variable. The domain consists of all values of x for which $f(x)$ is defined. The range consists of all values of $f(x)$ where x is in the domain of f.

EXAMPLE 5 Classify and evaluate functions

Tell whether the function is linear. Then evaluate the function when $x = -4$.

a. $f(x) = -x^2 - 2x + 7$

b. $g(x) = 5x + 8$

Solution

a. The function f is not linear because it has an x^2-term.

$f(\mathbf{x}) = -\mathbf{x}^2 - 2\mathbf{x} + 7$ **Write function.**

$f(\mathbf{-4}) = -(\mathbf{-4})^2 - 2(\mathbf{-4}) + 7$ **Substitute -4 for x.**

$= -1$ **Simplify.**

REPRESENT FUNCTIONS
Letters other than f, such as g or h, can also name functions.

b. The function g is linear because it has the form $g(x) = mx + b$.

$g(\mathbf{x}) = 5\mathbf{x} + 8$ **Write function.**

$g(\mathbf{-4}) = 5(\mathbf{-4}) + 8$ **Substitute -4 for x.**

$= -12$ **Simplify.**

✓ GUIDED PRACTICE for Examples 4 and 5

4. Graph the equation $y = 3x - 2$.

Tell whether the function is linear. Then evaluate the function when $x = -2$.

5. $f(x) = x - 1 - x^3$

6. $g(x) = -4 - 2x$

DOMAINS IN REAL LIFE In Example 5, the domain of each function is all real numbers because there is an output for every real number x. In real life, you may need to restrict the domain so that it is reasonable in the given situation.

EXAMPLE 6 Use a function in real life

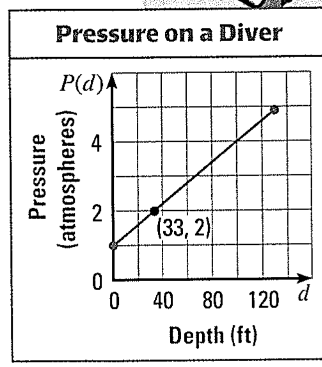
DIVING A diver using a Diver Propulsion Vehicle (DPV) descends to a depth of 130 feet. The pressure P (in atmospheres) on the diver is given by $P(d) = 1 + 0.03d$ where d is the depth (in feet). Graph the function, and determine a reasonable domain and range. What is the pressure on the diver at a depth of 33 feet?

Solution

The graph of $P(d)$ is shown. Because the depth varies from 0 feet to 130 feet, a reasonable domain is $0 \le d \le 130$.

The minimum value of $P(d)$ is $P(0) = 1$, and the maximum value of $P(d)$ is $P(130) = 4.9$. So, a reasonable range is $1 \le P(d) \le 4.9$.

▶ At a depth of 33 feet, the pressure on the diver is $P(33) = 1 + 0.03(33) \approx 2$ atmospheres, which you can verify from the graph.



✓ **GUIDED PRACTICE** for Example 6

7. **OCEAN EXPLORATION** In 1960, the deep-sea vessel *Trieste* descended to an estimated depth of 35,800 feet. Determine a reasonable domain and range of the function $P(d)$ in Example 6 for this trip.

2.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS2 for Exs. 7, 17, and 45
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 20, 24, 40, 46, and 49

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In the equation $y = x + 5$, x is the _?_ variable and y is the _?_ variable.

2. ★ **WRITING** *Describe* how to find the domain and range of a relation given by a set of ordered pairs.

EXAMPLE 1 on p. 72 for Exs. 3–9

REPRESENTING RELATIONS Identify the domain and range of the given relation. Then represent the relation using a graph and a mapping diagram.

3. $(-2, 3), (1, 2), (3, -1), (-4, -3)$
4. $(5, -2), (-3, -2), (3, 3), (-1, -1)$
5. $(6, -1), (-2, -3), (1, 8), (-2, 5)$
6. $(-7, 4), (2, -5), (1, -2), (-3, 6)$
7. $(5, 20), (10, 20), (15, 30), (20, 30)$
8. $(4, -2), (4, 2), (16, -4), (16, 4)$

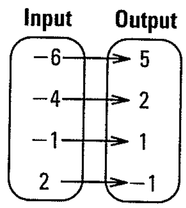
9. ★ **MULTIPLE CHOICE** What is the domain of the relation given by the ordered pairs (−4, 2), (−1, −3), (1, 4), (1, −3), and (2, 1)?

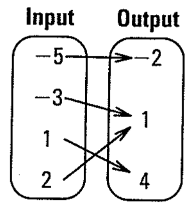
Ⓐ −3, 1, 2, and 4 Ⓑ −4, −1, 1, and 2

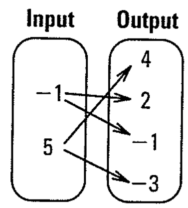
Ⓒ −4, −3, −1, and 2 Ⓓ −4, −3, −1, 1, 2, and 4

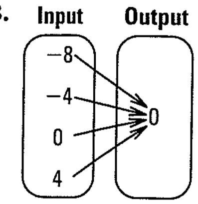
EXAMPLE 2
on p. 73
for Exs. 10–20

IDENTIFYING FUNCTIONS **Tell whether the relation is a function. *Explain.***

10.


11.


12.


13.


ERROR ANALYSIS ***Describe*** **and correct the error in the student's work.**

14.

The relation given by the ordered pairs (−4, 2), (−1, 5), (3, 6), and (7, 2) is not a function because the inputs −4 and 7 are both mapped to the output 2.

15.

x	0	1	2	1	0
y	5	6	7	8	9

The relation given by the table is a function because there is only one value of x for each value of y.

IDENTIFYING FUNCTIONS **Tell whether the relation is a function. *Explain.***

16. (3, −2), (0, 1), (1, 0), (−2, −1), (2, −1)

17. (2, −5), (−2, 5), (−1, 4), (−2, 0), (3, −4)

18. (0, 1), (1, 0), (2, 3), (3, 2), (4, 4)

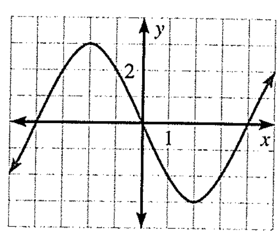
19. (−1, −1), (2, 5), (4, 8), (−5, −9), (−1, −5)

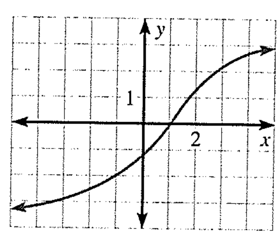
20. ★ **MULTIPLE CHOICE** The relation given by the ordered pairs (−6, 3), (−2, 4), (1, 5), and (4, 0) is a function. Which ordered pair can be included with this relation to form a new relation that is also a function?

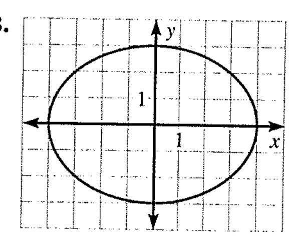
Ⓐ (1, −5) Ⓑ (6, 3) Ⓒ (−2, 19) Ⓓ (4, 4)

EXAMPLE 3
on p. 74
for Exs. 21–23

VERTICAL LINE TEST **Use the vertical line test to tell whether the relation is a function.**

21.


22.


23.


24. ★ **SHORT RESPONSE** *Explain* why a relation is not a function if a vertical line intersects the graph of the relation more than once.

EXAMPLE 4
on p. 75
for Exs. 25–33

GRAPHING EQUATIONS **Graph the equation.**

25. $y = x + 2$ 26. $y = -x + 5$ 27. $y = 3x + 1$

28. $y = 5x - 3$ 29. $y = 2x - 7$ 30. $y = -3x + 2$

31. $y = -2x$ 32. $y = \frac{1}{2}x + 2$ 33. $y = -\frac{3}{4}x - 1$

EXAMPLE 5
on p. 75
for Exs. 34–39

EVALUATING FUNCTIONS Tell whether the function is linear. Then evaluate the function for the given value of x.

34. $f(x) = x + 15; f(8)$

35. $f(x) = x^2 + 1; f(-3)$

36. $f(x) = |x| + 10; f(-4)$

37. $f(x) = 6; f(2)$

38. $g(x) = x^3 - 2x^2 + 5x - 8; g(-5)$

39. $h(x) = 7 - \frac{2}{3}x; h(15)$

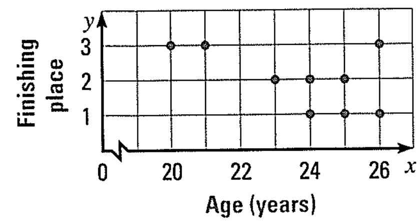
40. ★ **SHORT RESPONSE** Which, if any, of the relations described by the equations $y = |x|$, $x = |y|$, and $|y| = |x|$ represent functions? *Explain.*

41. **CHALLENGE** Let f be a function such that $f(a + b) = f(a) + f(b)$ for all real numbers a and b. Show that $f(2a) = 2 \cdot f(a)$ and that $f(0) = 0$.

PROBLEM SOLVING

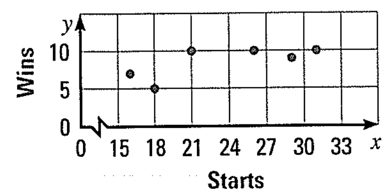
EXAMPLE 3
on p. 74
for Exs. 42–43

42. **BICYCLING** The graph shows the ages of the top three finishers in the Mt. Washington Auto Road Bicycle Hillclimb each year from 2002 through 2004. Do the ordered pairs (age, finishing place) represent a function? *Explain.*



@HomeTutor for problem solving help at classzone.com

43. **BASEBALL** The graph shows the number of games started and the number of wins for each starting pitcher on a baseball team during a regular season. Do the ordered pairs (starts, wins) represent a function? *Explain.*



@HomeTutor for problem solving help at classzone.com

44. **GEOMETRY** The volume V of a cube with edge length s is given by the function $V(s) = s^3$. Find $V(4)$. *Explain* what $V(4)$ represents.

45. **GEOMETRY** The volume V of a sphere with radius r is given by the function $V(r) = \frac{4}{3}\pi r^3$. Find $V(6)$. *Explain* what $V(6)$ represents.

EXAMPLE 6
on p. 76
for Exs. 46–48

46. ★ **SHORT RESPONSE** For the period 1999–2004, the average number of acres w (in thousands), used to grow watermelons in the United States can be modeled by the function $w(t) = -6.26t + 172$ where t is the number of years since 1999. Determine a reasonable domain and range for $w(t)$. *Explain* the meaning of the range.

47. **MULTI-STEP PROBLEM** Anthropologists can estimate a person's height from the length of certain bones. The height h (in inches) of an adult human female can be modeled by the function $h(\ell) = 1.95\ell + 28.7$ where ℓ is the length (in inches) of the femur, or thigh bone. The function is valid for femur lengths between 15 inches and 24 inches, inclusive.

a. Graph the function, and determine a reasonable domain and range.

b. Suppose a female's femur is 15.5 inches long. About how tall was she?

c. If an anthropologist estimates a female's height as 5 feet 11 inches, about how long is her femur?

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

48. **MOUNTAIN CLIMBING** A climber on Mount Rainier in Washington hikes from an elevation of 5400 feet above sea level to Camp Muir, which has an elevation of 10,100 feet. The elevation h (in feet) as the climber ascends can be modeled by $h(t) = 1000t + 5400$ where t is the time (in hours). Graph the function, and determine a reasonable domain and range. What is the climber's elevation after hiking 3.5 hours?

49. ★ **EXTENDED RESPONSE** The table shows the populations of several states and their electoral votes in the 2004 and 2008 U.S. presidential elections. The figures are based on U.S. census data for the year 2000.
   a. Identify the domain and range of the relation given by the ordered pairs (p, v).
   b. Is the relation from part (a) a function? *Explain.*
   c. Is the relation given by the ordered pairs (v, p) a function? *Explain.*

State	Population (millions), p	Electoral votes, v
California	33.87	55
Florida	15.98	27
Illinois	12.42	21
New York	18.98	31
Ohio	11.35	20
Pennsylvania	12.28	21
Texas	20.85	34

50. **CHALLENGE** The table shows ground shipping charges for an online retail store.
   a. Is the shipping cost a function of the merchandise cost? *Explain.*
   b. Is the merchandise cost a function of the shipping cost? *Explain.*

Merchandise cost	Shipping cost
$.01–$30.00	$4.50
$30.01–$60.00	$7.25
$60.01–$100.00	$9.50
Over $100.00	$12.50

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

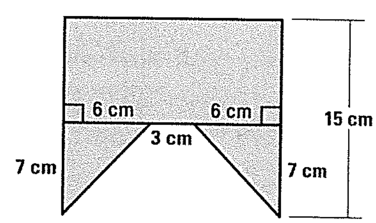
51. Kate is studying a bacteria culture in biology class. The table shows the number of bacteria, b, in the culture after t hours. How many bacteria are there after 10 hours?

Time (hours), t	0	1	2	3	4	5
Bacteria (billions), b	1	2	4	8	16	32

Ⓐ 64 billion Ⓑ 128 billion Ⓒ 256 billion Ⓓ 1024 billion

52. What is the area of the composite figure?

Ⓐ 138 cm^2 Ⓑ 141 cm^2
Ⓒ 162 cm^2 Ⓓ 210 cm^2



Extension
Use after Lesson 2.1

Use Discrete and Continuous Functions

GOAL Graph and classify discrete and continuous functions.

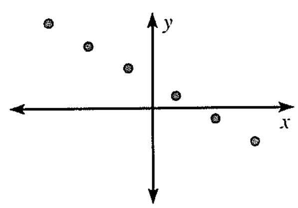
Key Vocabulary
- discrete function
- continuous function

The graph of a function may consist of *discrete*, or separate and unconnected, points in a plane. The graph of a function may also be a *continuous*, or unbroken, line or curve or part of a line or curve.

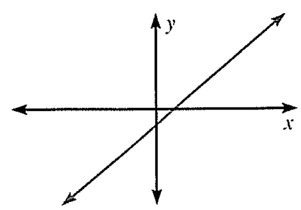
KEY CONCEPT *For Your Notebook*

Discrete and Continuous Functions

The graph of a **discrete function** consists of separate points.



The graph of a **continuous function** is unbroken.



EXAMPLE 1 Graph and classify functions

Graph the function $f(x) = 0.5x + 1$ for the given domain. Classify the function as *discrete* or *continuous* for the domain. Then identify the range.

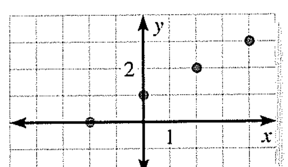
a. Domain: $x = -2, 0, 2, 4$

b. Domain: $x \geq -3$

Solution

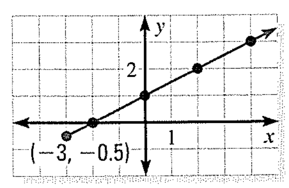
a. Make a table using the x-values in the domain.

x	−2	0	2	4
y	0	1	2	3



The graph consists of separate points, so the function is discrete. Its range is 0, 1, 2, 3.

b. Note that $f(x)$ is a linear function defined for $x \geq -3$, and that $f(-3) = -0.5$. So, the graph is the ray with endpoint $(-3, -0.5)$ that passes through all the points from the table in part (a).



The graph is unbroken, so the function is continuous. Its range is $y \geq -0.5$.

EXAMPLE 2 Graph and classify real-world functions

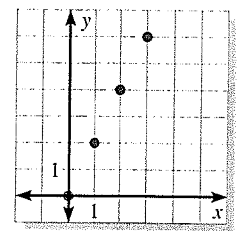
Write and graph the function described. Determine the domain and range. Then tell whether the function is *discrete* or *continuous*.

a. A student group is selling chocolate bars for \$2 each. The function $f(x)$ gives the amount of money collected after selling x chocolate bars.

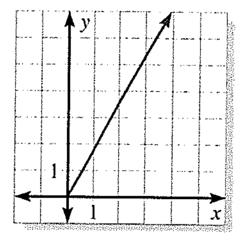
b. A low-flow shower head releases 1.8 gallons of water per minute. The function $V(x)$ gives the volume of water released after x minutes.

Solution

a. The function is $f(x) = 2x$. The first four points of the graph of $f(x)$ are shown. Only whole chocolate bars can be sold, so the domain is the set of whole numbers 0, 1, 2, 3, ... From the graph, you can see that the range is 0, 2, 4, 6, ... The graph consists of separate points, so the function is discrete.



b. The function is $V(x) = 1.8x$. You can run the shower any nonnegative amount of time, so the domain is $x \geq 0$. From the graph, you can see that the range is $y \geq 0$. The graph is unbroken, so the function is continuous.



PRACTICE

EXAMPLE 1 on p. 80 for Exs. 1–4

Graph the function for the given domain. Classify the function as *discrete* or *continuous*. Then identify the range of the function.

1. $y = 2x + 3$; domain: $-2, -1, 0, 1, 2$
2. $f(x) = 0.5x - 4$; domain: $-4, -2, 0, 2, 4$
3. $y = -3x + 9$; domain: $x < 5$
4. $f(x) = \frac{1}{3}x + 6$; domain: $x \geq -6$

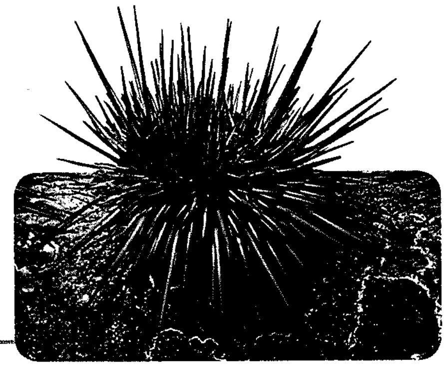
EXAMPLE 2 on p. 81 for Exs. 5–8

Write and graph the function described. Determine the domain and range. Then tell whether the function is *discrete* or *continuous*.

5. Amanda walks at an average speed of 3.5 miles per hour. The function $d(x)$ gives the distance (in miles) Amanda walks in x hours.
6. A token to ride a subway costs \$1.25. The function $s(x)$ gives the cost of riding the subway x times.
7. A family has 3 gallons of milk delivered every Thursday. The function $m(x)$ gives the total amount of milk that is delivered to the family after x weeks.
8. Steel cable that is $\frac{3}{8}$ inch in diameter weighs 0.24 pound per foot. The function $w(x)$ gives the weight of x feet of steel cable.
9. On a number line, the *signed distance* from a number a to a number b is given by $b - a$. The function $d(x)$ gives the signed distance from 3 to any number x.

2.2 Find Slope and Rate of Change

PA M11.D.3.2.1 Apply the formula for the slope of a line to solve problems.



Before	You graphed linear functions.
Now	You will find slopes of lines and rates of change.
Why?	So you can model growth rates, as in Ex. 46.

Key Vocabulary
- **slope**
- **parallel**
- **perpendicular**
- **rate of change**
- **reciprocal,** *p. 4*

KEY CONCEPT — *For Your Notebook*

Slope of a Line

Words

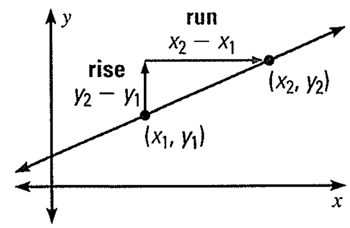
The **slope** m of a nonvertical line is the ratio of vertical change (the *rise*) to horizontal change (the *run*).

Algebra

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

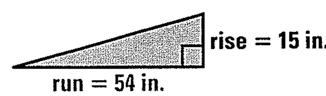
slope

Graph



EXAMPLE 1 Find slope in real life

SKATEBOARDING A skateboard ramp has a rise of 15 inches and a run of 54 inches. What is its slope?



Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{15}{54} = \frac{5}{18}$$

▸ The slope of the ramp is $\frac{5}{18}$.

★ EXAMPLE 2 Standardized Test Practice

What is the slope of the line passing through the points (−1, 3) and (2, −1)?

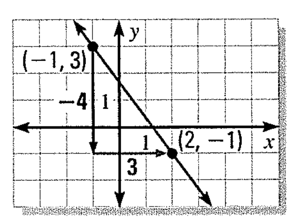
Ⓐ $-\frac{4}{3}$ Ⓑ $-\frac{3}{4}$ Ⓒ $\frac{3}{4}$ Ⓓ $\frac{4}{3}$

Solution

Let $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{2 - (-1)} = -\frac{4}{3}$$

▸ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ



AVOID ERRORS
When calculating slope, be sure to subtract the *x*- and *y*-coordinates in a consistent order.

✓ GUIDED PRACTICE for Examples 1 and 2

1. **WHAT IF?** In Example 1, suppose that the rise of the ramp is changed to 12 inches without changing the run. What is the slope of the ramp?

2. What is the slope of the line passing through the points $(-4, 9)$ and $(-8, 3)$?

   (A) $-\frac{2}{3}$ (B) $-\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

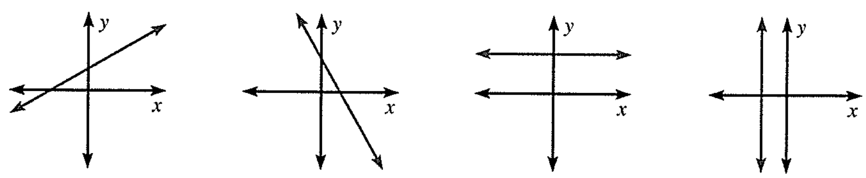
Find the slope of the line passing through the given points.

3. $(0, 3), (4, 8)$ 4. $(-5, 1), (5, -4)$ 5. $(-3, -2), (6, 1)$ 6. $(7, 3), (-1, 7)$

KEY CONCEPT — *For Your Notebook*

Classification of Lines by Slope

The slope of a line indicates whether the line rises from left to right, falls from left to right, is horizontal, or is vertical.



Positive slope Rises from left to right   **Negative slope** Falls from left to right   **Zero slope** Horizontal   **Undefined slope** Vertical

READING
A vertical line has "undefined slope" because for any two points, the slope formula's denominator becomes 0, and division by 0 is undefined.

EXAMPLE 3 Classify lines using slope

Without graphing, tell whether the line through the given points *rises, falls, is horizontal,* or *is vertical.*

a. $(-5, 1), (3, 1)$ b. $(-6, 0), (2, -4)$ c. $(-1, 3), (5, 8)$ d. $(4, 6), (4, -1)$

Solution

a. $m = \frac{1-1}{3-(-5)} = \frac{0}{8} = 0$ Because $m = 0$, the line is horizontal.

b. $m = \frac{-4-0}{2-(-6)} = \frac{-4}{8} = -\frac{1}{2}$ Because $m < 0$, the line falls.

c. $m = \frac{8-3}{5-(-1)} = \frac{5}{6}$ Because $m > 0$, the line rises.

d. $m = \frac{-1-6}{4-4} = \frac{-7}{0}$ Because m is undefined, the line is vertical.

✓ GUIDED PRACTICE for Example 3

Without graphing, tell whether the line through the given points *rises, falls, is horizontal,* or *is vertical.*

7. $(-4, 3), (2, -6)$ 8. $(7, 1), (7, -1)$ 9. $(3, -2), (5, -2)$ 10. $(5, 6), (1, -4)$

PARALLEL AND PERPENDICULAR LINES Recall that two lines in a plane are **parallel** if they do not intersect. Two lines in a plane are **perpendicular** if they intersect to form a right angle.

Slope can be used to determine whether two different nonvertical lines are parallel or perpendicular.

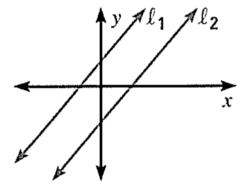
KEY CONCEPT *For Your Notebook*

Slopes of Parallel and Perpendicular Lines

Consider two different nonvertical lines ℓ_1 and ℓ_2 with slopes m_1 and m_2.

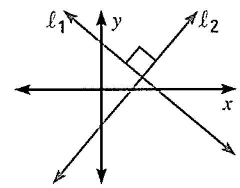
Parallel Lines The lines are parallel if and only if they have the same slope.

$$m_1 = m_2$$



Perpendicular Lines The lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$m_1 = -\frac{1}{m_2}, \text{ or } m_1m_2 = -1$$



EXAMPLE 4 Classify parallel and perpendicular lines

Tell whether the lines are *parallel, perpendicular,* or *neither.*

a. Line 1: through (−2, 2) and (0, −1)
Line 2: through (−4, −1) and (2, 3)

b. Line 1: through (1, 2) and (4, −3)
Line 2: through (−4, 3) and (−1, −2)

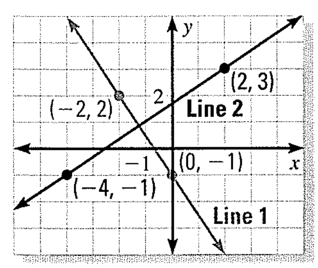
Solution

a. Find the slopes of the two lines.

$$m_1 = \frac{-1-2}{0-(-2)} = \frac{-3}{2} = -\frac{3}{2}$$

$$m_2 = \frac{3-(-1)}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$$

▶ Because $m_1m_2 = -\frac{3}{2} \cdot \frac{2}{3} = -1$, m_1 and m_2 are negative reciprocals of each other. So, the lines are perpendicular.

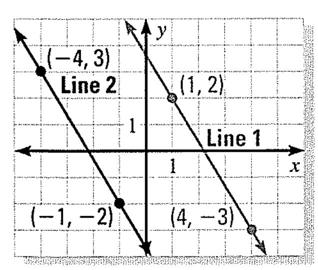


b. Find the slopes of the two lines.

$$m_1 = \frac{-3-2}{4-1} = \frac{-5}{3} = -\frac{5}{3}$$

$$m_2 = \frac{-2-3}{-1-(-4)} = \frac{-5}{3} = -\frac{5}{3}$$

▶ Because $m_1 = m_2$ (and the lines are different), you can conclude that the lines are parallel.



✓ GUIDED PRACTICE for Example 4

Tell whether the lines are *parallel, perpendicular,* or *neither.*

11. Line 1: through $(-2, 8)$ and $(2, -4)$
Line 2: through $(-5, 1)$ and $(-2, 2)$

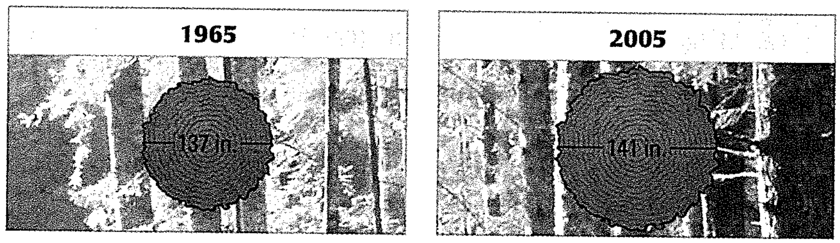
12. Line 1: through $(-4, -2)$ and $(1, 7)$
Line 2: through $(-1, -4)$ and $(3, 5)$

REVIEW RATES
Remember that a *rate* is a ratio of two quantities that have *different* units.

RATE OF CHANGE Slope can be used to represent an average **rate of change**, or how much one quantity changes, on average, relative to the change in another quantity. A slope that is a real-life rate of change involves units of measure such as miles per hour or degrees per day.

EXAMPLE 5 Solve a multi-step problem

FORESTRY Use the diagram, which illustrates the growth of a giant sequoia, to find the average rate of change in the diameter of the sequoia over time. Then predict the sequoia's diameter in 2065.



Solution

STEP 1 **Find** the average rate of change.

$$\text{Average rate of change} = \frac{\text{Change in diameter}}{\text{Change in time}}$$

$$= \frac{141 \text{ in.} - 137 \text{ in.}}{2005 - 1965}$$

$$= \frac{4 \text{ in.}}{40 \text{ years}}$$

$$= \mathbf{0.1} \text{ inch per year}$$

STEP 2 **Predict** the diameter of the sequoia in 2065.

Find the number of years from 2005 to 2065. Multiply this number by the average rate of change to find the total increase in diameter during the period 2005–2065.

Number of years $= 2065 - 2005 = \mathbf{60}$

Increase in diameter $= (\mathbf{60} \text{ years})(\mathbf{0.1} \text{ inch/year}) = 6$ inches

▶ In 2065, the diameter of the sequoia will be about $141 + 6 = 147$ inches.

✓ GUIDED PRACTICE for Example 5

13. WHAT IF? In Example 5, suppose that the diameter of the sequoia is 248 inches in 1965 and 251 inches in 2005. Find the average rate of change in the diameter, and use it to predict the diameter in 2105.

2.2 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS2 for Exs. 9, 19, and 45
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 35, 36, 44, 45, and 48

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The __?__ of a nonvertical line is the ratio of vertical change to horizontal change.

2. ★ **WRITING** How can you use slope to decide whether two nonvertical lines are parallel? whether two nonvertical lines are perpendicular?

EXAMPLES 2 and 3 on pp. 82–83 for Exs. 3–17

FINDING SLOPE Find the slope of the line passing through the given points. Then tell whether the line *rises, falls, is horizontal,* or *is vertical.*

3. $(2, -4), (4, -1)$
4. $(8, 9), (-4, 3)$
5. $(5, 1), (8, -4)$
6. $(-3, -2), (3, -2)$
7. $(-1, 4), (1, -4)$
8. $(-6, 5), (-6, -5)$
9. $(-5, -4), (-1, 3)$
10. $(-3, 6), (-7, 3)$
11. $(4, 4), (4, 9)$
12. $(5, 5), (7, 3)$
13. $(0, -3), (4, -3)$
14. $(1, -1), (-1, -4)$

Animated Algebra at classzone.com

ERROR ANALYSIS ***Describe*** **and correct the error in finding the slope of the line passing through the given points.**

15. $(-4, -3), (2, -1)$

$$m = \frac{-1-(-3)}{-4-2} = -\frac{1}{3}$$

16. $(-1, 4), (5, 1)$

$$m = \frac{5-(-1)}{1-4} = -2$$

17. ★ **MULTIPLE CHOICE** What is true about the line through $(2, -4)$ and $(5, 1)$?

Ⓐ It rises from left to right.
Ⓑ It falls from left to right.
Ⓒ It is horizontal.
Ⓓ It is vertical.

EXAMPLE 4 on p. 84 for Exs. 18–23

CLASSIFYING LINES Tell whether the lines are *parallel, perpendicular,* or *neither.*

18. Line 1: through $(3, -1)$ and $(6, -4)$
Line 2: through $(-4, 5)$ and $(-2, 7)$

19. Line 1: through $(1, 5)$ and $(3, -2)$
Line 2: through $(-3, 2)$ and $(4, 0)$

20. Line 1: through $(-1, 4)$ and $(2, 5)$
Line 2: through $(-6, 2)$ and $(0, 4)$

21. Line 1: through $(5, 8)$ and $(7, 2)$
Line 2: through $(-7, -2)$ and $(-4, -1)$

22. Line 1: through $(-3, 2)$ and $(5, 0)$
Line 2: through $(-1, -4)$ and $(3, -3)$

23. Line 1: through $(1, -4)$ and $(4, -2)$
Line 2: through $(8, 1)$ and $(14, 5)$

EXAMPLE 5 on p. 85 for Exs. 24–27

AVERAGE RATE OF CHANGE Find the average rate of change in y relative to x for the ordered pairs. Include units of measure in your answer.

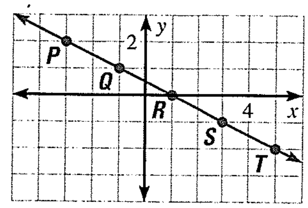
24. $(2, 12), (5, 30)$ — x is measured in hours and y is measured in dollars
25. $(0, 11), (3, 50)$ — x is measured in gallons and y is measured in miles
26. $(3, 10), (5, 18)$ — x is measured in seconds and y is measured in feet
27. $(1, 8), (7, 20)$ — x is measured in seconds and y is measured in meters

28. **REASONING** The Key Concept box on page 84 states that lines ℓ_1 and ℓ_2 must be nonvertical. *Explain* why this condition is necessary.

FINDING SLOPE Find the slope of the line passing through the given points.

29. $\left(-1, \frac{3}{2}\right), \left(0, \frac{7}{2}\right)$
30. $\left(-\frac{3}{4}, -2\right), \left(\frac{5}{4}, -3\right)$
31. $\left(-\frac{1}{2}, \frac{5}{2}\right), \left(\frac{5}{2}, 3\right)$
32. $(-4.2, 0.1), (-3.2, 0.1)$
33. $(-0.3, 2.2), (1.7, -0.8)$
34. $(3.5, -2), (4.5, 0.5)$

35. ★ **SHORT RESPONSE** Does it make a difference which two points on a line you choose when finding the slope? Does it make a difference which point is (x_1, y_1) and which point is (x_2, y_2) in the formula for slope? Support your answers using three different pairs of points on the line shown.



36. ★ **OPEN-ENDED MATH** Find two additional points on the line that passes through (0, 3) and has a slope of −4.

CHALLENGE Find the value of *k* so that the line through the given points has the given slope. Check your solution.

37. $(2, -3)$ and $(k, 7)$; $m = -2$
38. $(0, k)$ and $(3, 4)$; $m = 1$
39. $(-4, 2k)$ and $(k, -5)$; $m = -1$
40. $(-2, k)$ and $(2k, 2)$; $m = -0.25$

PROBLEM SOLVING

EXAMPLE 1 on p. 82 for Exs. 41–44

41. **ESCALATORS** An escalator in an airport rises 28 feet over a horizontal distance of 48 feet. What is the slope of the escalator?

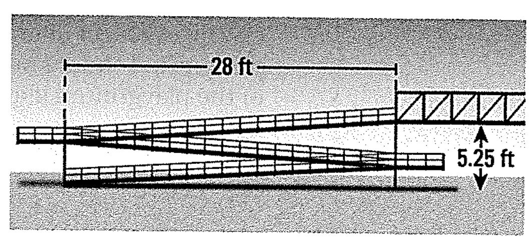
@HomeTutor for problem solving help at classzone.com

42. **INCLINE RAILWAY** The Duquesne Incline, a cable car railway, rises 400 feet over a horizontal distance of 685 feet on its ascent to an overlook of Pittsburgh, Pennsylvania. What is the slope of the incline?

@HomeTutor for problem solving help at classzone.com

43. **ROAD GRADE** A road's *grade* is its slope expressed as a percent. A road rises 195 feet over a horizontal distance of 3000 feet. What is the grade of the road?

44. ★ **SHORT RESPONSE** The diagram shows a three-section ramp to a bridge. For a person walking up the ramp, each section has the same positive slope. *Compare* this slope with the slope that a single-section ramp would have if it rose directly to the bridge from the same starting point. *Explain* the benefits of a three-section ramp in this situation.



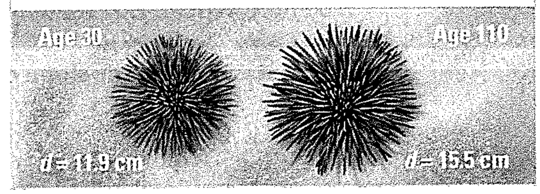
EXAMPLE 5 on p. 85 for Exs. 45–46

45. ★ **MULTIPLE CHOICE** Over a 30 day period, the amount of propane in a tank that stores propane for heating a home decreases from 400 gallons to 214 gallons. What is the average rate of change in the amount of propane?

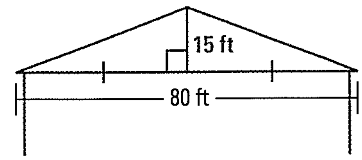
(A) −6.2 gallons per day
(B) −6 gallons per day
(C) −0.16 gallon per day
(D) 6 gallons per day

46. BIOLOGY A red sea urchin grows its entire life, which can last 200 years. The diagram gives information about the growth in the diameter d of one red sea urchin. What is the average growth rate of this urchin over the given period?

Growth of Red Sea Urchin



47. MULTI-STEP PROBLEM A building code requires the minimum slope, or *pitch*, of an asphalt-shingle roof to be a rise of 3 feet for each 12 feet of run. The asphalt-shingle roof of an apartment building has the dimensions shown.



a. **Calculate** What is the slope of the roof?

b. **Interpret** Does the roof satisfy the building code?

c. **Reasoning** If you answered "no" to part (b), by how much must the rise be increased to satisfy the code? If you answered "yes," by how much does the rise exceed the code minimum?

48. ★ EXTENDED RESPONSE Plans for a new water slide in an amusement park call for the slide to descend from a platform 80 feet tall. The slide will drop 1 foot for every 3 feet of horizontal distance.

a. What horizontal distance do you cover when descending the slide?

b. Use the Pythagorean theorem to find the length of the slide.

c. Engineers decide to shorten the slide horizontally by 5 feet to allow for a wider walkway at the slide's base. The plans for the platform remain unchanged. How will this affect the slope of the slide? *Explain.*

49. CHALLENGE A car travels 36 miles per gallon of gasoline in highway driving and 24 miles per gallon in city driving. If you drive the car equal distances on the highway and in the city, how many miles per gallon can you expect to average? (*Hint:* The average fuel efficiency for all the driving is the total distance traveled divided by the total amount of gasoline used.)

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

50. A city is building a rectangular playground in a community park. The city has 560 feet of fencing to enclose the playground. The length of the playground should be 40 feet longer than the width. What is the length of the playground if all of the fencing is used?

Ⓐ 120 ft　　Ⓑ 160 ft

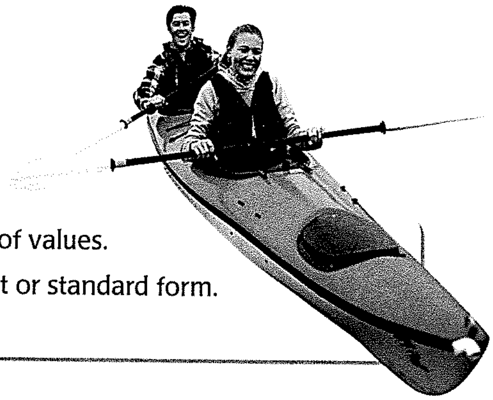
Ⓒ 200 ft　　Ⓓ 300 ft

51. A computer technician charges $185 for parts needed to fix a computer and $45 for each hour that he works on the computer. Which equation best represents the relationship between the number of hours, h, the technician works on the computer and the total charges, c?

Ⓐ $c = 45 - 185h$　　Ⓑ $c = 45 + 185h$

Ⓒ $c = 185 - 45h$　　Ⓓ $c = 185 + 45h$

2.3 Graph Equations of Lines



PA M11.D.2.1.2 Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

Before You graphed linear equations by making tables of values.

Now You will graph linear equations in slope-intercept or standard form.

Why? So you can model motion, as in Ex. 64.

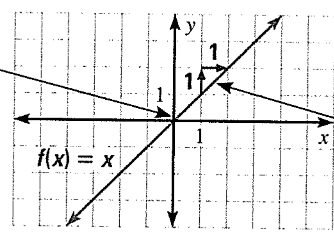
Key Vocabulary
- **parent function**
- ***y*-intercept**
- **slope-intercept form**
- **standard form of a linear equation**
- ***x*-intercept**

A *family* of functions is a group of functions with shared characteristics. The **parent function** is the most basic function in a family.

KEY CONCEPT *For Your Notebook*

Parent Function for Linear Functions

The parent function for the family of all linear functions is $f(x) = x$. The graph of $f(x) = x$ is shown.



In general, a ***y*-intercept** of a graph is the y-coordinate of a point where the graph intersects the y-axis.

DEFINE *Y*-INTERCEPT
A y-intercept is sometimes defined as a *point* where a graph intersects the y-axis. Using this definition, the y-intercept of the line $f(x) = x$ is (0, 0), not 0.

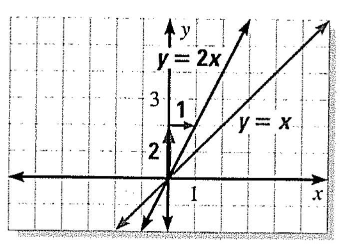
EXAMPLE 1 Graph linear functions

Graph the equation. Compare the graph with the graph of $y = x$.

a. $y = 2x$ **b.** $y = x + 3$

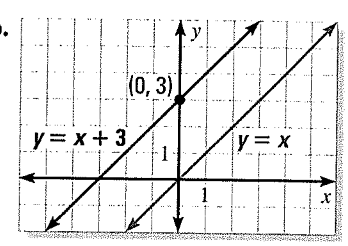
Solution

a.



The graphs of $y = 2x$ and $y = x$ both have a y-intercept of 0, but the graph of $y = 2x$ has a slope of 2 instead of 1.

b.



The graphs of $y = x + 3$ and $y = x$ both have a slope of 1, but the graph of $y = x + 3$ has a y-intercept of 3 instead of 0.

SLOPE-INTERCEPT FORM If you write the equations in Example 1 as $y = 2x + 0$ and $y = 1x + 3$, you can see that the x-coefficients, 2 and 1, are the slopes of the lines, while the constant terms, 0 and 3, are the y-intercepts. In general, a line with equation $y = mx + b$ has slope m and y-intercept b. The equation $y = mx + b$ is said to be in **slope-intercept form**.

KEY CONCEPT — *For Your Notebook*

Using Slope-Intercept Form to Graph an Equation

STEP 1 **Write** the equation in slope-intercept form by solving for y.

STEP 2 **Identify** the y-intercept b and use it to plot the point $(0, b)$ where the line crosses the y-axis.

STEP 3 **Identify** the slope m and use it to plot a second point on the line.

STEP 4 **Draw** a line through the two points.

EXAMPLE 2 Graph an equation in slope-intercept form

Graph $y = -\frac{2}{3}x - 1$.

Solution

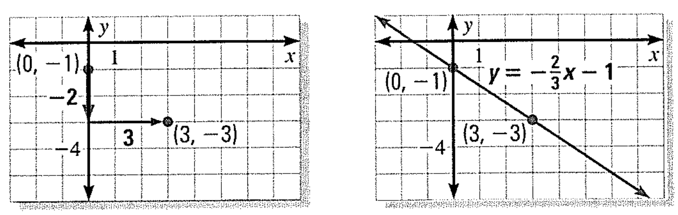
STEP 1 The equation is already in slope-intercept form.

STEP 2 **Identify** the y-intercept. The y-intercept is -1, so plot the point $(0, -1)$ where the line crosses the y-axis.

STEP 3 **Identify** the slope. The slope is $-\frac{2}{3}$, or $\frac{-2}{3}$, so plot a second point on the line by starting at $(0, -1)$ and then moving down 2 units and right 3 units. The second point is $(3, -3)$.

STEP 4 **Draw** a line through the two points.

ANOTHER WAY
Because $-\frac{2}{3} = \frac{2}{-3}$, you could also plot a second point by moving up 2 units and left 3 units.



Animated Algebra at classzone.com

GUIDED PRACTICE for Examples 1 and 2

Graph the equation. *Compare* the graph with the graph of $y = x$.

1. $y = -2x$
2. $y = x - 2$
3. $y = 4x$

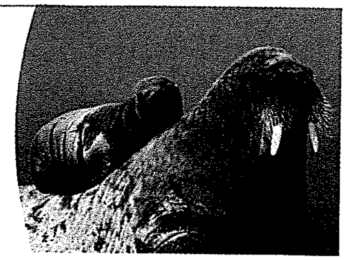
Graph the equation.

4. $y = -x + 2$
5. $y = \frac{2}{5}x + 4$
6. $y = \frac{1}{2}x - 3$
7. $y = 5 + x$
8. $f(x) = 1 - 3x$
9. $f(x) = 10 - x$

REAL-LIFE PROBLEMS In a real-life context, a line's slope can represent an average rate of change. The y-intercept in a real-life context is often an initial value.

EXAMPLE 3 Solve a multi-step problem

BIOLOGY The body length y (in inches) of a walrus calf can be modeled by $y = 5x + 42$ where x is the calf's age (in months).



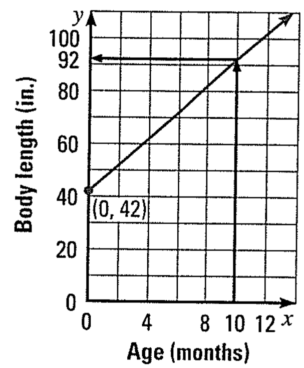
- Graph the equation.
- Describe what the slope and y-intercept represent in this situation.
- Use the graph to estimate the body length of a calf that is 10 months old.

Solution

STEP 1 **Graph** the equation.

STEP 2 **Interpret** the slope and y-intercept. The slope, 5, represents the calf's rate of growth in inches per month. The y-intercept, 42, represents a newborn calf's body length in inches.

STEP 3 **Estimate** the body length of the calf at age 10 months by starting at 10 on the x-axis and moving up until you reach the graph. Then move left to the y-axis. At age 10 months, the body length of the calf is about 92 inches.



ANOTHER WAY
You can check the result you obtained from the graph by substituting 10 for x in $y = 5x + 42$ and simplifying.

GUIDED PRACTICE for Example 3

10. WHAT IF? In Example 3, suppose that the body length of a fast-growing calf is modeled by $y = 6x + 48$. Repeat the steps of the example for the new model.

DEFINE X-INTERCEPT
An x-intercept is sometimes defined as a *point* where a graph intersects the x-axis, not the x-coordinate of such a point.

STANDARD FORM The **standard form** of a linear equation is $Ax + By = C$ where A and B are not both zero. You can graph an equation in standard form by identifying and plotting the x- and y-intercepts. An **x-intercept** is the x-coordinate of a point where a graph intersects the x-axis.

KEY CONCEPT *For Your Notebook*

Using Standard Form to Graph an Equation

STEP 1 **Write** the equation in standard form.

STEP 2 **Identify** the x-intercept by letting $y = 0$ and solving for x. Use the x-intercept to plot the point where the line crosses the x-axis.

STEP 3 **Identify** the y-intercept by letting $x = 0$ and solving for y. Use the y-intercept to plot the point where the line crosses the y-axis.

STEP 4 **Draw** a line through the two points.

EXAMPLE 4 Graph an equation in standard form

Graph $5x + 2y = 10$.

ANOTHER WAY

You can also graph $5x + 2y = 10$ by first solving for y to obtain $y = -\frac{5}{2}x + 5$ and then using the procedure for graphing an equation in slope-intercept form.

Solution

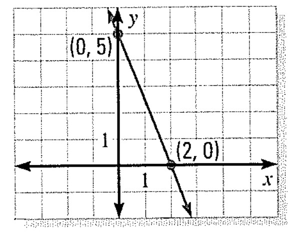
STEP 1 The equation is already in standard form.

STEP 2 **Identify** the x-intercept.

$5x + 2(\mathbf{0}) = 10$ **Let $y = 0$.**

$x = 2$ **Solve for x.**

The x-intercept is 2. So, plot the point (2, 0).



STEP 3 **Identify** the y-intercept.

$5(\mathbf{0}) + 2y = 10$ **Let $x = 0$.**

$y = 5$ **Solve for y.**

The y-intercept is 5. So, plot the point (0, 5).

STEP 4 **Draw** a line through the two points.

HORIZONTAL AND VERTICAL LINES The equation of a vertical line cannot be written in slope-intercept form because the slope is not defined. However, every linear equation—even that of a vertical line—can be written in standard form.

KEY CONCEPT *For Your Notebook*

Horizontal and Vertical Lines

Horizontal Lines The graph of $y = c$ is the horizontal line through $(0, c)$.

Vertical Lines The graph of $x = c$ is the vertical line through $(c, 0)$.

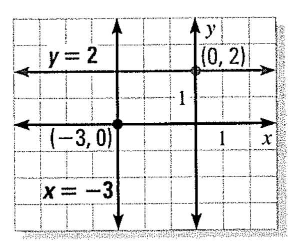
EXAMPLE 5 Graph horizontal and vertical lines

Graph (a) $y = 2$ and (b) $x = -3$.

Solution

a. The graph of $y = 2$ is the horizontal line that passes through the point (0, 2). Notice that every point on the line has a y-coordinate of 2.

b. The graph of $x = -3$ is the vertical line that passes through the point (−3, 0). Notice that every point on the line has an x-coordinate of −3.



GUIDED PRACTICE for Examples 4 and 5

Graph the equation.

11. $2x + 5y = 10$ **12.** $3x - 2y = 12$ **13.** $x = 1$ **14.** $y = -4$

2.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS3 for Exs. 15, 37, and 61

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 30, 55, 56, 63, and 68

◆ = **MULTIPLE REPRESENTATIONS** Ex. 67

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The linear equation $y = 2x + 5$ is written in _?_ form.

2. ★ **WRITING** *Describe* how to graph an equation of the form $Ax + By = C$.

EXAMPLE 1 on p. 89 for Exs. 3–8

GRAPHING LINEAR FUNCTIONS **Graph the equation.** ***Compare*** **the graph with the graph of** $y = x$.

3. $y = 3x$
4. $y = -x$
5. $y = x + 5$
6. $y = x - 2$
7. $y = 2x - 1$
8. $y = -3x + 2$

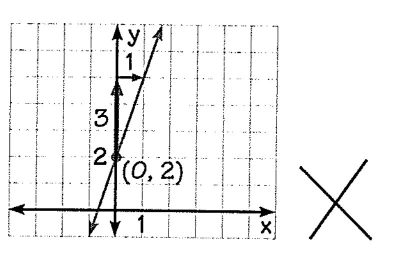
EXAMPLE 2 on p. 90 for Exs. 9–22

SLOPE-INTERCEPT FORM **Graph the equation.**

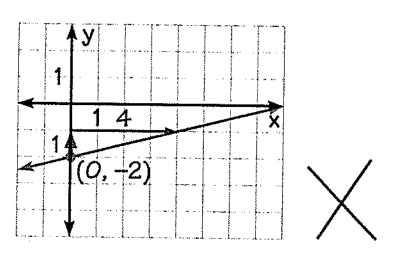
9. $y = -x - 3$
10. $y = x - 6$
11. $y = 2x + 6$
12. $y = 3x - 4$
13. $y = 4x - 1$
14. $y = \frac{2}{3}x - 2$
15. $f(x) = -\frac{1}{2}x - 1$
16. $f(x) = -\frac{5}{4}x + 1$
17. $f(x) = \frac{3}{2}x - 3$
18. $f(x) = \frac{5}{3}x + 4$
19. $f(x) = -1.5x + 2$
20. $f(x) = 3x - 1.5$

ERROR ANALYSIS ***Describe*** **and correct the error in graphing the equation.**

21. $y = 2x + 3$



22. $y = 4x - 2$



23. ★ **MULTIPLE CHOICE** What is the slope-intercept form of $4x - 3y = 18$?

Ⓐ $y = \frac{3}{4}x - 6$ Ⓑ $y = -\frac{3}{4}x - 6$ Ⓒ $y = \frac{4}{3}x - 6$ Ⓓ $y = -\frac{4}{3}x + 6$

EXAMPLES 4 and 5 on p. 92 for Exs. 24–42

FINDING INTERCEPTS **Find the** x**- and** y**-intercepts of the line with the given equation.**

24. $x - y = 4$
25. $x + 5y = -15$
26. $3x - 4y = -12$
27. $2x - y = 10$
28. $4x - 5y = 20$
29. $-6x + 8y = -36$

30. ★ **MULTIPLE CHOICE** What is the x-intercept of the graph of $5x - 6y = 30$?

Ⓐ -5 Ⓑ $\frac{5}{6}$ Ⓒ 6 Ⓓ 30

STANDARD FORM **Graph the equation. Label any intercepts.**

31. $x + 4y = 8$
32. $2x - 6y = -12$
33. $x = 4$
34. $y = -2$
35. $5x - y = 3$
36. $3x + 4y = 12$
37. $-5x + 10y = 20$
38. $-x - y = 6$
39. $y = 1.5$
40. $2.5x - 5y = -15$
41. $x = -\frac{5}{2}$
42. $\frac{1}{2}x + 2y = -2$

CHOOSING A METHOD **Graph the equation using any method.**

43. $6y = 3x + 6$
44. $-3 + x = 0$
45. $y + 7 = -2x$
46. $4y = 16$
47. $8y = -2x + 20$
48. $4x = -\frac{1}{2}y - 1$
49. $-4x = 8y + 12$
50. $3.5x = 10.5$
51. $y - 5.5x = 6$
52. $14 - 3x = 7y$
53. $2y - 5 = 0$
54. $5y = 7.5 - 2.5x$

55. ★ **OPEN-ENDED MATH** Write equations of two lines, one with an x-intercept but no y-intercept and one with a y-intercept but no x-intercept.

56. ★ **SHORT RESPONSE** Sketch $y = mx$ for several values of m, both positive and negative. *Describe* the relationship between m and the steepness of the line.

57. **REASONING** Consider the graph of $Ax + By = C$ where $B \neq 0$. What are the slope and y-intercept in terms of A, B, and C?

58. **CHALLENGE** Prove that the slope of the line $y = mx + b$ is m. (*Hint:* First find two points on the line by choosing convenient values of x.)

PROBLEM SOLVING

EXAMPLE 3 on p. 91 for Exs. 59–62

59. **FITNESS** The total cost y (in dollars) of a gym membership after x months is given by $y = 45x + 75$. Graph the equation. What is the total cost of the membership after 9 months?

@HomeTutor for problem solving help at classzone.com

60. **CAMPING** Your annual membership fee to a nature society lets you camp at several campgrounds. Your total annual cost y (in dollars) to use the campgrounds is given by $y = 5x + 35$ where x is the number of nights you camp. Graph the equation. What do the slope and y-intercept represent?

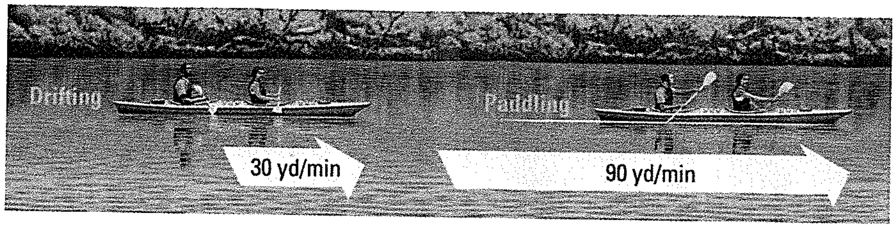
@HomeTutor for problem solving help at classzone.com

61. **SPORTS** Bowling alleys often charge a fixed fee to rent shoes and then charge for each game you bowl. The function $C(g) = 3g + 1.5$ gives the total cost C (in dollars) to bowl g games. Graph the function. What is the cost to rent shoes? What is the cost per game?

62. **PHONE CARDS** You purchase a 300 minute phone card. The function $M(w) = -30w + 300$ models the number M of minutes that remain on the card after w weeks. *Describe* how to determine a reasonable domain and range. Graph the function. How many minutes per week do you use the card?

63. ★ **SHORT RESPONSE** You receive a \$30 gift card to a shop that sells fruit smoothies for \$3. If you graph an equation of the line that represents the money y remaining on the card after you buy x smoothies, what will the y-intercept be? Will the line rise or fall from left to right? *Explain.*

64. **MULTI-STEP PROBLEM** You and a friend kayak 1800 yards down a river. You drift with the current partway at 30 yards per minute and paddle partway at 90 yards per minute. The trip is modeled by $30x + 90y = 1800$ where x is the drifting time and y is the paddling time (both in minutes).



a. Graph the equation, and determine a reasonable domain and range. What do the x- and y-intercepts represent?

b. If you paddle for 5 minutes, what is the total trip time?

c. If you paddle and drift equal amounts of time, what is the total trip time?

65. **VOLUNTEERING** You participate in a 14 mile run/walk for charity. You run partway at 6 miles per hour and walk partway at 3.5 miles per hour. A model for this situation is $6r + 3.5w = 14$ where r is the time you run and w is the time you walk (both in hours). Graph the equation. Give three possible combinations of running and walking times.

66. **TICKETS** An honor society has \$150 to buy science museum and art museum tickets for student awards. The numbers of tickets that can be bought are given by $5s + 7a = 150$ where s is the number of science museum tickets (at \$5 each) and a is the number of art museum tickets (at \$7 each). Graph the equation. Give two possible combinations of tickets that use all \$150.

67. ❖ **MULTIPLE REPRESENTATIONS** A hot air balloon is initially 200 feet above the ground. The burners are then turned on, causing the balloon to ascend at a rate of 150 feet per minute.

a. **Making a Table** Make a table showing the height h (in feet) of the balloon t minutes after the burners are turned on where $0 \le t \le 5$.

b. **Drawing a Graph** Plot the points from the table in part (a). Draw a line through the points for the domain $0 \le t \le 5$.

c. **Writing an Equation** The balloon's height is its initial height plus the product of the ascent rate and time. Write an equation representing this.

AnimatedAlgebra at classzone.com

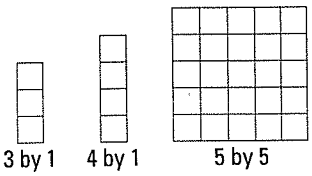
68. ★ **EXTENDED RESPONSE** You and a friend are each typing your research papers on computers. The function $y = 1400 - 50x$ models the number y of words you have left to type after x minutes. For your friend, $y = 1200 - 50x$ models the number y of words left to type after x minutes.

a. Graph the two equations in the same coordinate plane. *Describe* how the graphs are related geometrically.

b. What do the x-intercepts, y-intercepts, and slopes represent?

c. Who will finish first? *Explain.*

69. **CHALLENGE** You want to cover a five-by-five grid completely with x three-by-one rectangles and y four-by-one rectangles that do not overlap or extend beyond the grid.



a. *Explain* why x and y must be whole numbers that satisfy the equation $3x + 4y = 25$.

b. Find all solutions (x, y) of the equation in part (a) such that x and y are whole numbers.

c. Do all the solutions from part (b) represent combinations of rectangles that can actually cover the grid? Use diagrams to support your answer.

PA PENNSYLVANIA MIXED REVIEW

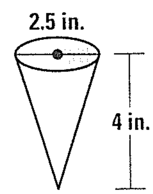
TEST PRACTICE at classzone.com

70. In isosceles triangle ABC, the interior angle A measures 110°. The measures of all three interior angles of triangle ABC are—

Ⓐ 110°, 110°, and 140° Ⓑ 110°, 110°, and 110°

Ⓒ 110°, 40°, and 30° Ⓓ 110°, 35°, and 35°

71. A paper cup is shaped like the cone shown. What is the approximate volume of this paper cup?

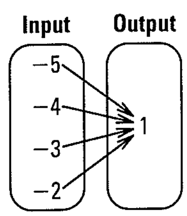


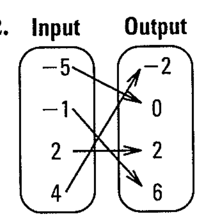
Ⓐ 6.5 in.^{3} Ⓑ 10.5 in.^{3}

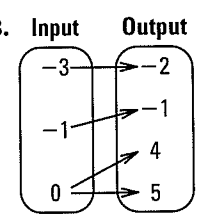
Ⓒ 26.2 in.^{3} Ⓓ 41.9 in.^{3}

QUIZ for Lessons 2.1–2.3

Tell whether the relation is a function. ***Explain.*** *(p. 72)*

1. 

2. 

3. 

Tell whether the lines are ***parallel, perpendicular,*** **or** ***neither.*** *(p. 82)*

4. Line 1: through (−3, −7) and (1, 9)
Line 2: through (−1, −4) and (0, −2)

5. Line 1: through (2, 7) and (−1, −2)
Line 2: through (3, −6) and (−6, −3)

Graph the equation. *(p. 89)*

6. $y = -5x + 3$

7. $x = 10$

8. $4x + 3y = -24$

9. **ROWING SPEED** In 1999, Tori Murden became the first woman to row across the Atlantic Ocean. She rowed a total of 3333 miles during her crossing. The distance d rowed (in miles) can be modeled by $d = 41t$ where t represents the time rowed (in days) at an average rate of 41 miles per day. Graph the function, and determine a reasonable domain and range. Then estimate how long it took Tori Murden to row 1000 miles. *(p. 72)*

EXTRA PRACTICE for Lesson 2.3, p. 1011 **ONLINE QUIZ** at classzone.com

Graphing Calculator ACTIVITY *Use after Lesson 2.3*

@HomeTutor
classzone.com
Keystrokes

2.3 Graph Equations

QUESTION **How can you use a graphing calculator to graph an equation?**

You can use a graphing calculator to graph equations in two variables. On most calculators, you must first write the equation in the form $y = f(x)$.

EXAMPLE **Graph a linear equation**

Graph the equation $x + 4y = 8$.

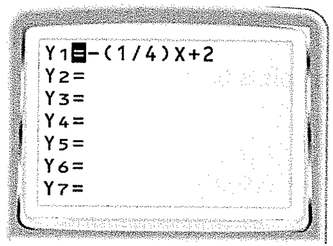
STEP 1 ***Solve for y***

First, solve the equation for y so that it can be entered into the calculator.

$$x + 4y = 8$$
$$4y = -x + 8$$
$$y = -\frac{1}{4}x + 2$$

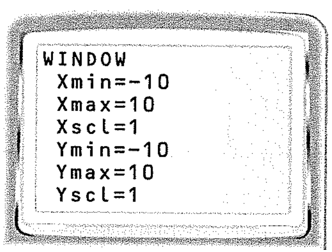
STEP 2 ***Enter equation***

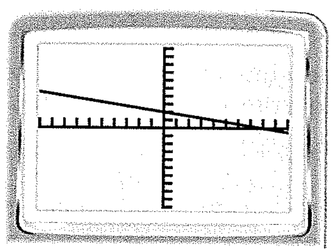
For fractional coefficients, use parentheses. So, enter the equation as $y = -(1/4)x + 2$.



STEP 3 ***Set viewing window and graph***

Enter minimum and maximum x- and y-values and x- and y-scales. The viewing window should show the intercepts. The *standard viewing window* settings and the corresponding graph are shown below.





PRACTICE

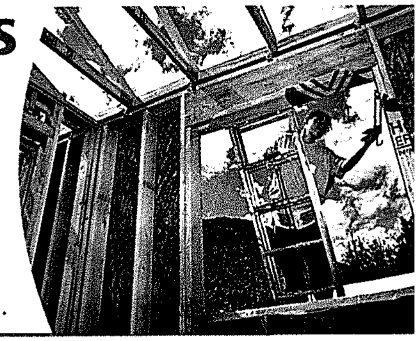
Graph the equation in a graphing calculator's standard viewing window.

1. $y + 14 = 17 - 2x$ **2.** $3x - y = 4$ **3.** $3x - 6y = -18$

Graph the equation using a graphing calculator. Use a viewing window that shows the x- and y-intercepts.

4. $8x = 5y + 16$ **5.** $4x = 25y - 240$ **6.** $1.25x + 4.2y = 28.7$

2.4 Write Equations of Lines



PA M11.D.3.2.2 Given the graph of the line, 2 points on the line, or the slope and a point on a line, write... the linear equation in point-slope...

Before You graphed linear equations.

Now You will write linear equations.

Why? So you can model a steady increase or decrease, as in Ex. 51.

Key Vocabulary
- point-slope form

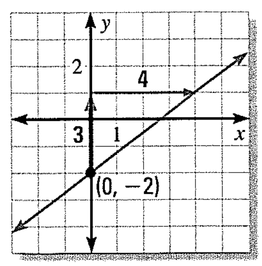
KEY CONCEPT *For Your Notebook*

Writing an Equation of a Line

Given slope m and y-intercept b	Use slope-intercept form: $y = mx + b$
Given slope m and a point (x_1, y_1)	Use **point-slope form**: $y - y_1 = m(x - x_1)$
Given points (x_1, y_1) and (x_2, y_2)	First use the slope formula to find m. Then use point-slope form with either given point.

EXAMPLE 1 Write an equation given the slope and y-intercept

Write an equation of the line shown.



Solution

From the graph, you can see that the slope is $m = \frac{3}{4}$ and the y-intercept is $b = -2$. Use slope-intercept form to write an equation of the line.

$y = mx + b$ — **Use slope-intercept form.**

$y = \frac{3}{4}x + (-2)$ — **Substitute $\frac{3}{4}$ for m and -2 for b.**

$y = \frac{3}{4}x - 2$ — **Simplify.**

Animated Algebra at classzone.com

✓ **GUIDED PRACTICE** for Example 1

Write an equation of the line that has the given slope and y-intercept.

1. $m = 3, b = 1$ **2.** $m = -2, b = -4$ **3.** $m = -\frac{3}{4}, b = \frac{7}{2}$

EXAMPLE 2 Write an equation given the slope and a point

Write an equation of the line that passes through (5, 4) and has a slope of −3.

Solution

Because you know the slope and a point on the line, use point-slope form to write an equation of the line. Let $(x_1, y_1) = (5, 4)$ and $m = -3$.

$y - y_1 = m(x - x_1)$	**Use point-slope form.**
$y - 4 = -3(x - 5)$	**Substitute for m, x_1, and y_1.**
$y - 4 = -3x + 15$	**Distributive property**
$y = -3x + 19$	**Write in slope-intercept form.**

SIMPLIFY EQUATIONS

In this book, equations written in point-slope form will be simplified to slope-intercept form.

EXAMPLE 3 Write equations of parallel or perpendicular lines

Write an equation of the line that passes through (−2, 3) and is (a) parallel to, and (b) perpendicular to, the line $y = -4x + 1$.

Solution

a. The given line has a slope of $m_1 = -4$. So, a line parallel to it has a slope of $m_2 = m_1 = -4$. You know the slope and a point on the line, so use the point-slope form with $(x_1, y_1) = (-2, 3)$ to write an equation of the line.

$y - y_1 = m_2(x - x_1)$	**Use point-slope form.**
$y - 3 = -4(x - (-2))$	**Substitute for m_2, x_1, and y_1.**
$y - 3 = -4(x + 2)$	**Simplify.**
$y - 3 = -4x - 8$	**Distributive property**
$y = -4x - 5$	**Write in slope-intercept form.**

b. A line perpendicular to a line with slope $m_1 = -4$ has a slope of $m_2 = -\frac{1}{m_1} = \frac{1}{4}$. Use point-slope form with $(x_1, y_1) = (-2, 3)$.

$y - y_1 = m_2(x - x_1)$	**Use point-slope form.**
$y - 3 = \frac{1}{4}(x - (-2))$	**Substitute for m_2, x_1, and y_1.**
$y - 3 = \frac{1}{4}(x + 2)$	**Simplify.**
$y - 3 = \frac{1}{4}x + \frac{1}{2}$	**Distributive property**
$y = \frac{1}{4}x + \frac{7}{2}$	**Write in slope-intercept form.**

✓ GUIDED PRACTICE for Examples 2 and 3

4. Write an equation of the line that passes through (−1, 6) and has a slope of 4.
5. Write an equation of the line that passes through (4, −2) and is **(a)** parallel to, and **(b)** perpendicular to, the line $y = 3x - 1$.

EXAMPLE 4 Write an equation given two points

Write an equation of the line that passes through (5, −2) and (2, 10).

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 105 for the **Problem Solving Workshop**.

Solution

The line passes through $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (2, 10)$. Find its slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - (-2)}{2 - 5} = \frac{12}{-3} = -4$$

You know the slope and a point on the line, so use point-slope form with either given point to write an equation of the line. Choose $(x_1, y_1) = (2, 10)$.

$y - y_1 = m(x - x_1)$ **Use point-slope form.**

$y - \mathbf{10} = \mathbf{-4}(x - \mathbf{2})$ **Substitute for m, x_1, and y_1.**

$y - 10 = -4x + 8$ **Distributive property**

$y = -4x + 18$ **Write in slope-intercept form.**

EXAMPLE 5 Write a model using slope-intercept form

SPORTS In the school year ending in 1993, 2.00 million females participated in U.S. high school sports. By 2003, the number had increased to 2.86 million. Write a linear equation that models female sports participation.

Solution

STEP 1 **Define** the variables. Let x represent the time (in years) since 1993 and let y represent the number of participants (in millions).

STEP 2 **Identify** the initial value and rate of change. The initial value is 2.00. The rate of change is the slope m.

AVOID ERRORS
Because time is defined in years since 1993 in Step 1, 1993 corresponds to $x_1 = 0$ and 2003 corresponds to $x_2 = 10$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.86 - 2.00}{10 - 0} = \frac{0.86}{10} = 0.086$$

Use $(x_1, y_1) = (0, 2.00)$ and $(x_2, y_2) = (10, 2.86)$.

STEP 3 **Write** a verbal model. Then write a linear equation.

Participants (millions)	=	Initial number	+	Rate of change	·	Years since 1993
y	=	**2.00**	+	**0.086**	·	x

▸ In slope-intercept form, a linear model is $y = 0.086x + 2.00$.

✓ GUIDED PRACTICE for Examples 4 and 5

Write an equation of the line that passes through the given points.

6. (−2, 5), (4, −7)
7. (6, 1), (−3, −8)
8. (−1, 2), (10, 0)

9. **SPORTS** In Example 5, the corresponding data for males are 3.42 million participants in 1993 and 3.99 million participants in 2003. Write a linear equation that models male participation in U.S. high school sports.

EXAMPLE 6 Write a model using standard form

ONLINE MUSIC You have \$30 to spend on downloading songs for your digital music player. Company A charges \$.79 per song, and company B charges \$.99 per song. Write an equation that models this situation.

Solution

Write a verbal model. Then write an equation.

Company A song price (dollars/song)	•	Songs from company A (songs)	+	Company B song price (dollars/song)	•	Songs from company B (songs)	=	Your budget (dollars)
↓		↓		↓		↓		↓
0.79	•	x	+	**0.99**	•	y	=	**30**

▸ An equation for this situation is $0.79x + 0.99y = 30$.

✓ GUIDED PRACTICE for Example 6

10. **WHAT IF?** In Example 6, suppose that company A charges \$.69 per song and company B charges \$.89 per song. Write an equation that models this situation.

2.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS3 for Exs. 15, 35, and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 26, 39, 47, and 53

◆ = **MULTIPLE REPRESENTATIONS** Ex. 57

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The linear equation $6x + 8y = 72$ is written in _?_ form.

2. ★ **WRITING** Given two points on a line, explain how you can use point-slope form to write an equation of the line.

EXAMPLE 1 on p. 98 for Exs. 3–8

SLOPE-INTERCEPT FORM Write an equation of the line that has the given slope and y-intercept.

3. $m = 0, b = 2$
4. $m = 3, b = -4$
5. $m = 6, b = 0$
6. $m = \frac{2}{3}, b = 4$
7. $m = -\frac{5}{4}, b = 7$
8. $m = -5, b = -1$

EXAMPLE 2 on p. 99 for Exs. 9–19

POINT-SLOPE FORM Write an equation of the line that passes through the given point and has the given slope.

9. $(0, -2), m = 4$
10. $(3, -1), m = -3$
11. $(-4, 3), m = 2$
12. $(-5, -6), m = 0$
13. $(8, 13), m = -9$
14. $(12, 0), m = \frac{3}{4}$
15. $(7, -3), m = -\frac{4}{7}$
16. $(-4, 2), m = \frac{3}{2}$
17. $(9, -5), m = -\frac{1}{3}$

ERROR ANALYSIS *Describe* **and correct the error in writing an equation of the line that passes through the given point and has the given slope.**

18. $(-4, 2)$, $m = 3$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 3(x - 4)$$
$$y - 2 = 3x - 12$$
$$y = 3x - 10$$

19. $(5, 1)$, $m = -2$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -2(x - 1)$$
$$y - 5 = -2x + 2$$
$$y = -2x + 7$$

EXAMPLE 3
on p. 99
for Exs. 20–26

PARALLEL AND PERPENDICULAR LINES **Write an equation of the line that passes through the given point and satisfies the given condition.**

20. $(-3, -5)$; parallel to $y = -4x + 1$

21. $(7, 1)$; parallel to $y = -x + 3$

22. $(2, 8)$; parallel to $y = 3x - 2$

23. $(4, 1)$; perpendicular to $y = \frac{1}{3}x + 3$

24. $(-6, 2)$; perpendicular to $y = -2$

25. $(3, -1)$; perpendicular to $y = 4x + 1$

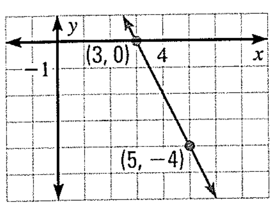
26. ★ **MULTIPLE CHOICE** What is an equation of the line that passes through $(1, 4)$ and is perpendicular to the line $y = 2x - 3$?

(A) $y = 2x + 2$ (B) $y = \frac{1}{2}x + \frac{7}{2}$ (C) $y = -\frac{1}{2}x + \frac{9}{2}$ (D) $y = -\frac{1}{2}x + 4$

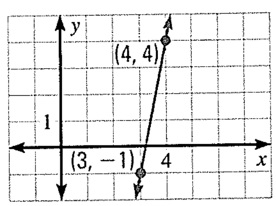
EXAMPLE 4
on p. 100
for Exs. 27–38

VISUAL THINKING **Write an equation of the line.**

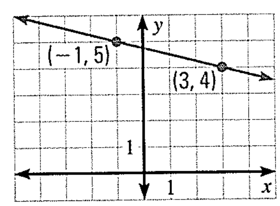
27.



28.



29.



WRITING EQUATIONS **Write an equation of the line that passes through the given points.**

30. $(-1, 3)$, $(2, 9)$

31. $(4, -1)$, $(6, -7)$

32. $(-2, -3)$, $(2, -1)$

33. $(0, 7)$, $(3, 5)$

34. $(-1, 2)$, $(3, -4)$

35. $(-5, -2)$, $(-3, 8)$

36. $(15, 20)$, $(-12, 29)$

37. $(3.5, 7)$, $(-1, 20.5)$

38. $(0.6, 0.9)$, $(3.4, -2.6)$

39. ★ **MULTIPLE CHOICE** Which point lies on the line that passes through the point $(9, -5)$ and has a slope of -6?

(A) $(6, 10)$ (B) $(6, 6)$ (C) $(7, 7)$ (D) $(6, -4)$

STANDARD FORM **Write an equation in standard form $Ax + By = C$ of the line that satisfies the given conditions. Use integer values for A, B, and C.**

40. $m = -3$, $b = 5$

41. $m = 4$, $b = -3$

42. $m = -\frac{3}{2}$, passes through $(4, -7)$

43. $m = \frac{4}{5}$, passes through $(2, 3)$

44. passes through $(-1, 3)$ and $(-6, -7)$

45. passes through $(2, 8)$ and $(-4, 16)$

Animated Algebra at classzone.com

46. **REASONING** Write an equation of the line that passes through (3, 4) and satisfies the given condition.

a. Parallel to $y = -2$

b. Perpendicular to $y = -2$

c. Parallel to $x = -2$

d. Perpendicular to $x = -2$

47. ★ **OPEN-ENDED MATH** Write an equation of a line ℓ such that ℓ and the lines $y = -3x + 5$ and $y = 2x + 1$ form a right triangle.

48. **REASONING** Consider two distinct nonvertical lines $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$. Show that the following statements are true.

a. If the lines are parallel, then $A_1B_2 = A_2B_1$.

b. If the lines are perpendicular, then $A_1A_2 + B_1B_2 = 0$.

49. **CHALLENGE** Show that an equation of the line with x-intercept a and y-intercept b is $\frac{x}{a} + \frac{y}{b} = 1$. This is the *intercept form* of a linear equation.

PROBLEM SOLVING

EXAMPLE 5 on p. 100 for Exs. 50–51

50. **CAR EXPENSES** You buy a used car for $6500. The monthly cost of owning the car (including insurance, fuel, maintenance, and taxes) averages $350. Write an equation that models the total cost of buying and owning the car.

@HomeTutor for problem solving help at classzone.com

51. **HOUSING** Since its founding, a volunteer group has restored 50 houses. It plans to restore 15 houses per year in the future. Write an equation that models the total number n of restored houses t years from now.

@HomeTutor for problem solving help at classzone.com

EXAMPLE 6 on p. 101 for Exs. 52–54

52. **GARDENING** You have a rectangular plot measuring 16 feet by 25 feet in a community garden. You want to grow tomato plants that each need 8 square feet of space and pepper plants that each need 5 square feet. Write an equation that models how many tomato plants and how many pepper plants you can grow. How many pepper plants can you grow if you grow 15 tomato plants?

53. ★ **SHORT RESPONSE** Concert tickets cost $15 for general admission, but only $9 with a student ID. Ticket sales total $4500. Write and graph an equation that models this situation. *Explain* how to use your graph to find how many student tickets were sold if 200 general admission tickets were sold.

54. **MULTI-STEP PROBLEM** A company will lease office space in two buildings. The annual cost is $21.75 per square foot in the first building and $17 per square foot in the second. The company has $86,000 budgeted for rent.

a. Write an equation that models the possible amounts of space rented in the buildings.

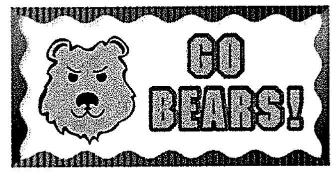
b. How many square feet of space can be rented in the first building if 2500 square feet are rented in the second?

c. If the company wants to rent equal amounts of space in the buildings, what is the total number of square feet that can be rented?

55. **CABLE TELEVISION** In 1994, the average monthly cost for expanded basic cable television service was $21.62. In 2004, this cost had increased to $38.23. Write a linear equation that models the monthly cost as a function of the number of years since 1994. Predict the average monthly cost of expanded basic cable television service in 2010.

56. **TIRE PRESSURE** Automobile tire pressure increases about 1 psi (pound per square inch) for each 10°F increase in air temperature. At an air temperature of 55°F, a tire's pressure is 30 psi. Write an equation that models the tire's pressure as a function of air temperature.

57. **MULTIPLE REPRESENTATIONS** Your class wants to make a rectangular spirit display, and has 24 feet of decorative border to enclose the display



w

ℓ

a. **Writing an Equation** Write an equation in standard form relating the possible lengths ℓ and widths w of the display.

b. **Drawing a Graph** Graph the equation from part (a).

c. **Making a Table** Make a table of at least five possible pairs of dimensions for the display.

58. **CHALLENGE** You are participating in a dance-a-thon to raise money for a class trip. Donors can pledge an amount of money for each hour you dance, a fixed amount of money that does not depend on how long you dance, or both. The table shows the amounts pledged by four donors. Write an equation that models the total amount y of money you will raise from the donors if you dance for x hours.

Donor	Hourly amount	Fixed amount
Clare	$4	$15
Emilia	$8	None
Julio	None	$35
Max	$3	$20

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

59. At the end of the week, John has $180 in his bank account. During the week he withdrew $30 for lunches, deposited a $125 paycheck, and withdrew $22 to buy a shirt. How much money did John have in his account at the beginning of the week?

(A) $95 (B) $100 (C) $107 (D) $117

60. Use the table to determine the expression that best represents the total measure of the interior angles of any convex polygon having n sides.

Number of sides, n	3	4	5	6	7
Total measure of interior angles (in degrees)	180	360	540	720	900

(A) $90(n-1)$ (B) $180(n-2)$

(C) $360(n-3)$ (D) $\frac{360}{n-1}$

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 100

MULTIPLE REPRESENTATIONS In Example 4 on page 100, you wrote an equation of a line through two given points by first writing the equation in point-slope form and then rewriting it in slope-intercept form. You can also write an equation of a line through two points by using the slope-intercept form to solve for the y-intercept.

PROBLEM

Write an equation of the line that passes through $(5, -2)$ and $(2, 10)$.

METHOD

Solving for the y-Intercept To write an equation of a line through two points, you can substitute the slope and the coordinates of one of the points into $y = mx + b$ and solve for the y-intercept b.

STEP 1 **Find** the slope of the line. $m = \frac{10 - (-2)}{2 - 5} = \frac{12}{-3} = -4$

STEP 2 **Substitute** the slope and the coordinates of one point into the slope-intercept form. Use the point $(\mathbf{5}, \mathbf{-2})$.

$y = mx + b$

$\mathbf{-2} = \mathbf{-4}(\mathbf{5}) + b$

STEP 3 **Solve** for b.

$-2 = -20 + b$

$18 = b$

STEP 4 **Substitute** m and b into the slope-intercept form.

$y = -4x + 18$

PRACTICE

1. **WRITE AN EQUATION** Use the method above to write an equation of the line that passes through $(2, 15)$ and $(7, 35)$.

2. **FITNESS** At a speed of 45 yards per minute, a 120 pound swimmer burns 420 calories per hour and a 172 pound swimmer burns 600 calories per hour. Use two different methods to write a linear equation that models the number of calories burned per hour as a function of a swimmer's weight.

3. **SAFETY** A motorist lights an emergency flare after having a flat tire. After burning for 6 minutes, the flare is 13 inches long. After burning for 20 minutes, it is 6 inches long. Use two different methods to write a linear equation that models the flare's length as a function of time.

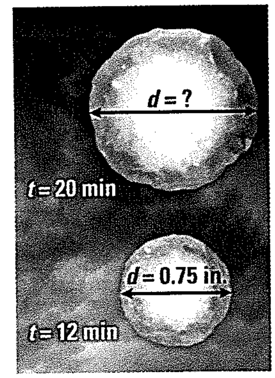
4. **SNOWFALL** After 4 hours of snowfall, the snow depth is 8 inches. After 6 hours of snowfall, the snow depth is 9.5 inches. Use two different methods to write a linear equation that models the snow depth as a function of time.

5. **ARCHAEOLOGY** Ancient cities often rose in elevation through time as citizens built on top of accumulating rubble and debris. An archaeologist at a site dates artifacts from a depth of 54 feet as 3500 years old and artifacts from a depth of 26 feet as 2600 years old. Use two different methods to write a linear equation that models an artifact's age as a function of depth.

6. **REASONING** Suppose a line has slope m and passes through (x_1, y_1). Write an expression for the y-intercept b in terms of m, x_1, and y_1.

EXAMPLE 2 Write and apply a model for direct variation

METEOROLOGY Hailstones form when strong updrafts support ice particles high in clouds, where water droplets freeze onto the particles. The diagram shows a hailstone at two different times during its formation.



a. Write an equation that gives the hailstone's diameter d (in inches) after t minutes if you assume the diameter varies directly with the time the hailstone takes to form.

b. Using your equation from part (a), predict the diameter of the hailstone after 20 minutes.

Solution

a. Use the given values of t and d to find the constant of variation.

$d = at$	**Write direct variation equation.**
$\mathbf{0.75} = a(\mathbf{12})$	**Substitute 0.75 for *d* and 12 for *t*.**
$0.0625 = a$	**Solve for *a*.**

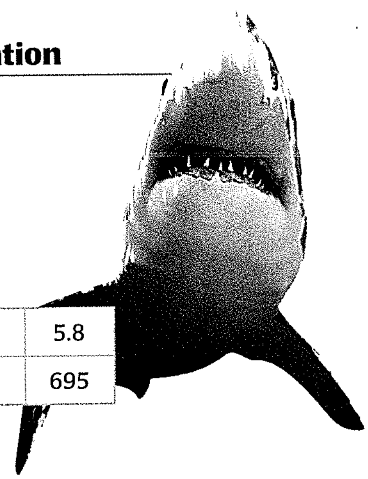
An equation that relates t and d is $d = 0.0625t$.

b. After $t = 20$ minutes, the predicted diameter of the hailstone is $d = 0.0625(20) = 1.25$ inches.

RATIOS AND DIRECT VARIATION Because the direct variation equation $y = ax$ can be written as $\frac{y}{x} = a$, a set of data pairs (x, y) shows direct variation if the ratio of y to x is constant.

EXAMPLE 3 Use ratios to identify direct variation

SHARKS Great white sharks have triangular teeth. The table below gives the length of a side of a tooth and the body length for each of six great white sharks. Tell whether tooth length and body length show direct variation. If so, write an equation that relates the quantities.



Tooth length, t (cm)	1.8	2.4	2.9	3.6	4.7	5.8
Body length, b (cm)	215	290	350	430	565	695

Solution

Find the ratio of the body length b to the tooth length t for each shark.

AVOID ERRORS
For real-world data, the ratios do not have to be *exactly* the same to show that direct variation is a plausible model.

$\frac{215}{1.8} \approx 119$ $\quad\frac{290}{2.4} \approx 121$ $\quad\frac{350}{2.9} = 121$

$\frac{430}{3.6} \approx 119$ $\quad\frac{565}{4.7} \approx 120$ $\quad\frac{695}{5.8} \approx 120$

▶ Because the ratios are approximately equal, the data show direct variation. An equation relating tooth length and body length is $\frac{b}{t} = 120$, or $b = 120t$.

✓ **GUIDED PRACTICE** for Examples 2 and 3

5. **WHAT IF?** In Example 2, suppose that a hailstone forming in a cloud has a radius of 0.6 inch. Predict how long it has been forming.

6. **SHARKS** In Example 3, the respective body masses m (in kilograms) of the great white sharks are 80, 220, 375, 730, 1690, and 3195. Tell whether tooth length and body mass show direct variation. If so, write an equation that relates the quantities.

2.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS3 for Exs. 5, 15, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 30, 40, and 44

SKILL PRACTICE

1. **VOCABULARY** Define the constant of variation for two variables x and y that vary directly.

2. ★ **WRITING** Given a table of ordered pairs (x, y), describe how to determine whether x and y show direct variation.

EXAMPLE 1 on p. 107 for Exs. 3–10

WRITING AND GRAPHING Write and graph a direct variation equation that has the given ordered pair as a solution.

3. $(2, 6)$
4. $(-3, 12)$
5. $(6, -21)$
6. $(4, 10)$
7. $(-5, -1)$
8. $(24, -8)$
9. $\left(\frac{4}{3}, -4\right)$
10. $(12.5, 5)$

EXAMPLE 2 on p. 108 for Exs. 11–17

WRITING AND EVALUATING The variables x and y vary directly. Write an equation that relates x and y. Then find y when $x = 12$.

11. $x = 4, y = 8$
12. $x = -3, y = -5$
13. $x = 35, y = -7$
14. $x = -18, y = 4$
15. $x = -4.8, y = -1.6$
16. $x = \frac{2}{3}, y = -10$

17. ★ **MULTIPLE CHOICE** Which equation is a direct variation equation that has $(3, 18)$ as a solution?

Ⓐ $y = 2x^2$ Ⓑ $y = \frac{1}{6}x$ Ⓒ $y = 6x$ Ⓓ $y = 4x + 6$

IDENTIFYING DIRECT VARIATION Tell whether the equation represents direct variation. If so, give the constant of variation.

18. $y = -8x$
19. $y - 4 = 3x$
20. $3y - 7 = 10x$
21. $2y - 5x = 0$
22. $5y = -4x$
23. $6y = x$

WRITING AND SOLVING The variables x and y vary directly. Write an equation that relates x and y. Then find x when $y = -4$.

24. $x = 5, y = -15$
25. $x = -6, y = 8$
26. $x = -18, y = -2$
27. $x = -12, y = 84$
28. $x = -\frac{20}{3}, y = -\frac{15}{8}$
29. $x = -0.5, y = 3.6$

30. ★ **OPEN-ENDED MATH** Give an example of two real-life quantities that show direct variation. *Explain* your reasoning.

EXAMPLE 3 on p. 108 for Exs. 31–34

IDENTIFYING DIRECT VARIATION Tell whether the data in the table show direct variation. If so, write an equation relating *x* and *y*.

31.

x	3	6	9	12	15
y	−1	−2	−3	−4	−5

32.

x	1	2	3	4	5
y	7	9	11	13	15

33.

x	−5	−4	−3	−2	−1
y	20	16	12	8	4

34.

x	−8	−4	4	8	12
y	8	4	−4	−8	−12

35. **ERROR ANALYSIS** A student tried to determine whether the data pairs (1, 24), (2, 12), (3, 8), and (4, 6) show direct variation. *Describe* and correct the error in the student's work.

$1 \cdot 24 = 24$ $\quad 2 \cdot 12 = 24$

$3 \cdot 8 = 24$ $\quad 4 \cdot 6 = 24$

Because the products *xy* are constant, *y* varies directly with *x*.

36. **REASONING** Let (x_1, y_1) be a solution, other than (0, 0), of a direct variation equation. Write a second direct variation equation whose graph is perpendicular to the graph of the first equation.

37. **CHALLENGE** Let (x_1, y_1) and (x_2, y_2) be any two distinct solutions of a direct variation equation. Show that $\frac{x_2}{x_1} = \frac{y_2}{y_1}$.

PROBLEM SOLVING

EXAMPLE 2 on p. 108 for Exs. 38–40

38. **SCUBA DIVING** The time *t* it takes a diver to ascend safely to the surface varies directly with the depth *d*. It takes a minimum of 0.75 minute for a safe ascent from a depth of 45 feet. Write an equation that relates *d* and *t*. Then predict the minimum time for a safe ascent from a depth of 100 feet.

@HomeTutor for problem solving help at classzone.com

39. **WEATHER** Hail 0.5 inch deep and weighing 1800 pounds covers a roof. The hail's weight *w* varies directly with its depth *d*. Write an equation that relates *d* and *w*. Then predict the weight on the roof of hail that is 1.75 inches deep.

@HomeTutor for problem solving help at classzone.com

40. ★ **MULTIPLE CHOICE** Your weight *M* on Mars varies directly with your weight *E* on Earth. If you weigh 116 pounds on Earth, you would weigh 44 pounds on Mars. Which equation relates *E* and *M*?

Ⓐ $M = E - 72$ Ⓑ $44M = 116E$ Ⓒ $M = \frac{29}{11}E$ Ⓓ $M = \frac{11}{29}E$

EXAMPLE 3 on p. 108 for Exs. 41–43

41. **INTERNET DOWNLOADS** The ordered pairs (4.5, 23), (7.8, 40), and (16.0, 82) are in the form (*s*, *t*) where *t* represents the time (in seconds) needed to download an Internet file of size *s* (in megabytes). Tell whether the data show direct variation. If so, write an equation that relates *s* and *t*.

○ = **WORKED-OUT SOLUTIONS on p. WS1** ★ = **STANDARDIZED TEST PRACTICE**

GEOMETRY In Exercises 42 and 43, consider squares with side lengths of 1, 2, 3, and 4 centimeters.

42. Copy and complete the table.

Side length, s (cm)	1	2	3	4
Perimeter, *P* (cm)	?	?	?	?
Area, *A* (cm^2)	?	?	?	?

43. Tell whether the given variables show direct variation. If so, write an equation relating the variables. If not, explain why not.

a. s and P **b.** s and A **c.** P and A

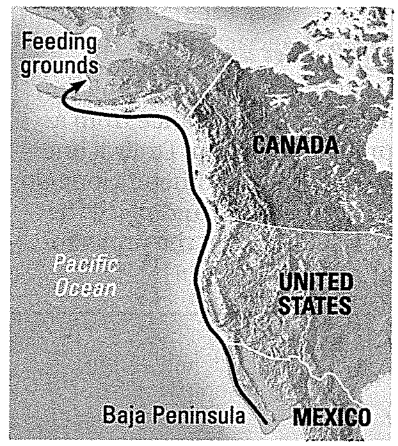
44. ★ **EXTENDED RESPONSE** Each year, gray whales migrate from Mexico's Baja Peninsula to feeding grounds near Alaska. A whale may travel 6000 miles at an average rate of 75 miles per day.

a. Write an equation that gives the distance d_1 traveled in t days of migration.

b. Write an equation that gives the distance d_2 that remains to be traveled after t days of migration.

c. Tell whether the equations from parts (a) and (b) represent direct variation. *Explain* your answers.

Animated Algebra at classzone.com



45. **CHALLENGE** At a jewelry store, the price p of a gold necklace varies directly with its length ℓ. Also, the weight w of a necklace varies directly with its length. Show that the price of a necklace varies directly with its weight.

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

46. An Internet service provider has a 15% off sale on a 6 month subscription. Which statement best represents the functional relationship between the sale price of the subscription and the original price?

(A) The original price is dependent on the sale price.

(B) The sale price is dependent on the original price.

(C) The sale price and the original price are independent of each other.

(D) The relationship cannot be determined.

47. Rose works as a salesperson at a car stereo store. She earns an 8% commission on every sale. She wants to earn $300 from commissions in the next 5 days. What is the average amount of car stereo sales Rose must make per day to reach her goal?

(A) $480 (B) $750 (C) $1000 (D) $3750

TRANSFORMATIONS OF ANY GRAPH You can perform transformations on the graph of *any* function f in the same way as for absolute value graphs.

KEY CONCEPT *For Your Notebook*

Transformations of General Graphs

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of any function $y = f(x)$ by performing these steps:

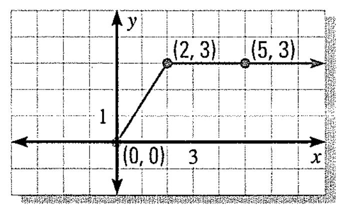
STEP 1 **Stretch or shrink** the graph of $y = f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.

STEP 2 **Reflect** the resulting graph from Step 1 in the x-axis if $a < 0$.

STEP 3 **Translate** the resulting graph from Step 2 horizontally h units and vertically k units.

EXAMPLE 5 Apply transformations to a graph

The graph of a function $y = f(x)$ is shown. Sketch the graph of the given function.



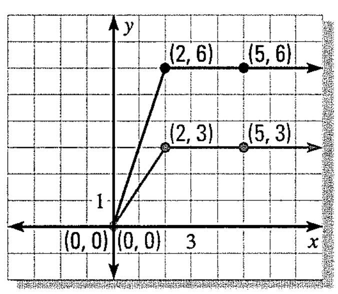
a. $y = 2 \cdot f(x)$

b. $y = -f(x + 2) + 1$

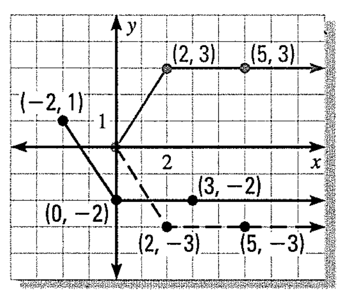
Solution

AVOID ERRORS
In Example 5, part (b), the value of h is -2 because $-f(x + 2) + 1 = -f(x - (-2)) + 1$. Because $-2 < 0$, the horizontal translation is to the left.

a. The graph of $y = 2 \cdot f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. (There is no reflection or translation.) To draw the graph, multiply the y-coordinate of each labeled point on the graph of $y = f(x)$ by 2 and connect their images.



b. The graph of $y = -f(x + 2) + 1$ is the graph of $y = f(x)$ reflected in the x-axis, then translated left 2 units and up 1 unit. To draw the graph, first reflect the labeled points and connect their images. Then translate and connect these points to form the final image.



GUIDED PRACTICE for Examples 4 and 5

4. WHAT IF? In Example 4, suppose the reference beam originates at (3, 0) and reflects off a mirror at (5, 4). Write an equation for the path of the beam.

Use the graph of $y = f(x)$ from Example 5 to graph the given function.

5. $y = 0.5 \cdot f(x)$ **6.** $y = -f(x - 2) - 5$ **7.** $y = 2 \cdot f(x + 3) - 1$

2.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS4 for Exs. 13, 19, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 27, 28, 31, 32, 33, 38, and 40

◆ = **MULTIPLE REPRESENTATIONS** Ex. 41

SKILL PRACTICE

1. **VOCABULARY** The point (h, k) is the __?__ of the graph of $y = a|x - h| + k$.

2. ★ **WRITING** *Describe* three different types of transformations.

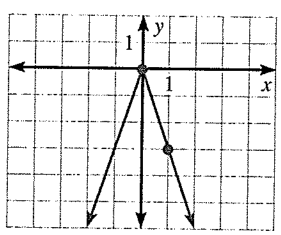
EXAMPLES 1, 2, and 3 on pp. 124–125 for Exs. 3–14

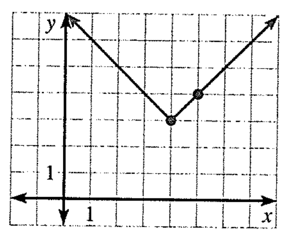
GRAPHING FUNCTIONS Graph the function. *Compare* the graph with the graph of $y = |x|$.

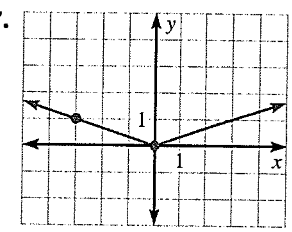
3. $y = |x| - 7$
4. $y = |x + 2|$
5. $y = |x + 4| - 2$
6. $f(x) = |x - 1| + 4$
7. $f(x) = 2|x|$
8. $f(x) = -3|x|$
9. $y = -\frac{1}{3}|x|$
10. $y = \frac{3}{4}|x|$
11. $y = 2|x + 1| - 6$
12. $f(x) = -4|x + 2| - 3$
13. $f(x) = -\frac{1}{2}|x - 1| + 5$
14. $f(x) = \frac{1}{4}|x - 4| + 3$

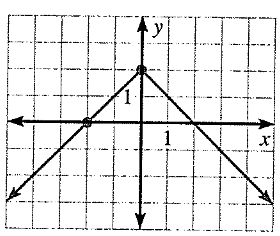
EXAMPLE 4 on p. 125 for Exs. 15–20

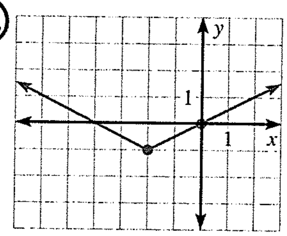
WRITING EQUATIONS Write an equation of the graph.

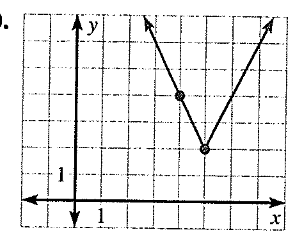
15. 

16. 

17. 

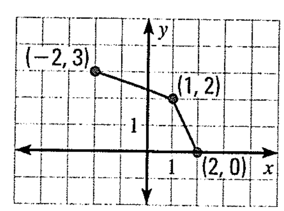
18. 

19. 

20. 

EXAMPLE 5 on p. 126 for Exs. 21–28

TRANSFORMATIONS Use the graph of $y = f(x)$ shown to sketch the graph of the given function.



21. $y = f(x + 2) - 3$
22. $y = f(x - 4) + 1$
23. $y = \frac{1}{2} \cdot f(x)$
24. $y = -3 \cdot f(x)$
25. $y = -f(x - 1) + 4$
26. $y = 2 \cdot f(x + 3) - 1$

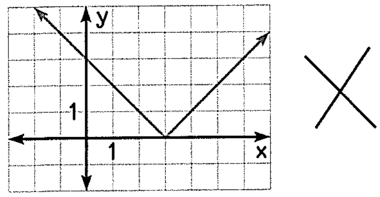
27. ★ **OPEN-ENDED MATH** Create a graph of a function $y = f(x)$. Then sketch the graphs of **(a)** $y = f(x + 3) - 4$, **(b)** $y = 2 \cdot f(x)$, and **(c)** $y = -f(x)$.

28. ★ **MULTIPLE CHOICE** The highest point on the graph of $y = f(x)$ is $(-1, 6)$. What is the highest point on the graph of $y = 4 \cdot f(x - 3) + 5$?

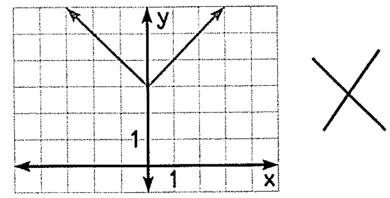
Ⓐ $(-11, 6)$ Ⓑ $(8, 11)$ Ⓒ $(-4, 29)$ Ⓓ $(2, 29)$

ERROR ANALYSIS *Describe* and correct the error in graphing $y = |x + 3|$.

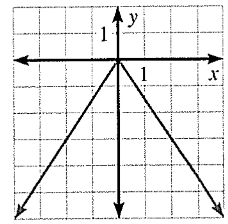
29.



30.



31. ★ **MULTIPLE CHOICE** Which equation has the graph shown?

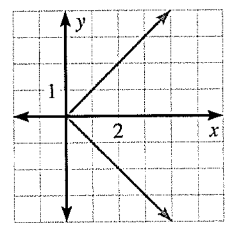


Ⓐ $y = \frac{3}{2}|x|$ Ⓑ $y = \frac{2}{3}|x|$

Ⓒ $y = -\frac{2}{3}|x|$ Ⓓ $y = -\frac{3}{2}|x|$

32. ★ **WRITING** *Describe* how the signs of h and k affect how to obtain the graph of $y = f(x - h) + k$ from the graph of $y = f(x)$.

33. ★ **SHORT RESPONSE** The graph of the relation $x = |y|$ is shown at the right. Is the relation a function? *Explain.*



34. REASONING Is it true in general that $|x + h| = |x| + |h|$? *Justify* your answer by considering how the graphs of $y = |x + h|$ and $y = |x| + |h|$ are related to the graph of $y = |x|$.

35. CHALLENGE The graph of $y = a|x - h| + k$ passes through $(-2, 4)$ and $(4, 4)$. *Describe* the possible values of h and k.

PROBLEM SOLVING

EXAMPLE 1 on p. 124 for Ex. 36

36. SPEEDOMETER A car's speedometer reads 60 miles per hour. The error E in this measurement is $E = |a - 60|$ where a is the actual speed. Graph the function. For what value(s) of a will E be 2.5 miles per hour?

@HomeTutor for problem solving help at classzone.com

EXAMPLE 3 on p. 125 for Ex. 37

37. SALES Weekly sales s (in thousands) of a new basketball shoe increase steadily for a while and then decrease as described by the function $s = -2|t - 15| + 50$ where t is the time (in weeks). Graph the function. What is the greatest number of pairs of shoes sold in one week?

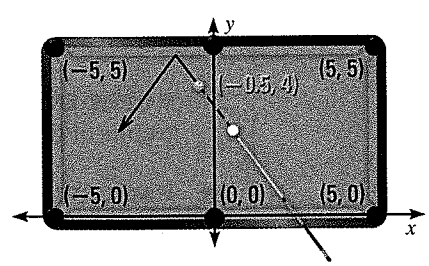
@HomeTutor for problem solving help at classzone.com

EXAMPLE 4 on p. 125 for Exs. 38–39

38. ★ **SHORT RESPONSE** On the pool table shown, you bank the five ball off the side at $(-1.25, 5)$. You want the ball to go in the pocket at $(-5, 0)$.

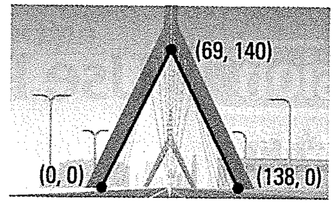
a. Write an equation for the path of the ball.

b. Do you make the shot? *Explain* how you found your answer.



◯ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

39. **ENGINEERING** The Leonard P. Zakim Bunker Hill Bridge spans the Charles River in Boston. The bridge is suspended from two towers. Each tower has the dimensions shown. Write an absolute value function that represents the inverted V-shaped portion of a tower.



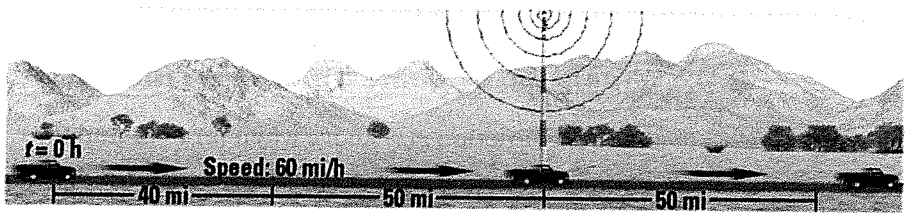
40. ★ **EXTENDED RESPONSE** A snowstorm begins with light snow that increases to very heavy snow before decreasing again. The snowfall rate r (in inches per hour) is given by $r(t) = -0.5|t - 4| + 2$ where t is the time (in hours).

a. **Graph** Graph the function.

b. **Interpret** When is the snowfall heaviest? What is the maximum snowfall rate? How are your answers related to the function's graph?

c. **Extend** The total snowfall is given by the area of the triangle formed by the graph of $r(t)$ and the t-axis. What is the total snowfall?

41. ❖ **MULTIPLE REPRESENTATIONS** The diagram shows a truck driving toward a radio station transmitter that has a broadcasting range of 50 miles.



a. **Making a Table** Make a table that shows the truck's distance d (in miles) from the transmitter after $t = 0, 0.5, 1, 1.5, 2, 2.5,$ and 3 hours.

b. **Drawing a Graph** Use your table from part (a) to draw a graph that shows d as a function of t.

c. **Writing an Equation** Write an equation that gives d as a function of t. During what driving times is the truck within range of the transmitter?

42. **CHALLENGE** A hiker walks up and down a hill. The hill has a cross section that can be modeled by $y = -\frac{4}{3}|x - 300| + 400$ where x and y are measured in feet and $0 \le x \le 600$. How far does the hiker walk?

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

43. Which expression is equivalent to $12(n^2 + n) - 5(n^2 + 3n - 2)$?

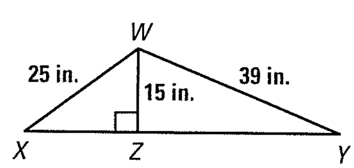
Ⓐ $-7n^2 + 3n - 10$ Ⓑ $7n^2 - 3n + 10$

Ⓒ $17n^2 + 27n - 10$ Ⓓ $17n^2 - 13n + 10$

44. In the figure shown, what is the length of $\overline{YX}$ in inches?

Ⓐ 20 in. Ⓑ 36 in.

Ⓒ 56 in. Ⓓ 3136 in.



TEST PREPARATION

MULTIPLE CHOICE

1. Which equation below does not represent a function?

   A $x = 4$ B $y = \pi$

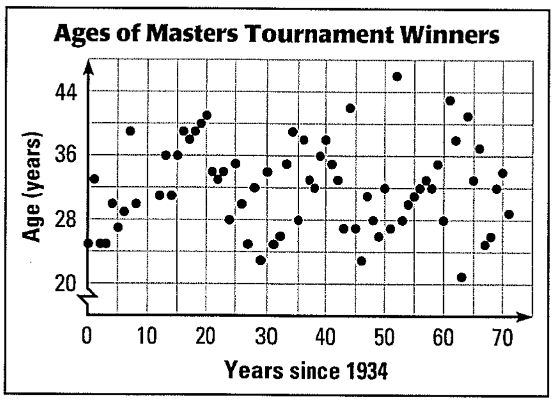
   C $y = |x|$ D $y = x^2 + 4$

2. Water boils at 100° Celsius at sea level. The boiling temperature changes depending on altitude. For each increase of 1 kilometer in altitude, the water boils at about 3.5°C lower. Which equation relates the boiling temperature (C) in degrees Celsius to the altitude (a) in kilometers?

   A $C = -3.5a$ B $C = 100 - 3.5a$

   C $C = 100 - |3.5a|$ D $C = 3.5a + 100$

3. The scatter plot shows the ages of the winners of professional golf's annual Masters Tournament for the years 1934–2005. (There was no tournament in the years 1943–1945.) What type of correlation does the graph show?



   A The age of the winner is positively correlated to the year.

   B The age of the winner is not correlated to the year.

   C The age of the winner is negatively correlated to the year.

   D The correlation of the age of the winner to the year is undefined.

4. The Wellness Center increased the number of holistic medicine providers between 2003 and 2007, as shown in the table below. If the trend continues, how many providers will there be x years after 2007?

Year	Number of Providers
2003	32
2004	34
2005	36
2006	38
2007	40

   A $32x$ B $32 + 2x$

   C $40 + 2x$ D $42 + 2x$

5. A house's roof has a length of 25 feet from its peak to its lowest edge. The height of the roof at the peak is 15 feet. A special chimney protrudes perpendicularly from the roof. What is the slope of the chimney?

   A $\frac{5}{3}$ B $\frac{3}{4}$

   C $\frac{4}{3}$ D $\frac{3}{5}$

6. Which of the ordered pairs below is not on the graph of $-3x + 2y = 12$?

   A $(-4, 0)$ B $(0, 6)$

   C $(2, 9)$ D $(6, -3)$

7. A direct variation equation $y = ax$ includes the point $(-3, -6)$. What is the value of the constant of variation?

   A -2 B -1

   C 1 D 2

8. What is the value of k so that the line passing through $(k, 2)$ and $(2, 7)$ has a slope of -1?

   A -5 B -3

   C 3 D 7

MULTIPLE CHOICE

9. Which ordered pair is a solution of the inequality $-x + 2y > 10$?

A $(-10, 0)$ B $(0, 5)$
C $(5, 5)$ D $(5, 10)$

10. The graph of which equation passes through the point $(1, -3)$ and is perpendicular to the line $x + y = 10$?

A $x + y = -2$ B $x - y = -2$
C $x + y = 4$ D $x - y = 4$

11. What is the slope of the line passing through the points $(0, -4)$ and $(-3, 2)$?

A -2 B $-\frac{1}{2}$
C $\frac{1}{2}$ D 2

12. Which is the x-intercept of the graph of $2x + 3y = 36$?

A 6 B 12
C 18 D 36

13 Which choice best describes the relationship between the lines $y = x + 2$ and $y = -x + 2$?

A The lines are perpendicular.
B The lines are parallel.
C The lines have the same x-intercept.
D The lines are the same.

14. The graph of $g(x) = -3 \cdot f(x) + 5$ is obtained from the graph of $y = f(x)$ through several transformations. The point $(-7, -6)$ lies on the graph of $y = f(x)$. What is $g(-7)$?

A 18 B 21
C 23 D 26

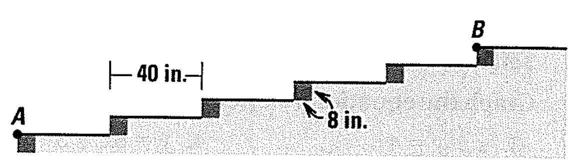
OPEN-ENDED

15. The number of women elected to the U.S. House of Representatives has increased nearly every Congress since 1985. In the 99th Congress beginning in 1985, there were 22 female representatives. In the 109th Congress beginning in 2005, there were 68 female representatives.

A. Write a linear equation that models the number y of female members of the House of Representatives x years after 1985.

B. Given that the number of representatives is fixed at 435, is it reasonable to think that the model could be a fairly accurate predictor of the number of female representatives in 2030? *Explain.*

16. A park trail leading up a hill has been improved by building steps from wooden timbers, as shown in the diagram of a section of the trail. Each timber measures 8 inches on a side.



A. What is the average rate of change in elevation on the trail from point A to point B?

B. If the trail continues as shown up a hill that is 120 feet high, what is the horizontal distance covered by climbing the trail?

C. If the trail had used 6-inch timbers for each step instead of 8-inch timbers, what horizontal distance would each step cover? *Explain.*

TEST PREPARATION

3 Linear Systems and Matrices

PA

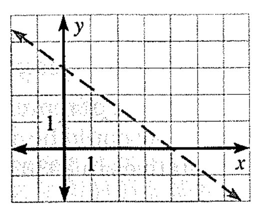
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 3: graphing equations, solving equations, and graphing inequalities.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.



1. The **linear inequality** that represents the graph shown at the right is _?_.
2. The **graph of a linear inequality** in two variables is the set of all points in a coordinate plane that are _?_ of the inequality.

SKILLS CHECK

Graph the equation. *(Review p. 89 for 3.1.)*

3. $x + y = 4$
4. $y = 3x - 3$
5. $-2x + 3y = -12$

Solve the equation. *(Review p. 18 for 3.2, 3.4.)*

6. $2x - 12 = 16$
7. $-3x - 7 = 12$
8. $-2x + 5 = 2x - 5$

Graph the inequality in a coordinate plane. *(Review p. 132 for 3.3.)*

9. $y \geq -x + 2$
10. $x + 4y < -16$
11. $3x + 5y > -5$