3 Linear Systems and Matrices

PA

M11.D.2.1.4
M11.D.2.1.4
M11.D.2.1.2

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 3: graphing equations, solving equations, and graphing inequalities.

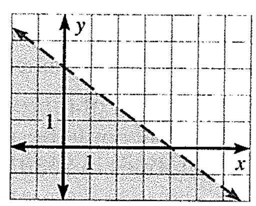
Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **linear inequality** that represents the graph shown at the right is _?_.

2. The **graph of a linear inequality** in two variables is the set of all points in a coordinate plane that are _?_ of the inequality.



SKILLS CHECK

Graph the equation. *(Review p. 89 for 3.1.)*

3. $x + y = 4$
4. $y = 3x - 3$
5. $-2x + 3y = -12$

Solve the equation. *(Review p. 18 for 3.2, 3.4.)*

6. $2x - 12 = 16$
7. $-3x - 7 = 12$
8. $-2x + 5 = 2x - 5$

Graph the inequality in a coordinate plane. *(Review p. 132 for 3.3.)*

9. $y \geq -x + 2$
10. $x + 4y < -16$
11. $3x + 5y > -5$

MULTIPLE CHOICE

9. Which ordered pair is a solution of the inequality $-x + 2y > 10$?

A (−10, 0) B (0, 5)

C (5, 5) D (5, 10)

10. The graph of which equation passes through the point (1, −3) and is perpendicular to the line $x + y = 10$?

A $x + y = -2$ B $x - y = -2$

C $x + y = 4$ D $x - y = 4$

11. What is the slope of the line passing through the points (0, −4) and (−3, 2)?

A −2 B $-\frac{1}{2}$

C $\frac{1}{2}$ D 2

12. Which is the x-intercept of the graph of $2x + 3y = 36$?

A 6 B 12

C 18 D 36

13 Which choice best describes the relationship between the lines $y = x + 2$ and $y = -x + 2$?

A The lines are perpendicular.

B The lines are parallel.

C The lines have the same x-intercept.

D The lines are the same.

14. The graph of $g(x) = -3 \cdot f(x) + 5$ is obtained from the graph of $y = f(x)$ through several transformations. The point (−7, −6) lies on the graph of $y = f(x)$. What is $g(-7)$?

A 18 B 21

C 23 D 26

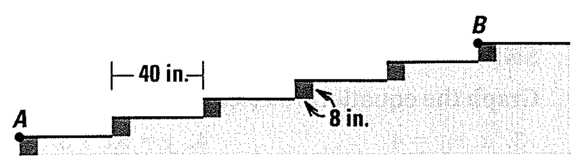
OPEN-ENDED

15. The number of women elected to the U.S. House of Representatives has increased nearly every Congress since 1985. In the 99th Congress beginning in 1985, there were 22 female representatives. In the 109th Congress beginning in 2005, there were 68 female representatives.

A. Write a linear equation that models the number y of female members of the House of Representatives x years after 1985.

B. Given that the number of representatives is fixed at 435, is it reasonable to think that the model could be a fairly accurate predictor of the number of female representatives in 2030? *Explain.*

16. A park trail leading up a hill has been improved by building steps from wooden timbers, as shown in the diagram of a section of the trail. Each timber measures 8 inches on a side.



A. What is the average rate of change in elevation on the trail from point A to point B?

B. If the trail continues as shown up a hill that is 120 feet high, what is the horizontal distance covered by climbing the trail?

C. If the trail had used 6-inch timbers for each step instead of 8-inch timbers, what horizontal distance would each step cover? *Explain.*

TEST PREPARATION

Now

In Chapter 3, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 221. You will also use the key vocabulary listed below.

Big Ideas

1. **Solving systems of equations using a variety of methods**
2. **Graphing systems of equations and inequalities**
3. **Using matrices**

KEY VOCABULARY

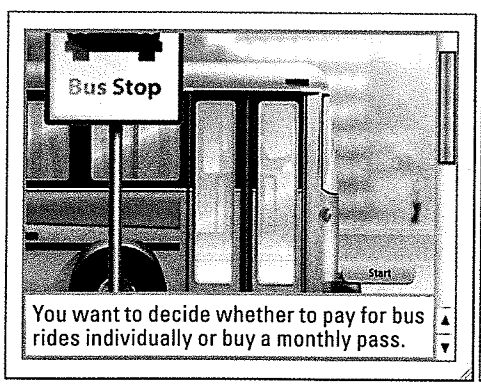
- system of two linear equations, *p. 153*
- consistent, *p. 154*
- inconsistent, *p. 154*
- independent, *p. 154*
- dependent, *p. 154*
- substitution method, *p. 160*
- elimination method, *p. 161*
- system of linear inequalities, *p. 168*
- system of three linear equations, *p. 178*
- ordered triple, *p. 178*
- matrix, *p. 187*
- determinant, *p. 203*
- Cramer's rule, *p. 205*
- identity matrix, *p. 210*
- inverse matrices, *p. 210*

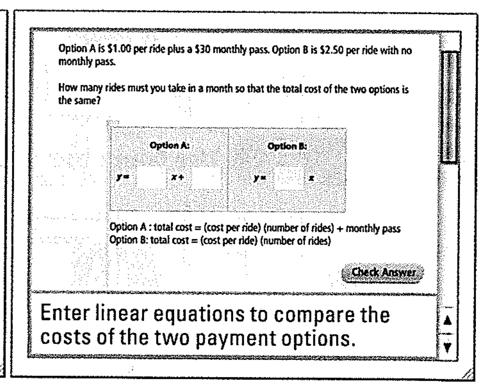
Why?

You can use systems of linear equations to solve real-world problems. For example, you can determine which of two payment options for riding a bus is more cost-effective.

Animated Algebra

The animation illustrated below for Example 4 on page 155 helps you answer this question: After how many bus rides will the cost of two payment options be the same?





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Other animations for Chapter 3: pages 153, 161, 168, 196, 211, and 212

Investigating Algebra ACTIVITY *Use before Lesson 3.1*

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Keystrokes

3.1 Solving Linear Systems Using Tables

MATERIALS • graphing calculator

QUESTION **How can you solve a system of linear equations using a table?**

An example of a *system of linear equations* in two variables x and y is the following:

$y = 2x + 4$ **Equation 1**
$y = -3x + 44$ **Equation 2**

A *solution* of a system of equations in two variables is an ordered pair (x, y) that is a solution of both equations. One way to solve a system is to use the *table* feature of a graphing calculator.

EXPLORE **Solve a system**

Use a table to solve the system of equations above.

STEP 1 ***Enter equations***

Press [Y=] to enter the equations. Enter Equation 1 as y_1 and Equation 2 as y_2.

```
Y1=2X+4
Y2=-3X+44
Y3=
Y4=
Y5=
Y6=
Y7=
```

STEP 2 ***Make a table***

Set the starting x-value of the table to 0 and the step value to 1. Then use the *table* feature to make a table.

X	Y1	Y2
0	4	44
1	6	41
2	8	38
3	10	35
4	12	32

X=0

STEP 3 ***Find the solution***

Scroll through the table until you find an x-value for which y_1 and y_2 are equal. The table shows $y_1 = y_2 = 20$ when $x = 8$.

X	Y1	Y2
4	12	32
5	14	29
6	16	26
7	18	23
8	20	20

X=8

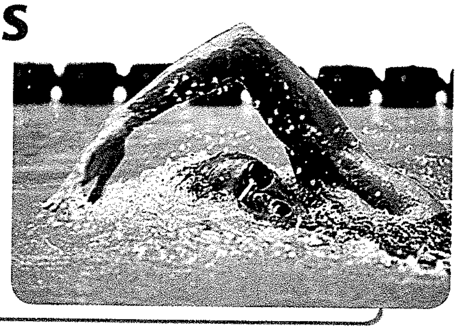
▸ The solution of the system is (8, 20).

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Use a table to solve the system. If you are using a graphing calculator, you may need to first solve the equations in the system for y before entering them.

1. $y = 2x + 5$
   $y = -x + 2$
2. $y = 4x + 1$
   $y = 4x - 8$
3. $y = 4x - 3$
   $y = \frac{3}{2}x + 2$
4. $8x - 4y = 16$
   $-6x + 3y = 3$
5. $6x - 2y = -2$
   $-3x - 7y = 17$
6. $x + y = 11$
   $-x - y = -11$
7. Based on your results in Exercises 1–6, make a conjecture about the number of solutions a system of linear equations can have.

3.1 Solve Linear Systems by Graphing



PA M11.D.2.1.4 Write and/or solve systems of equations using graphing, substitution and/or elimination (limit systems to 2 equations).

Before You solved linear equations.

Now You will solve systems of linear equations.

Why? So you can compare swimming data, as in Ex. 39.

Key Vocabulary
- **system of two linear equations**
- **solution of a system**
- **consistent**
- **inconsistent**
- **independent**
- **dependent**

A **system of two linear equations** in two variables x and y, also called a *linear system*, consists of two equations that can be written in the following form.

$$Ax + By = C \quad \textbf{Equation 1}$$
$$Dx + Ey = F \quad \textbf{Equation 2}$$

A **solution** of a system of linear equations in two variables is an ordered pair (x, y) that satisfies each equation. Solutions correspond to points where the graphs of the equations in a system intersect.

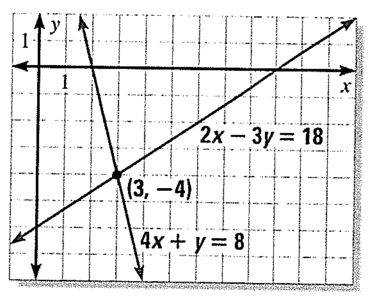
EXAMPLE 1 Solve a system graphically

Graph the linear system and estimate the solution. Then check the solution algebraically.

$$4x + y = 8 \quad \textbf{Equation 1}$$
$$2x - 3y = 18 \quad \textbf{Equation 2}$$

Solution

Begin by graphing both equations, as shown at the right. From the graph, the lines *appear* to intersect at $(3, -4)$. You can check this algebraically as follows.



AVOID ERRORS
Remember to check the graphical solution in *both* equations before concluding that it is a solution of the system.

Equation 1	**Equation 2**
$4x + y = 8$	$2x - 3y = 18$
$4(3) + (-4) \stackrel{?}{=} 8$	$2(3) - 3(-4) \stackrel{?}{=} 18$
$12 - 4 \stackrel{?}{=} 8$	$6 + 12 \stackrel{?}{=} 18$
$8 = 8$ ✓	$18 = 18$ ✓

▶ The solution is $(3, -4)$.

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✓ **GUIDED PRACTICE** for Example 1

Graph the linear system and estimate the solution. Then check the solution algebraically.

1. $3x + 2y = -4$
   $x + 3y = 1$

2. $4x - 5y = -10$
   $2x - 7y = 4$

3. $8x - y = 8$
   $3x + 2y = -16$

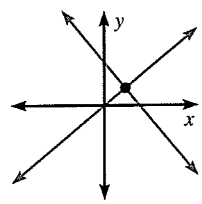
CLASSIFYING SYSTEMS A system that has at least one solution is **consistent.** If a system has no solution, the system is **inconsistent.** A consistent system that has exactly one solution is **independent,** and a consistent system that has infinitely many solutions is **dependent.** The system in Example 1 is consistent and independent.

KEY CONCEPT — *For Your Notebook*

Number of Solutions of a Linear System

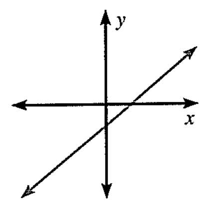
The relationship between the graph of a linear system and the system's number of solutions is described below.

Exactly one solution



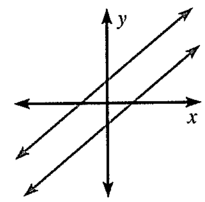
Lines intersect at one point; consistent and independent

Infinitely many solutions



Lines coincide; consistent and dependent

No solution



Lines are parallel; inconsistent

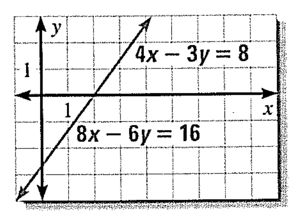
EXAMPLE 2 Solve a system with many solutions

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.*

$4x - 3y = 8$ **Equation 1**
$8x - 6y = 16$ **Equation 2**

Solution

The graphs of the equations are the same line. So, each point on the line is a solution, and the system has infinitely many solutions. Therefore, the system is consistent and dependent.



CHECK SOLUTION

To check your solution in Example 2, observe that both equations have the same slope-intercept form:

$$y = \frac{4}{3}x - \frac{8}{3}$$

So the graphs are the same line.

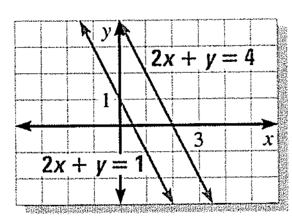
EXAMPLE 3 Solve a system with no solution

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.*

$2x + y = 4$ **Equation 1**
$2x + y = 1$ **Equation 2**

Solution

The graphs of the equations are two parallel lines. Because the two lines have no point of intersection, the system has no solution. Therefore, the system is inconsistent.



CHECK SOLUTION

To verify that the graphs in Example 3 are parallel lines, write the equations in slope-intercept form and observe that the lines have the same slope, −2, but different *y*-intercepts, 4 and 1.

★ EXAMPLE 4 Standardized Test Practice

You ride an express bus from the center of town to your street. You have two payment options. Option A is to buy a monthly pass and pay \$1 per ride. Option B is to pay \$2.50 per ride. A monthly pass costs \$30. After how many rides will the total costs of the two options be the same?

Ⓐ 12 rides Ⓑ 20 rides Ⓒ 24 rides Ⓓ 28 rides

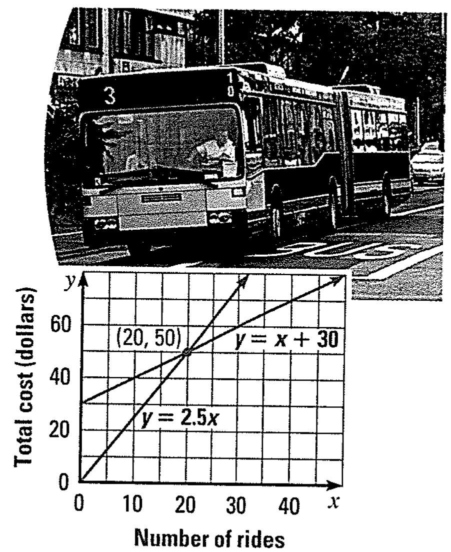
Solution

Equation 1 (Option A)

Total cost (dollars)	=	Cost per ride (dollars/ride)	•	Number of rides (rides)	+	Monthly fee (dollars)
↓		↓		↓		↓
y	=	**1**	•	x	+	**30**

Equation 2 (Option B)

Total cost (dollars)	=	Cost per ride (dollars/ride)	•	Number of rides (rides)
↓		↓		↓
y	=	**2.5**	•	x



To solve the system, graph the equations $y = x + 30$ and $y = 2.5x$, as shown at the right.

Notice that you need to graph the equations only in the first quadrant because only nonnegative values of x and y make sense in this situation.

The lines appear to intersect at about the point (20, 50). You can check this algebraically as follows.

$50 = 20 + 30$ ✓ **Equation 1 checks.**
$50 = 2.5(20)$ ✓ **Equation 2 checks.**

The total costs are equal after 20 rides.

▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

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✓ GUIDED PRACTICE for Examples 2, 3, and 4

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.*

4. $2x + 5y = 6$
$4x + 10y = 12$

5. $3x - 2y = 10$
$3x - 2y = 2$

6. $-2x + y = 5$
$y = -x + 2$

7. **WHAT IF?** In Example 4, suppose the cost of the monthly pass is increased to \$36. How does this affect the solution?

3.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS4 for Exs. 9, 21, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 29, 30, 37, and 39

◆ = **MULTIPLE REPRESENTATIONS** Ex. 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A consistent system that has exactly one solution is called _?_.

2. ★ **WRITING** *Explain* how to identify the solution(s) of a system from the graphs of the equations in the system.

EXAMPLE 1 on p. 153 for Exs. 3–16

GRAPH AND CHECK Graph the linear system and estimate the solution. Then check the solution algebraically.

3. $y = -3x + 2$
   $y = 2x - 3$

4. $y = 5x + 2$
   $y = 3x$

5. $y = -x + 3$
   $-x - 3y = -1$

6. $x + 2y = 2$
   $x - 4y = 14$

7. $y = 2x - 10$
   $x - 4y = 5$

8. $-x + 6y = -12$
   $x + 6y = 12$

9. $y = -3x - 2$
   $5x + 2y = -2$

10. $y = -3x - 13$
    $-x - 2y = -4$

11. $x - 7y = 6$
    $-3x + 21y = -18$

12. $y = 4x + 3$
    $20x - 5y = -15$

13. $5x - 4y = 3$
    $3x + 2y = 15$

14. $7x + y = -17$
    $3x - 10y = 24$

15. ★ **MULTIPLE CHOICE** What is the solution of the system?

$$-4x - y = 2$$
$$7x + 2y = -5$$

Ⓐ $(2, -6)$ Ⓑ $(-1, 6)$ Ⓒ $(1, -6)$ Ⓓ $(-3, 8)$

16. **ERROR ANALYSIS** A student used the check shown to conclude that $(0, -1)$ is a solution of this system:

$$3x - 2y = 2$$
$$x + 2y = 6$$

$3x - 2y = 2$
$3(0) - 2(-1) \stackrel{?}{=} 2$
$2 = 2$ ✗

Describe and correct the student's error.

EXAMPLES 2 and 3 on p. 154 for Exs. 17–29

SOLVE AND CLASSIFY Solve the system. Then classify the system as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

17. $y = -1$
    $3x + y = 5$

18. $2x - y = 4$
    $x - 2y = -1$

19. $y = 3x + 2$
    $y = 3x - 2$

20. $y = 2x - 1$
    $-6x + 3y = -3$

21. $-20x + 12y = -24$
    $5x - 3y = 6$

22. $4x - 5y = 0$
    $3x - 5y = -5$

23. $3x + 7y = 6$
    $2x + 9y = 4$

24. $4x + 5y = 3$
    $6x + 9y = 9$

25. $8x + 9y = 15$
    $5x - 2y = 17$

26. $\frac{1}{2}x - 3y = 10$
    $\frac{1}{4}x + 2y = -2$

27. $3x - 2y = -15$
    $x - \frac{2}{3}y = -5$

28. $\frac{5}{2}x - y = -4$
    $5x - 2y = \frac{1}{4}$

29. ★ **MULTIPLE CHOICE** How would you classify the system?

$$-12x + 16y = 10$$
$$3x + 4y = -6$$

Ⓐ Consistent and independent Ⓑ Consistent and dependent

Ⓒ Inconsistent Ⓓ None of these

30. ★ **OPEN-ENDED MATH** Write a system of two linear equations that has the given number of solutions.

a. One solution **b.** No solution **c.** Infinitely many solutions

GRAPH AND CHECK Graph the system and estimate the solution(s). Then check the solution(s) algebraically.

31. $y = |x + 2|$, $y = x$

32. $y = |x - 1|$, $y = -x + 4$

33. $y = |x| - 2$, $y = 2$

34. **CHALLENGE** State the conditions on the constants a, b, c, and d for which the system below is **(a)** consistent and independent, **(b)** consistent and dependent, and **(c)** inconsistent.

$$y = ax + b$$
$$y = cx + d$$

PROBLEM SOLVING

EXAMPLE 4 on p. 155 for Exs. 35–39

35. **WORK SCHEDULE** You worked 14 hours last week and earned a total of \$96 before taxes. Your job as a lifeguard pays \$8 per hour, and your job as a cashier pays \$6 per hour. How many hours did you work at each job?

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36. **LAW ENFORCEMENT** During one calendar year, a state trooper issued a total of 375 citations for warnings and speeding tickets. Of these, there were 37 more warnings than speeding tickets. How many warnings and how many speeding tickets were issued?

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37. ★ **SHORT RESPONSE** A gym offers two options for membership plans. Option A includes an initiation fee of \$121 and costs \$1 per day. Option B has no initiation fee but costs \$12 per day. After how many days will the total costs of the gym membership plans be equal? How does your answer change if the daily cost of Option B increases? *Explain.*

38. ◆ **MULTIPLE REPRESENTATIONS** The price of refrigerator A is \$600, and the price of refrigerator B is \$1200. The cost of electricity needed to operate the refrigerators is \$50 per year for refrigerator A and \$40 per year for refrigerator B.

a. Writing Equations Write an equation for the cost of owning refrigerator A and an equation for the cost of owning refrigerator B.

b. Graphing Equations Graph the equations from part (a). After how many years are the total costs of owning the refrigerators equal?

c. Checking Reasonableness Is your solution from part (b) reasonable in this situation? *Explain.*

39. ★ **EXTENDED RESPONSE** The table below gives the winning times (in seconds) in the Olympic 100 meter freestyle swimming event for the period 1972–2000.

Years since 1972, x	0	4	8	12	16	20	24	28
Men's time, m	51.2	50.0	50.4	49.8	48.6	49.0	48.7	48.3
Women's time, w	58.6	55.7	54.8	55.9	54.9	54.6	54.4	53.8

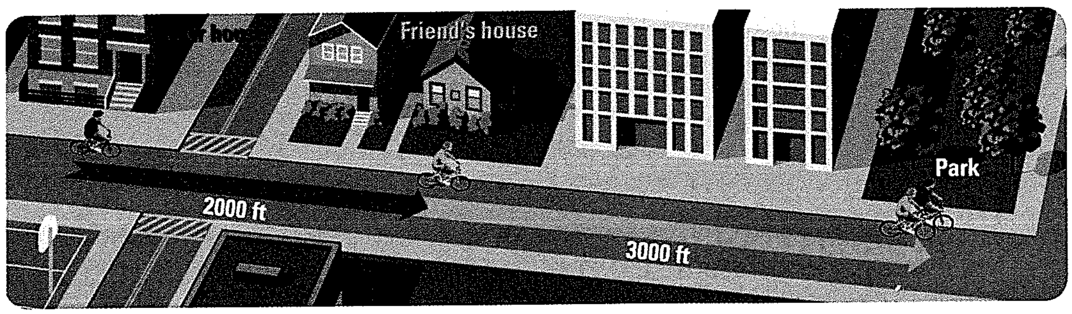
a. Use a graphing calculator to fit a line to the data pairs (x, m).

b. Use a graphing calculator to fit a line to the data pairs (x, w).

c. Graph the lines and predict when the women's performance will catch up to the men's performance.

d. Do you think your prediction from part (c) is reasonable? *Explain.*

40. **CHALLENGE** Your house and your friend's house are both on a street that passes by a park, as shown below.



At 1:00 P.M., you and your friend leave your houses on bicycles and head toward the park. You travel at a speed of 25 feet per second, and your friend also travels at a constant speed. You both reach the park at the same time.

a. Write and graph an equation giving your distance d (in feet) from the park after t seconds.

b. At what speed does your friend travel to the park? *Explain* how you found your answer.

c. Write an equation giving your friend's distance d (in feet) from the park after t seconds. Graph the equation in the same coordinate plane you used for part (a).

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

41. A realtor earns a base salary of $31,000 plus 2.5% of the value of any real estate sold. Which equation best represents the realtor's total salary, s, in terms of the value, x, of the real estate sold?

Ⓐ $s = 31{,}000 - 0.025x$ Ⓑ $s = 31{,}000x + 0.025$

Ⓒ $s = 31{,}000 + 0.025x$ Ⓓ $s = 31{,}000 + 2.5x$

42. In $\triangle MNP$, the measure of $\angle M$ is 40°. The measure of $\angle N$ is four times the measure of $\angle P$. What is $m\angle P$?

Ⓐ 28° Ⓑ 35° Ⓒ 45° Ⓓ 112°

Graphing Calculator **ACTIVITY** *Use after Lesson 3.1*

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3.1 Graph Systems of Equations

QUESTION **How can you solve a system of linear equations using a graphing calculator?**

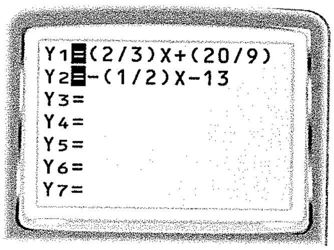
In Lesson 3.1, you learned to *estimate* the solution of a linear system by graphing. You can use the *intersect* feature of a graphing calculator to get an answer that is very close to, and sometimes *exactly* equal to, the actual solution.

EXAMPLE **Solve a system**

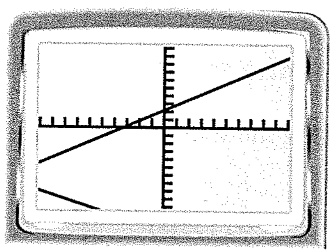
Use a graphing calculator to solve the system.

$6x - 9y = -20$ **Equation 1**
$2x + 4y = -52$ **Equation 2**

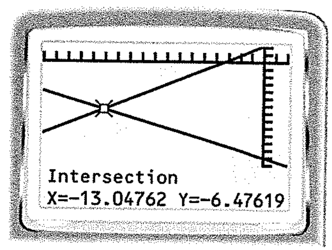
STEP 1 ***Enter equations***
Solve each equation for y. Then enter the revised equations into a graphing calculator.



STEP 2 ***Graph equations***
Graph the equations in the standard viewing window.



STEP 3 ***Find the solution***
Adjust the viewing window, and use the *intersect* feature to find the intersection point.



▸ The solution is about (−13.05, −6.48).

PRACTICE

Solve the linear system using a graphing calculator.

1. $y = -x + 2$
$y = 2x - 5$

2. $y = -2x + 15$
$y = 5x - 4$

3. $-9x + 7y = 14$
$-3x + y = -17$

4. $-11x - 6y = -6$
$4x + 2y = 10$

5. $5x + 8y = -48$
$x + 3y = 27$

6. $-2x + 16y = 56$
$4x + 7y = -35$

7. **VACATION** Your family is planning a 7 day trip to Texas. You estimate that it will cost \$275 per day in San Antonio and \$400 per day in Dallas. Your budget for the 7 days is \$2300. How many days should you spend in each city?

8. **MOVIE TICKETS** In one day, a movie theater collected \$4600 from 800 people. The price of admission is \$7 for an adult and \$5 for a child. How many adults and how many children were admitted to the movie theater that day?

3.2 Solve Linear Systems Algebraically

PA M11.D.2.1.4 Write and/or solve systems of equations using graphing, substitution and/or elimination (limit systems to 2 equations).

Before You solved linear systems graphically.

Now You will solve linear systems algebraically.

Why? So you can model guitar sales, as in Ex. 55.

Key Vocabulary
- **substitution method**
- **elimination method**

In this lesson, you will study two algebraic methods for solving linear systems. The first method is called the **substitution method**.

KEY CONCEPT — *For Your Notebook*

The Substitution Method

STEP 1 **Solve** one of the equations for one of its variables.

STEP 2 **Substitute** the expression from Step 1 into the other equation and solve for the other variable.

STEP 3 **Substitute** the value from Step 2 into the revised equation from Step 1 and solve.

EXAMPLE 1 Use the substitution method

Solve the system using the substitution method.

$2x + 5y = -5$ **Equation 1**

$x + 3y = 3$ **Equation 2**

Solution

STEP 1 **Solve** Equation 2 for x.

$x = -3y + 3$ **Revised Equation 2**

STEP 2 **Substitute** the expression for x into Equation 1 and solve for y.

$2x + 5y = -5$ **Write Equation 1.**

$2(\mathbf{-3y + 3}) + 5y = -5$ **Substitute $-3y + 3$ for x.**

$y = 11$ **Solve for y.**

STEP 3 **Substitute** the value of y into revised Equation 2 and solve for x.

$x = -3y + 3$ **Write revised Equation 2.**

$x = -3(\mathbf{11}) + 3$ **Substitute 11 for y.**

$x = -30$ **Simplify.**

▶ The solution is $(-30, 11)$.

CHECK Check the solution by substituting into the original equations.

$2(\mathbf{-30}) + 5(\mathbf{11}) \stackrel{?}{=} -5$ **Substitute for x and y.** $\mathbf{-30} + 3(\mathbf{11}) \stackrel{?}{=} 3$

$-5 = -5$ ✓ **Solution checks.** $3 = 3$ ✓

ELIMINATION METHOD Another algebraic method that you can use to solve a system of equations is the **elimination method**. The goal of this method is to eliminate one of the variables by adding equations.

KEY CONCEPT ***For Your Notebook***

The Elimination Method

STEP 1 **Multiply** one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.

STEP 2 **Add** the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

STEP 3 **Substitute** the value obtained in Step 2 into either of the original equations and solve for the other variable.

EXAMPLE 2 Use the elimination method

Solve the system using the elimination method.

$$\begin{aligned} 3x - 7y &= 10 \quad \textbf{Equation 1} \\ 6x - 8y &= 8 \quad \textbf{Equation 2} \end{aligned}$$

Solution

STEP 1 **Multiply** Equation 1 by **−2** so that the coefficients of x differ only in sign.

$$\begin{array}{rcr} 3x - 7y = 10 & \xrightarrow{\times -2} & -6x + 14y = -20 \\ 6x - 8y = 8 & \longrightarrow & 6x - 8y = 8 \\ \hline & & 6y = -12 \\ & & y = -2 \end{array}$$

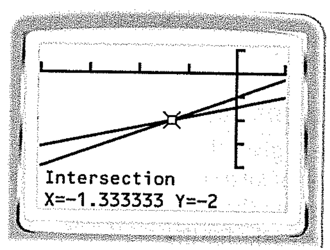
STEP 2 **Add** the revised equations and solve for y.

STEP 3 **Substitute** the value of y into one of the original equations. Solve for x.

$$\begin{aligned} 3x - 7y &= 10 && \textbf{Write Equation 1.} \\ 3x - 7(-2) &= 10 && \textbf{Substitute −2 for } y. \\ 3x + 14 &= 10 && \textbf{Simplify.} \\ x &= -\frac{4}{3} && \textbf{Solve for } x. \end{aligned}$$

▶ The solution is $\left(-\frac{4}{3}, -2\right)$.

CHECK You can check the solution algebraically using the method shown in Example 1. You can also use a graphing calculator to check the solution.



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SOLVE SYSTEMS
In Example 2, one coefficient of x is a multiple of the other. In this case, it is easier to eliminate the x-terms because you need to multiply only one equation by a constant.

✓ **GUIDED PRACTICE** for Examples 1 and 2

Solve the system using the substitution or the elimination method.

1. $4x + 3y = -2$
   $x + 5y = -9$

2. $3x + 3y = -15$
   $5x - 9y = 3$

3. $3x - 6y = 9$
   $-4x + 7y = -16$

3.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 5, 29, and 59
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 40, 50, 57, 58, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To solve a linear system where one of the coefficients is 1 or −1, it is usually easiest to use the _?_ method.

2. ★ **WRITING** *Explain* how to use the elimination method to solve a linear system.

EXAMPLES 1 and 4 on pp. 160–163 for Exs. 3–14

SUBSTITUTION METHOD **Solve the system using the substitution method.**

3. $2x + 5y = 7$
   $x + 4y = 2$

4. $3x + y = 16$
   $2x - 3y = -4$

5. $6x - 2y = 5$
   $-3x + y = 7$

6. $x + 4y = 1$
   $3x + 2y = -12$

7. $3x - y = 2$
   $6x + 3y = 14$

8. $3x - 4y = -5$
   $-x + 3y = -5$

9. $3x + 2y = 6$
   $x - 4y = -12$

10. $6x - 3y = 15$
    $-2x + y = -5$

11. $3x + y = -1$
    $2x + 3y = 18$

12. $2x - y = 1$
    $8x + 4y = 6$

13. $3x + 7y = 13$
    $x + 3y = -7$

14. $2x + 5y = 10$
    $-3x + y = 36$

EXAMPLES 2 and 4 on pp. 161–163 for Exs. 15–27

ELIMINATION METHOD **Solve the system using the elimination method.**

15. $2x + 6y = 17$
    $2x - 10y = 9$

16. $4x - 2y = -16$
    $-3x + 4y = 12$

17. $3x - 4y = -10$
    $6x + 3y = -42$

18. $4x - 3y = 10$
    $8x - 6y = 20$

19. $5x - 3y = -3$
    $2x + 6y = 0$

20. $10x - 2y = 16$
    $5x + 3y = -12$

21. $2x + 5y = 14$
    $3x - 2y = -36$

22. $7x + 2y = 11$
    $-2x + 3y = 29$

23. $3x + 4y = 18$
    $6x + 8y = 18$

24. $2x + 5y = 13$
    $6x + 2y = -13$

25. $4x - 5y = 13$
    $6x + 2y = 48$

26. $6x - 4y = 14$
    $2x + 8y = 21$

27. **ERROR ANALYSIS** *Describe* and correct the error in the first step of solving the system.

    $3x + 2y = 7$
    $5x + 4y = 15$

$$\begin{aligned} -6x - 4y &= 7 \\ 5x + 4y &= 15 \\ \hline -x \qquad &= 22 \\ x &= -22 \end{aligned}$$

CHOOSING A METHOD **Solve the system using any algebraic method.**

28. $3x + 2y = 11$
    $4x + y = -2$

29. $2x - 3y = 8$
    $-4x + 5y = -10$

30. $3x + 7y = -1$
    $2x + 3y = 6$

31. $4x - 10y = 18$
    $-2x + 5y = -9$

32. $3x - y = -2$
    $5x + 2y = 15$

33. $x + 2y = -8$
    $3x - 4y = -24$

34. $2x + 3y = -6$
    $3x - 4y = 25$

35. $3x + y = 15$
    $-x + 2y = -19$

36. $4x - 3y = 8$
    $-8x + 6y = 16$

37. $4x - y = -10$
    $6x + 2y = -1$

38. $7x + 5y = -12$
    $3x - 4y = 1$

39. $2x + y = -1$
    $-4x + 6y = 6$

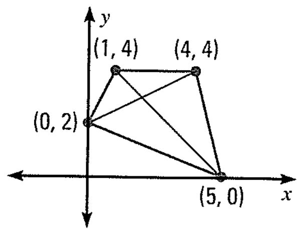
40. ★ **MULTIPLE CHOICE** What is the solution of the linear system?

$$3x + 2y = 4$$
$$6x - 3y = -27$$

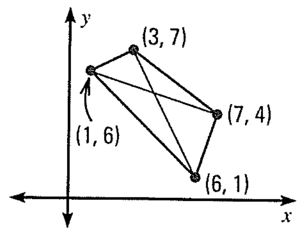
Ⓐ $(-2, -5)$ Ⓑ $(-2, 5)$ Ⓒ $(2, -5)$ Ⓓ $(2, 5)$

GEOMETRY Find the coordinates of the point where the diagonals of the quadrilateral intersect.

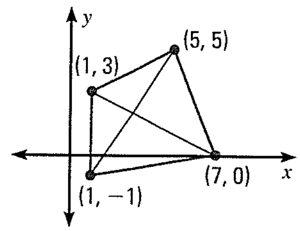
41.



42.



43.



SOLVING LINEAR SYSTEMS Solve the system using any algebraic method.

44. $0.02x - 0.05y = -0.38$
$0.03x + 0.04y = 1.04$

45. $0.05x - 0.03y = 0.21$
$0.07x + 0.02y = 0.16$

46. $\frac{2}{3}x + 3y = -34$
$x - \frac{1}{2}y = -1$

47. $\frac{1}{2}x + \frac{2}{3}y = \frac{5}{6}$
$\frac{5}{12}x + \frac{7}{12}y = \frac{3}{4}$

48. $\frac{x+3}{4} + \frac{y-1}{3} = 1$
$2x - y = 12$

49. $\frac{x-1}{2} + \frac{y+2}{3} = 4$
$x - 2y = 5$

50. ★ **OPEN-ENDED MATH** Write a system of linear equations that has $(-1, 4)$ as its only solution. Verify that $(-1, 4)$ is a solution using either the substitution method or the elimination method.

SOLVING NONLINEAR SYSTEMS Use the elimination method to solve the system.

51. $7y + 18xy = 30$
$13y - 18xy = 90$

52. $xy - x = 14$
$5 - xy = 2x$

53. $2xy + y = 44$
$32 - xy = 3y$

54. **CHALLENGE** Find values of r, s, and t that produce the indicated solution(s).

$$-3x - 5y = 9$$
$$rx + sy = t$$

a. No solution **b.** Infinitely many solutions **c.** A solution of $(2, -3)$

PROBLEM SOLVING

EXAMPLE 3 on p. 162 for Exs. 55–59

55. **GUITAR SALES** In one week, a music store sold 9 guitars for a total of \$3611. Electric guitars sold for \$479 each and acoustic guitars sold for \$339 each. How many of each type of guitar were sold?

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56. **COUNTY FAIR** An adult pass for a county fair costs \$2 more than a children's pass. When 378 adult and 214 children's passes were sold, the total revenue was \$2384. Find the cost of an adult pass.

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57. ★ **SHORT RESPONSE** A company produces gas mowers and electric mowers at two factories. The company has orders for 2200 gas mowers and 1400 electric mowers. The production capacity of each factory (in mowers per week) is shown in the table.

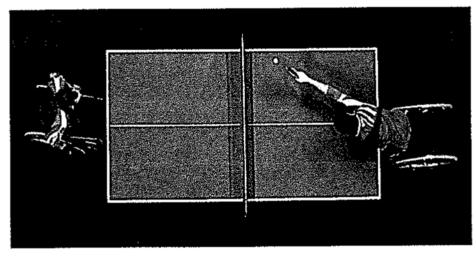
	Factory A	Factory B
Gas mowers	200	400
Electric mowers	100	300

Describe how the company can fill its orders by operating the factories simultaneously at full capacity. Write and solve a linear system to support your answer.

58. ★ **MULTIPLE CHOICE** The cost of 11 gallons of regular gasoline and 16 gallons of premium gasoline is $58.55. Premium costs $.20 more per gallon than regular. What is the cost of a gallon of premium gasoline?

(A) $2.05 (B) $2.25 (C) $2.29 (D) $2.55

59. **TABLE TENNIS** One evening, 76 people gathered to play doubles and singles table tennis. There were 26 games in progress at one time. A doubles game requires 4 players and a singles game requires 2 players. How many games of each kind were in progress at one time if all 76 people were playing?



60. ★ **EXTENDED RESPONSE** A local hospital is holding a two day marathon walk to raise funds for a new research facility. The total distance of the marathon is 26.2 miles. On the first day, Martha starts walking at 10:00 A.M. She walks 4 miles per hour. Carol starts two hours later than Martha but decides to run to catch up to Martha. Carol runs at a speed of 6 miles per hour.

a. Write an equation to represent the distance Martha travels.

b. Write an equation to represent the distance Carol travels.

c. Solve the system of equations to find when Carol will catch up to Martha.

d. Carol wants to reduce the time she takes to catch up to Martha by 1 hour. How can she do this by changing her starting time? How can she do this by changing her speed? *Explain* whether your answers are reasonable.

61. **BUSINESS** A nut wholesaler sells a mix of peanuts and cashews. The wholesaler charges $2.80 per pound for peanuts and $5.30 per pound for cashews. The mix is to sell for $3.30 per pound. How many pounds of peanuts and how many pounds of cashews should be used to make 100 pounds of the mix?

62. **AVIATION** Flying with the wind, a plane flew 1000 miles in 5 hours. Flying against the wind, the plane could fly only 500 miles in the same amount of time. Find the speed of the plane in calm air and the speed of the wind.

63. **CHALLENGE** For a recent job, an electrician earned $50 per hour, and the electrician's apprentice earned $20 per hour. The electrician worked 4 hours more than the apprentice, and together they earned a total of $550. How much money did each person earn?

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

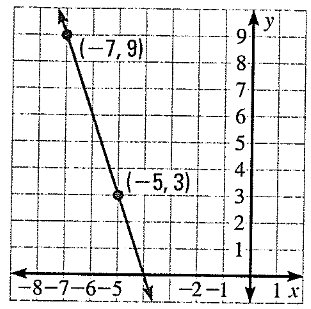
64. What is the y-intercept of the line shown?

Ⓐ $b = -18$

Ⓑ $b = -12$

Ⓒ $b = -8$

Ⓓ $b = -4$



65. Which two lines are parallel?

Ⓐ $3x + 2y = 8$ and $6x - 4y = -18$

Ⓑ $2x + 6y = 9$ and $4x + 12y = -15$

Ⓒ $3x + 2y = 8$ and $2x + 3y = 10$

Ⓓ $2x + 6y = 9$ and $-4x + 12y = 12$

66. Which ordered pair represents the x-intercept of the equation $4x - 5y = 20$?

Ⓐ $(-4, 0)$ Ⓑ $(0, -4)$ Ⓒ $(0, 5)$ Ⓓ $(5, 0)$

QUIZ *for Lessons 3.1–3.2*

Graph the linear system and estimate the solution. Then check the solution algebraically. ***(p. 153)***

1. $3x + y = 11$
   $x - 2y = -8$

2. $2x + y = -5$
   $-x + 3y = 6$

3. $x - 2y = -2$
   $3x + y = -20$

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.* ***(p. 153)***

4. $4x + 8y = 8$
   $x + 2y = 6$

5. $-5x + 3y = -5$
   $y = \frac{5}{3}x + 1$

6. $x - 2y = 2$
   $2x - y = -5$

Solve the system using the substitution method. ***(p. 160)***

7. $3x - y = -4$
   $x + 3y = -28$

8. $x + 5y = 1$
   $-3x + 4y = 16$

9. $6x + y = -6$
   $4x + 3y = 17$

Solve the system using the elimination method. ***(p. 160)***

10. $2x - 3y = -1$
    $2x + 3y = -19$

11. $3x - 2y = 10$
    $-6x + 4y = -20$

12. $2x + 3y = 17$
    $5x + 8y = 20$

13. **HOME ELECTRONICS** To connect a VCR to a television set, you need a cable with special connectors at both ends. Suppose you buy a 6 foot cable for \$15.50 and a 3 foot cable for \$10.25. Assuming that the cost of a cable is the sum of the cost of the two connectors and the cost of the cable itself, what would you expect to pay for a 4 foot cable? *Explain* how you got your answer.

3.3 Graph Systems of Linear Inequalities

PA M11.D.2.1.2 Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

Before You graphed linear inequalities.

Now You will graph systems of linear inequalities.

Why? So you can model heart rates during exercise, as in Ex. 39.

Key Vocabulary
- **system of linear inequalities**
- **solution of a system of inequalities**
- **graph of a system of inequalities**

The following is an example of a **system of linear inequalities** in two variables.

$x + y \leq 8$ **Inequality 1**
$4x - y > 6$ **Inequality 2**

A **solution** of a system of inequalities is an ordered pair that is a solution of each inequality in the system. For example, $(5, -2)$ is a solution of the system above. The **graph** of a system of inequalities is the graph of all solutions of the system.

KEY CONCEPT *For Your Notebook*

Graphing a System of Linear Inequalities

To graph a system of linear inequalities, follow these steps:

STEP 1 **Graph** each inequality in the system. You may want to use colored pencils to distinguish the different half-planes.

STEP 2 **Identify** the region that is common to all the graphs of the inequalities. This region is the graph of the system. If you used colored pencils, the graph of the system is the region that has been shaded with every color.

EXAMPLE 1 Graph a system of two inequalities

Graph the system of inequalities.

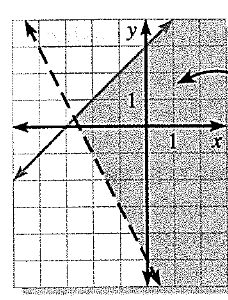
$y > -2x - 5$ **Inequality 1**
$y \leq x + 3$ **Inequality 2**

REVIEW INEQUALITIES
For help with graphing linear inequalities in two variables, see p. 132.

Solution

STEP 1 **Graph** each inequality in the system. Use **red** for $y > -2x - 5$ and **blue** for $y \leq x + 3$.

STEP 2 **Identify** the region that is common to both graphs. It is the region that is shaded **purple**.



The graph of the system is the intersection of the red and blue regions.

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EXAMPLE 2 Graph a system with no solution

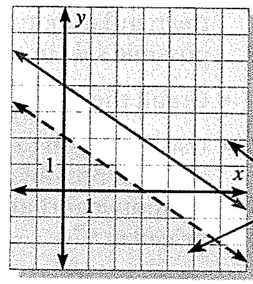
Graph the system of inequalities.

$2x + 3y < 6$ **Inequality 1**

$y \geq -\frac{2}{3}x + 4$ **Inequality 2**

Solution

STEP 1 **Graph** each inequality in the system. Use **red** for $2x + 3y < 6$ and **blue** for $y \geq -\frac{2}{3}x + 4$.



The red and blue regions do not intersect.

STEP 2 **Identify** the region that is common to both graphs. There is no region shaded both red and blue. So, the system has no solution.

EXAMPLE 3 Graph a system with an absolute value inequality

Graph the system of inequalities.

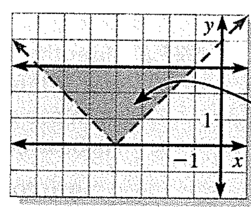
$y \leq 3$ **Inequality 1**

$y > |x + 4|$ **Inequality 2**

REVIEW ABSOLUTE VALUE
For help with graphing absolute value inequalities, see p. 132.

Solution

STEP 1 **Graph** each inequality in the system. Use **red** for $y \leq 3$ and **blue** for $y > |x + 4|$.



The graph of the system is the intersection of the red and blue regions.

STEP 2 **Identify** the region that is common to both graphs. It is the region that is shaded **purple**.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the system of inequalities.

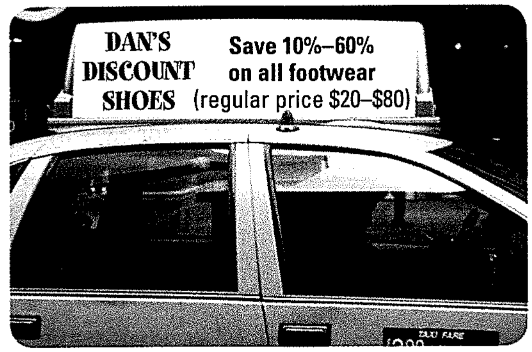
1. $y \leq 3x - 2$ $y > -x + 4$	**2.** $2x - \frac{1}{2}y \geq 4$ $4x - y \leq 5$	**3.** $x + y > -3$ $-6x + y < 1$
4. $y \leq 4$ $y \geq \lvert x - 5\rvert$	**5.** $y > -2$ $y \leq -\lvert x + 2\rvert$	**6.** $y \geq 2\lvert x + 1\rvert$ $y < x + 1$

SYSTEMS OF THREE OR MORE INEQUALITIES You can also graph a system of three or more linear inequalities, as shown in Example 4.

EXAMPLE 4 Solve a multi-step problem

SHOPPING A discount shoe store is having a sale, as described in the advertisement shown.

- Use the information in the ad to write a system of inequalities for the regular footwear prices and possible sale prices.
- Graph the system of inequalities.
- Use the graph to estimate the range of possible sale prices for footwear that is regularly priced at $70.



Solution

STEP 1 **Write** a system of inequalities. Let x be the regular footwear price and let y be the sale price. From the information in the ad, you can write the following four inequalities.

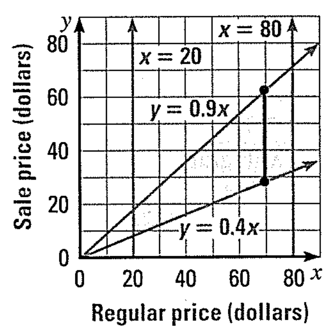
$x \geq 20$ **Regular price must be at least $20.**

$x \leq 80$ **Regular price can be at most $80.**

$y \geq 0.4x$ **Sale price is at least (100 − 60)% = 40% of regular price.**

$y \leq 0.9x$ **Sale price is at most (100 − 10)% = 90% of regular price.**

STEP 2 **Graph** each inequality in the system. Then identify the region that is common to all the graphs. It is the region that is shaded.



STEP 3 **Identify** the range of possible sale prices for $70 footwear. From the graph you can see that when $x = 70$, the value of y is between these values:

$0.4(70) = 28$ and $0.9(70) = 63$

So, the value of y satisfies $28 \leq y \leq 63$.

▶ Therefore, footwear regularly priced at $70 sells for between $28 and $63, inclusive, during the sale.

✓ GUIDED PRACTICE for Example 4

7. **WHAT IF?** In Example 4, suppose the advertisement showed a range of discounts of 20%–50% and a range of regular prices of $40–$100.
   a. Write and graph a system of inequalities for the regular footwear prices and possible sale prices.
   b. Use the graph to estimate the range of possible sale prices for footwear that is regularly priced at $60.

3.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 9, 19, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 26, 27, 36, and 39

◆ = **MULTIPLE REPRESENTATIONS** Ex. 37

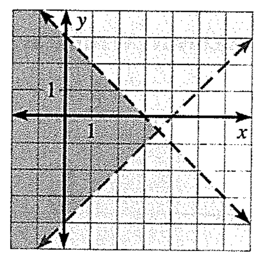
SKILL PRACTICE

1. **VOCABULARY** What must be true in order for an ordered pair to be a solution of a system of linear inequalities?

2. ★ **WRITING** *Describe* how to graph a system of linear inequalities.

EXAMPLES 1, 2, and 3 on pp. 168–169 for Exs. 3–16

3. ★ **MULTIPLE CHOICE** Which system of inequalities is represented by the graph?



Ⓐ $x + y > 3$
$-x + y < -4$

Ⓑ $-x + y \geq -4$
$x + y \leq 3$

Ⓒ $-2x + y > -4$
$2x + y < 3$

Ⓓ $-x + y > -4$
$x + y < 3$

SYSTEMS OF TWO INEQUALITIES Graph the system of inequalities.

4. $x > -1$
$x < 3$

5. $x \leq 2$
$y \leq 5$

6. $y \geq 5$
$y \leq 1$

7. $-x + y < -3$
$-x + y > 4$

8. $y < 10$
$y > |x|$

9. $4x - 4y \geq -16$
$-x + 2y \geq -4$

10. $-x \geq y$
$-x + y \geq -5$

11. $y > |x| - 4$
$3y < -2x + 9$

12. $x + y \geq -3$
$-6x + 4y < 14$

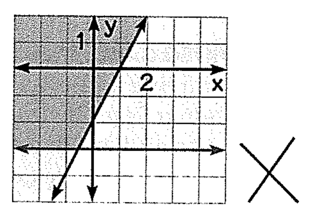
13. $2y < -5x - 10$
$5x + 2y > -2$

14. $3x - y > 12$
$-x + 8y > -4$

15. $x - 4y \leq -10$
$y \leq 3|x - 1|$

16. **ERROR ANALYSIS** *Describe* and correct the error in graphing the system of inequalities.

$y \geq -3$
$y \leq 2x - 2$



EXAMPLE 4 on p. 170 for Exs. 17–25

SYSTEMS OF THREE OR MORE INEQUALITIES Graph the system of inequalities.

17. $x < 6$
$y > -1$
$y < x$

18. $x \geq -8$
$y \leq -1$
$y < -2x - 4$

19. $3x + 2y > -6$
$-5x + 2y > -2$
$y < 5$

20. $x + y < 5$
$2x - y > 0$
$-x + 5y > -20$

21. $x \geq 2$
$-3x + y < -1$
$4x + 3y < 12$

22. $y \geq x$
$x + 3y < 5$
$2x + y \geq -3$

23. $y \geq 0$
$x > 3$
$x + y \geq -2$
$y < 4x$

24. $x + y < 5$
$x + y > -5$
$x - y < 4$
$x - y > -2$

25. $x \leq 10$
$x \geq -2$
$3x + 2y < 6$
$6x + 4y > -12$

26. ★ **MULTIPLE CHOICE** Which quadrant of the coordinate plane contains no solutions of the system of inequalities?

$$y \le -|x-3| + 2$$
$$4x - 5y \le 20$$

Ⓐ Quadrant I Ⓑ Quadrant II Ⓒ Quadrant III Ⓓ Quadrant IV

27. ★ **OPEN-ENDED MATH** Write a system of two linear inequalities that has (2, −1) as a solution.

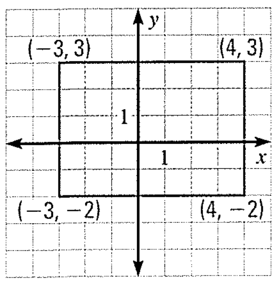
ABSOLUTE VALUE SYSTEMS Graph the system of inequalities.

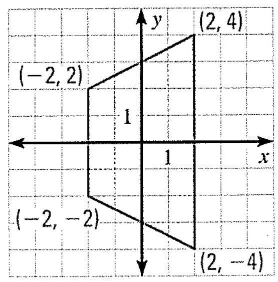
28. $y < |x|$
$y > -|x|$

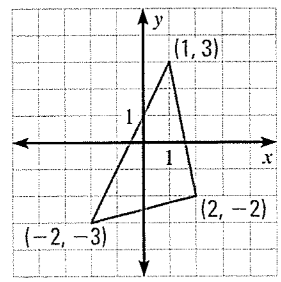
29. $y \le |x - 2|$
$y \ge |x| - 2$

30. $y \le -|x - 3| + 2$
$y > |x - 3| - 1$

CHALLENGE Write a system of linear inequalities for the shaded region.

31. 

32. 

33. 

PROBLEM SOLVING

EXAMPLE 4
on p. 170
for Exs. 34–39

34. **SUMMER JOBS** You can work at most 20 hours next week. You need to earn at least \$92 to cover your weekly expenses. Your dog-walking job pays \$7.50 per hour and your job as a car wash attendant pays \$6 per hour. Write a system of linear inequalities to model the situation.

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35. **VIDEO GAME SALE** An online media store is having a sale, as described in the ad shown. Use the information in the ad to write and graph a system of inequalities for the regular video game prices and possible sale prices. Then use the graph to estimate the range of possible sale prices for games that are regularly priced at \$20.

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36. ★ **SHORT RESPONSE** A book on the care of tropical fish states that the pH level of the water should be between 8.0 and 8.3 pH units and the temperature of the water should be between 76°F and 80°F. Let x be the pH level and y be the temperature. Write and graph a system of inequalities that describes the proper pH level and temperature of the water. *Compare* this graph to the graph you would obtain if the temperatures were given in degrees Celsius.

○ = **WORKED-OUT SOLUTIONS on p. WS1** ★ = **STANDARDIZED TEST PRACTICE** ◆ = **MULTIPLE REPRESENTATIONS**

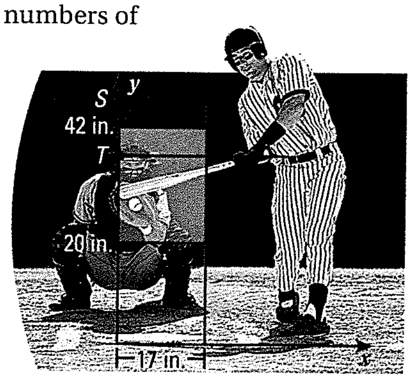
37. ❖ **MULTIPLE REPRESENTATIONS** The Junior-Senior Prom Committee must consist of 5 to 8 representatives from the junior and senior classes. The committee must include at least 2 juniors and at least 2 seniors. Let x be the number of juniors and y be the number of seniors.

a. **Writing a System** Write a system of inequalities to describe the situation.

b. **Graphing a System** Graph the system you wrote in part (a).

c. **Finding Solutions** Give two possible solutions for the numbers of juniors and seniors on the prom committee.

38. **BASEBALL** In baseball, the strike zone is a rectangle the width of home plate that extends from the batter's knees to a point halfway between the shoulders S and the top T of the uniform pants. The width of home plate is 17 inches. Suppose a batter's knees are 20 inches above the ground and the point halfway between his shoulders and the top of his pants is 42 inches above the ground. Write and graph a system of inequalities that represents the strike zone.



39. ★ **EXTENDED RESPONSE** A person's theoretical maximum heart rate (in heartbeats per minute) is $220 - x$ where x is the person's age in years ($20 \le x \le 65$). When a person exercises, it is recommended that the person strive for a heart rate that is at least 50% of the maximum and at most 75% of the maximum.

a. Write a system of linear inequalities that describes the given information.

b. Graph the system you wrote in part (a).

c. A 40-year-old person has a heart rate of 158 heartbeats per minute when exercising. Is the person's heart rate in the target zone? *Explain.*

40. **CHALLENGE** You and a friend are trying to guess the number of pennies in a jar. You both agree that the jar contains at least 500 pennies. You guess that there are x pennies, and your friend guesses that there are y pennies. The actual number of pennies in the jar is 1000. Write and graph a system of inequalities describing the values of x and y for which your guess is closer than your friend's guess to the actual number of pennies.

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

41. What is the value of x in the equation $-6(-2x + 1) = -12(x - 3) - 6x$?

Ⓐ -7 Ⓑ $-\frac{7}{5}$ Ⓒ $\frac{7}{5}$ Ⓓ 7

42. Rick enlarges a 4 inch by 6 inch digital photo using his computer. The dimensions of the resulting photo are 175% of the dimensions of the original photo. What are the dimensions of the enlarged photo?

Ⓐ 4.1 in. by 6.15 in. Ⓑ 5.3 in. by 8 in.

Ⓒ 7 in. by 10.5 in. Ⓓ 11 in. by 16.5 in.

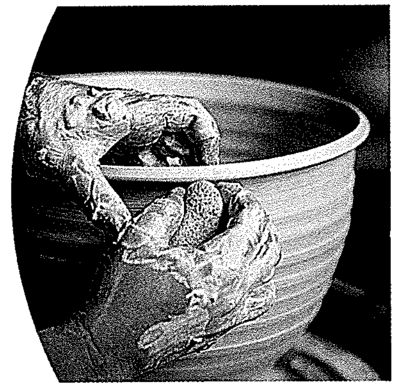
Extension
Use after Lesson 3.3

Use Linear Programming

GOAL Solve linear programming problems.

Key Vocabulary
- **constraints**
- **objective function**
- **linear programming**
- **feasible region**

BUSINESS A potter wants to make and sell serving bowls and plates. A bowl uses 5 pounds of clay. A plate uses 4 pounds of clay. The potter has 40 pounds of clay and wants to make at least 4 bowls.



Let x be the number of bowls made and let y be the number of plates made. You can represent the information above using linear inequalities called **constraints**.

$x \geq 4$	**Make at least 4 bowls.**
$y \geq 0$	**Number of plates cannot be negative.**
$5x + 4y \leq 40$	**Can use up to 40 pounds of clay.**

The profit on a bowl is \$35 and the profit on a plate is \$30. The potter's total profit P is given by the equation below, called the **objective function**.

$$P = 35x + 30y$$

It is reasonable for the potter to want to maximize profit subject to the given constraints. The process of maximizing or minimizing a linear objective function subject to constraints that are linear inequalities is called **linear programming**.

If the constraints are graphed, all of the points in the intersection are the combinations of bowls and plates that the potter can make. The intersection of the graphs is called the **feasible region**.

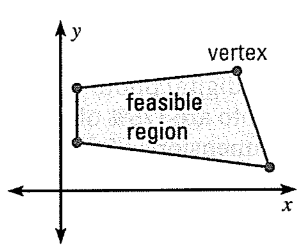
The following result tells you how to determine the optimal solution of a linear programming problem.

READING
A feasible region is *bounded* if it is completely enclosed by line segments.

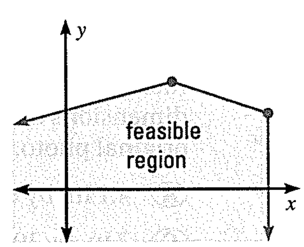
KEY CONCEPT — *For Your Notebook*

Optimal Solution of a Linear Programming Problem

If the feasible region for a linear programming problem is bounded, then the objective function has both a maximum value and a minimum value on the region. Moreover, the maximum and minimum values each occur at a vertex of the feasible region.



Bounded region



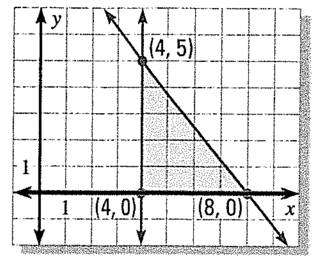
Unbounded region

EXAMPLE 1 Use linear programming to maximize profit

BUSINESS How many bowls and how many plates should the potter described on page 174 make in order to maximize profit?

Solution

STEP 1 **Graph** the system of constraints:



$x \geq 4$	**Make at least 4 bowls.**
$y \geq 0$	**Number of plates cannot be negative.**
$5x + 4y \leq 40$	**Can use up to 40 pounds of clay.**

STEP 2 **Evaluate** the profit function $P = 35x + 30y$ at each vertex of the feasible region.

At (4, 0): $P = 35(\mathbf{4}) + 30(\mathbf{0}) = 140$
At (8, 0): $P = 35(\mathbf{8}) + 30(\mathbf{0}) = 280$
At (4, 5): $P = 35(\mathbf{4}) + 30(\mathbf{5}) = 290$ ← **Maximum**

▶ The potter can maximize profit by making 4 bowls and 5 plates.

EXAMPLE 2 Solve a linear programming problem

Find the minimum value and the maximum value of the objective function $C = 4x + 5y$ subject to the following constraints.

$x \geq 0$

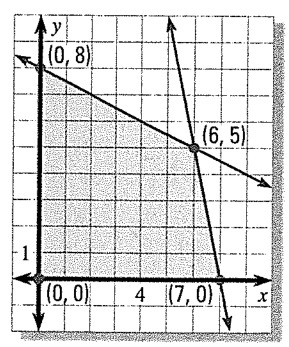
$y \geq 0$

$x + 2y \leq 16$

$5x + y \leq 35$

Solution

STEP 1 **Graph** the system of constraints. Find the coordinates of the vertices of the feasible region by solving systems of two linear equations. For example, the solution of the system



$$x + 2y = 16$$
$$5x + y = 35$$

gives the vertex (6, 5). The other three vertices are (0, 0), (7, 0), and (0, 8).

STEP 2 **Evaluate** the function $C = 4x + 5y$ at each of the vertices.

At (0, 0): $C = 4(\mathbf{0}) + 5(\mathbf{0}) = 0$ ← **Minimum**
At (7, 0): $C = 4(\mathbf{7}) + 5(\mathbf{0}) = 28$
At (6, 5): $C = 4(\mathbf{6}) + 5(\mathbf{5}) = 49$ ← **Maximum**
At (0, 8): $C = 4(\mathbf{0}) + 5(\mathbf{8}) = 40$

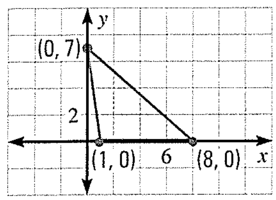
▶ The minimum value of C is 0. It occurs when $x = 0$ and $y = 0$.
The maximum value of C is 49. It occurs when $x = 6$ and $y = 5$.

PRACTICE

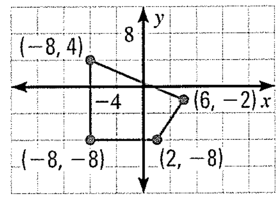
EXAMPLES 1 and 2 on p. 175 for Exs. 1–9

CHECKING VERTICES **Find the minimum and maximum values of the objective function for the given feasible region.**

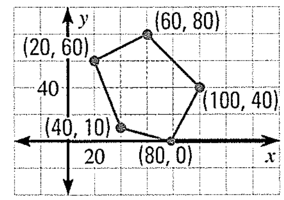
1. $C = x + 2y$



2. $C = 4x - 2y$



3. $C = 3x + 5y$



FINDING VALUES **Find the minimum and maximum values of the objective function subject to the given constraints.**

4. **Objective function:**
$C = 3x + 4y$

Constraints:
$x \geq 0$
$y \geq 0$
$x + y \leq 5$

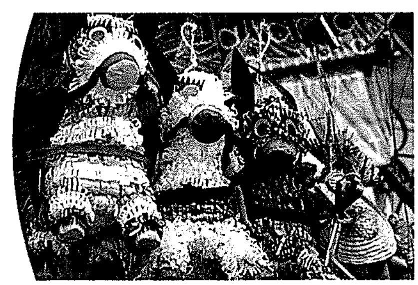
5. **Objective function:**
$C = 2x + 5y$

Constraints:
$x \leq 5$
$y \geq 3$
$-3x + 5y \leq 30$

6. **Objective function:**
$C = 3x + y$

Constraints:
$x \geq 0$
$y \geq -2$
$y \geq -x$
$x - 4y \geq -16$

7. **CRAFT FAIR** Piñatas are made to sell at a craft fair. It takes 2 hours to make a mini piñata and 3 hours to make a regular-sized piñata. The owner of the craft booth will make a profit of \$12 for each mini piñata sold and \$24 for each regular-sized piñata sold. If the craft booth owner has no more than 30 hours available to make piñatas and wants to have at least 12 piñatas to sell, how many of each size piñata should be made to maximize profit?



8. **MANUFACTURING** A company manufactures two types of printers, an inkjet printer and a laser printer. The company can make a total of 60 printers per day, and it has 120 labor-hours per day available. It takes 1 labor-hour to make an inkjet printer and 3 labor-hours to make a laser printer. The profit is \$40 per inkjet printer and \$60 per laser printer. How many of each type of printer should the company make to maximize its daily profit?

9. **FARM STAND SALES** You have 140 tomatoes and 13 onions left over from your garden. You want to use these to make jars of tomato sauce and jars of salsa to sell at a farm stand. A jar of tomato sauce requires 10 tomatoes and 1 onion, and a jar of salsa requires 5 tomatoes and $\frac{1}{4}$ onion. You will make a profit of \$2 on every jar of tomato sauce sold and a profit of \$1.50 on every jar of salsa sold. The owner of the farm stand wants at least three times as many jars of tomato sauce as jars of salsa. How many jars of each should you make to maximize profit?

10. **CHALLENGE** Consider the objective function $C = 2x + 3y$. Draw a feasible region that satisfies the given condition.
    a. C has a maximum value but no minimum value on the region.
    b. C has a minimum value but no maximum value on the region.

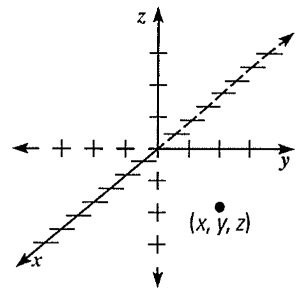
3.4 Graphing Linear Equations in Three Variables

MATERIALS • graph paper • ruler

QUESTION **What is the graph of a linear equation in three variables?**

A *linear equation in three variables* has the form $ax + by + cz = d$. You can graph this type of equation in a three-dimensional coordinate system formed by three axes that divide space into eight *octants*. Each point in space is represented by an *ordered triple* (x, y, z).

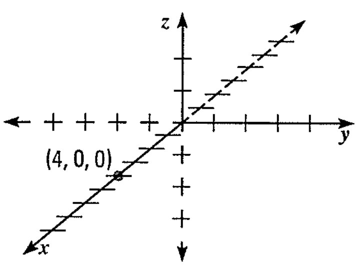
The graph of any equation in three variables is the set of all points (x, y, z) whose coordinates make the equation true. For a linear equation in three variables, the graph is a plane.



EXPLORE **Graph $3x + 4y + 6z = 12$**

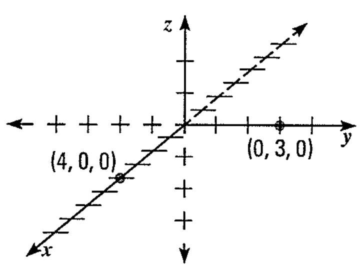
STEP 1 ***Find x-intercept***

Find the x-intercept by setting y and z equal to 0 and solving the resulting equation, $3x = 12$. The x-intercept is 4, so plot $(4, 0, 0)$.



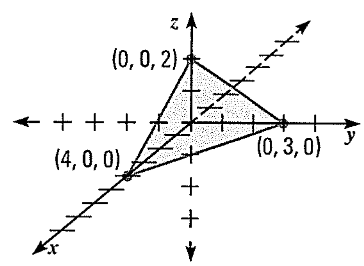
STEP 2 ***Find y-intercept***

Find the y-intercept by setting x and z equal to 0 and solving the resulting equation, $4y = 12$. The y-intercept is 3, so plot $(0, 3, 0)$.



STEP 3 ***Find z-intercept***

Find the z-intercept by setting x and y equal to 0 and solving the resulting equation, $6z = 12$. The z-intercept is 2, so plot $(0, 0, 2)$. Then connect the points.



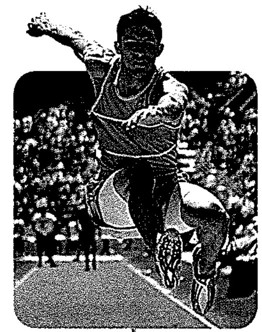
The triangular region shown in Step 3 is the portion of the graph of $3x + 4y + 6z = 12$ that lies in the first octant.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Sketch the graph of the equation.

1. $4x + 3y + 2z = 12$
2. $2x + 2y + 3z = 6$
3. $x + 5y + 3z = 15$
4. $5x - y - 2z = 10$
5. $-7x + 7y + 2z = 14$
6. $2x + 9y - 3z = -18$
7. Suppose three linear equations in three variables are graphed in the same coordinate system. In how many different ways can the planes intersect? *Explain* your reasoning.

3.4 Solve Systems of Linear Equations in Three Variables



Before You solved systems of equations in two variables.

Now You will solve systems of equations in three variables.

Why? So you can model the results of a sporting event, as in Ex. 45.

Key Vocabulary
- linear equation in three variables
- system of three linear equations
- solution of a system of three linear equations
- ordered triple

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.

The following is an example of a **system of three linear equations** in three variables.

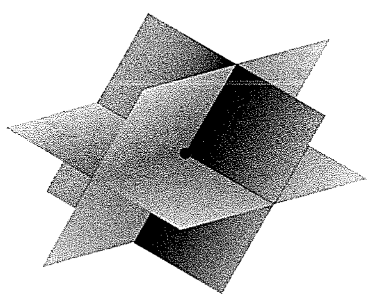
$$2x + y - z = 5 \quad \textbf{Equation 1}$$
$$3x - 2y + z = 16 \quad \textbf{Equation 2}$$
$$4x + 3y - 5z = 3 \quad \textbf{Equation 3}$$

A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space. The graphs of three such equations that form a system are three planes whose intersection determines the number of solutions of the system, as shown in the diagrams below.

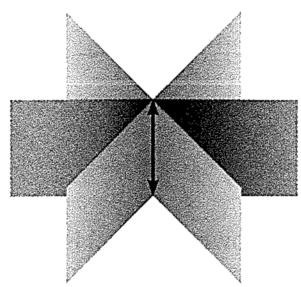
Exactly one solution

The planes intersect in a single point.



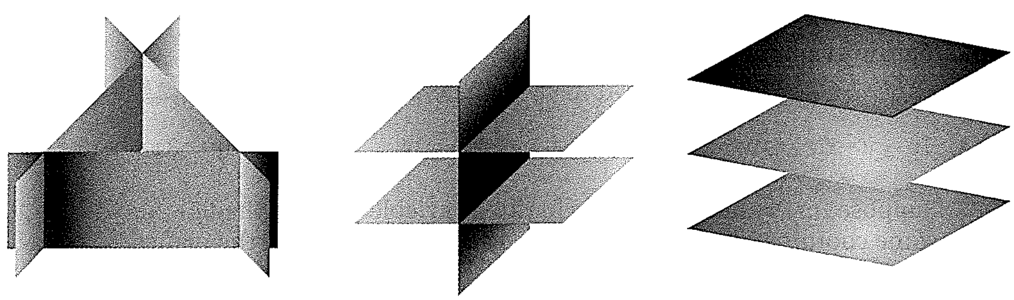
Infinitely many solutions

The planes intersect in a line or are the same plane.



No solution

The planes have no common point of intersection.



ELIMINATION METHOD The elimination method you studied in Lesson 3.2 can be extended to solve a system of linear equations in three variables.

KEY CONCEPT *For Your Notebook*

The Elimination Method for a Three-Variable System

STEP 1 **Rewrite** the linear system in three variables as a linear system in two variables by using the elimination method.

STEP 2 **Solve** the new linear system for both of its variables.

STEP 3 **Substitute** the values found in Step 2 into one of the original equations and solve for the remaining variable.

If you obtain a false equation, such as $0 = 1$, in any of the steps, then the system has no solution.

If you do not obtain a false equation, but obtain an identity such as $0 = 0$, then the system has infinitely many solutions.

EXAMPLE 1 Use the elimination method

Solve the system.

$$\begin{aligned} 4x + 2y + 3z &= 1 && \textbf{Equation 1}\\ 2x - 3y + 5z &= -14 && \textbf{Equation 2}\\ 6x - y + 4z &= -1 && \textbf{Equation 3} \end{aligned}$$

Solution

ANOTHER WAY
In Step 1, you could also eliminate x to get two equations in y and z, or you could eliminate z to get two equations in x and y.

STEP 1 **Rewrite** the system as a linear system in *two* variables.

$$\begin{array}{rl} 4x + 2y + 3z = 1 & \textbf{Add 2 times Equation 3}\\ \underline{12x - 2y + 8z = -2} & \textbf{to Equation 1.}\\ 16x \qquad + 11z = -1 & \textbf{New Equation 1} \end{array}$$

$$\begin{array}{rl} 2x - 3y + 5z = -14 & \textbf{Add } -3 \textbf{ times Equation 3}\\ \underline{-18x + 3y - 12z = 3} & \textbf{to Equation 2.}\\ -16x \qquad - 7z = -11 & \textbf{New Equation 2} \end{array}$$

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{array}{rl} 16x + 11z = -1 & \textbf{Add new Equation 1}\\ \underline{-16x - 7z = -11} & \textbf{and new Equation 2.}\\ 4z = -12 & \\ z = -3 & \textbf{Solve for } z.\\ x = 2 & \textbf{Substitute into new Equation 1 or 2 to find } x. \end{array}$$

STEP 3 **Substitute** $x = 2$ and $z = -3$ into an original equation and solve for y.

$$\begin{array}{rl} 6x - y + 4z = -1 & \textbf{Write original Equation 3.}\\ 6(2) - y + 4(-3) = -1 & \textbf{Substitute 2 for } x \textbf{ and } -3 \textbf{ for } z.\\ y = 1 & \textbf{Solve for } y. \end{array}$$

▶ The solution is $x = 2$, $y = 1$, and $z = -3$, or the ordered triple $(2, 1, -3)$. Check this solution in each of the original equations.

3.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 11, 25, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 24, 34, 45, and 47

SKILL PRACTICE

1. **VOCABULARY** Write a linear equation in three variables. What is the graph of such an equation?

2. ★ **WRITING** *Explain* how to use the substitution method to solve a system of three linear equations in three variables.

EXAMPLES 1, 2, and 3 on pp. 179–180 for Exs. 3–14

CHECKING SOLUTIONS Tell whether the given ordered triple is a solution of the system.

3. $(1, 4, -3)$
$2x - y + z = -5$
$5x + 2y - 2z = 19$
$x - 3y + z = -5$

4. $(-1, -2, 5)$
$4x - y + 3z = 13$
$x + y + z = 2$
$x + 3y - 2z = -17$

5. $(6, 0, -3)$
$x + 4y - 2z = 12$
$3x - y + 4z = 6$
$-x + 3y + z = -9$

6. $(-5, 1, 0)$
$3x + 4y - 2z = -11$
$2x + y - z = 11$
$x + 4y + 3z = -1$

7. $(2, 8, 4)$
$3x - y + 5z = 34$
$x + 3y - 6z = 2$
$-3x + y - 2z = -6$

8. $(0, -4, 7)$
$2x + 4y - z = -23$
$x - 5y - 3z = -1$
$-x + y + 4z = 24$

ELIMINATION METHOD Solve the system using the elimination method.

9. $3x + y + z = 14$
$-x + 2y - 3z = -9$
$5x - y + 5z = 30$

10. $2x - y + 2z = -7$
$-x + 2y - 4z = 5$
$x + 4y - 6z = -1$

11. $3x - y + 2z = 4$
$6x - 2y + 4z = -8$
$2x - y + 3z = 10$

12. $4x - y + 2z = -18$
$-x + 2y + z = 11$
$3x + 3y - 4z = 44$

13. $5x + y - z = 6$
$x + y + z = 2$
$3x + y = 4$

14. $2x + y - z = 9$
$-x + 6y + 2z = -17$
$5x + 7y + z = 4$

EXAMPLE 4 on p. 181 for Exs. 15–20

SUBSTITUTION METHOD Solve the system using the substitution method.

15. $x + y - z = 4$
$3x + 2y + 4z = 17$
$-x + 5y + z = 8$

16. $2x - y - z = 15$
$4x + 5y + 2z = 10$
$-x - 4y + 3z = -20$

17. $4x + y + 5z = -40$
$-3x + 2y + 4z = 10$
$x - y - 2z = -2$

18. $x + 3y - z = 12$
$2x + 4y - 2z = 6$
$-x - 2y + z = -6$

19. $2x - y + z = -2$
$6x + 3y - 4z = 8$
$-3x + 2y + 3z = -6$

20. $3x + 5y - z = 12$
$x + y + z = 0$
$-x + 2y + 2z = -27$

ERROR ANALYSIS ***Describe*** **and correct the error in the first step of solving the system.**

$$2x + y - 2z = 23$$
$$3x + 2y + z = 11$$
$$x - y + z = -2$$

21.
$$\begin{array}{r} 2x + y - 2z = 23 \\ 6x + 2y + 2z = 22 \\ \hline 8x + 3y = 45 \end{array}$$

22.
$z = 11 + 3x + 2y$

$2x + y - 2(11 + 3x + 2y) = 23$

$-4x - 3y = 45$

23. ★ **MULTIPLE CHOICE** Which ordered triple is a solution of the system?

$$\begin{aligned} 2x + 5y + 3z &= 10 \\ 3x - y + 4z &= 8 \\ 5x - 2y + 7z &= 12 \end{aligned}$$

Ⓐ (7, 1, −3) Ⓑ (7, −1, −3) Ⓒ (7, 1, 3) Ⓓ (−7, 1, −3)

24. ★ **MULTIPLE CHOICE** Which ordered triple describes all of the solutions of the system?

$$\begin{aligned} 2x - 2y - z &= 6 \\ -x + y + 3z &= -3 \\ 3x - 3y + 2z &= 9 \end{aligned}$$

Ⓐ $(-x, x + 2, 0)$ Ⓑ $(x, x - 3, 0)$ Ⓒ $(x + 2, x, 0)$ Ⓓ $(0, y, y + 4)$

CHOOSING A METHOD Solve the system using any algebraic method.

25. $x + 5y - 2z = -1$, $-x - 2y + z = 6$, $-2x - 7y + 3z = 7$

26. $4x + 5y + 3z = 15$, $x - 3y + 2z = -6$, $-x + 2y - z = 3$

27. $6x + y - z = -2$, $x + 6y + 3z = 23$, $-x + y + 2z = 5$

28. $x + 2y = -1$, $3x - y + 4z = 17$, $-4x + 2y - 3z = -30$

29. $2x - y + 2z = -21$, $x + 5y - z = 25$, $-3x + 2y + 4z = 6$

30. $4x - 8y + 2z = 10$, $-3x + y - 2z = 6$, $2x - 4y + z = 8$

31. $-x + 5y - z = -16$, $2x + 3y + 4z = 18$, $x + y - z = -8$

32. $2x - y + 4z = 19$, $-x + 3y - 2z = -7$, $4x + 2y + 3z = 37$

33. $x + y + z = 3$, $3x - 4y + 2z = -28$, $-x + 5y + z = 23$

34. ★ **OPEN-ENDED MATH** Write a system of three linear equations in three variables that has the given number of solutions.

a. One solution **b.** No solution **c.** Infinitely many solutions

SYSTEMS WITH FRACTIONS Solve the system using any algebraic method.

35. $x + \frac{1}{2}y + \frac{1}{2}z = \frac{5}{2}$, $\frac{3}{4}x + \frac{1}{4}y + \frac{3}{2}z = \frac{7}{4}$, $\frac{1}{3}x + \frac{3}{2}y + \frac{2}{3}z = \frac{13}{6}$

36. $\frac{1}{3}x + \frac{5}{6}y + \frac{2}{3}z = \frac{4}{3}$, $\frac{1}{6}x + \frac{2}{3}y + \frac{1}{4}z = \frac{5}{6}$, $\frac{2}{3}x + \frac{1}{6}y + \frac{3}{2}z = \frac{4}{3}$

37. **REASONING** For what values of a, b, and c does the linear system shown have $(-1, 2, -3)$ as its only solution? *Explain* your reasoning.

$$\begin{aligned} x + 2y - 3z &= a \\ -x - y + z &= b \\ 2x + 3y - 2z &= c \end{aligned}$$

CHALLENGE Solve the system of equations. *Describe* each step of your solution.

38. $w + x + y + z = 2$, $2w - x + 2y - z = 1$, $-w + 2x - y + 2z = -2$, $3w + x + y - z = -5$

39. $2w + x - 3y + z = 4$, $w - 3x + y + z = 32$, $-w + 2x + 2y - z = -10$, $w + x - y + 3z = 14$

40. $w + 2x + 5y = 11$, $-2w + x + 4y + 2z = -7$, $w + 2x - 2y + 5z = 3$, $-3w + x = -1$

41. $2w + 7x - 3y = 41$, $-w - 2x + y = -13$, $-2w + 4x + z = 12$, $-w - x + y = -8$

PROBLEM SOLVING

EXAMPLE 4
on p. 181
for Exs. 42–47

42. **PIZZA SPECIALS** At a pizza shop, two small pizzas, a liter of soda, and a salad cost \$14; one small pizza, a liter of soda, and three salads cost \$15; and three small pizzas and a liter of soda cost \$16. What is the cost of one small pizza? of one liter of soda? of one salad?

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43. **HEALTH CLUB** The juice bar at a health club receives a delivery of juice at the beginning of each month. Over a three month period, the health club received 1200 gallons of orange juice, 900 gallons of pineapple juice, and 1000 gallons of grapefruit juice. The table shows the composition of each juice delivery. How many gallons of juice did the health club receive in each delivery?

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Juice	1st delivery	2nd delivery	3rd delivery
Orange	70%	50%	30%
Pineapple	20%	30%	30%
Grapefruit	10%	20%	40%

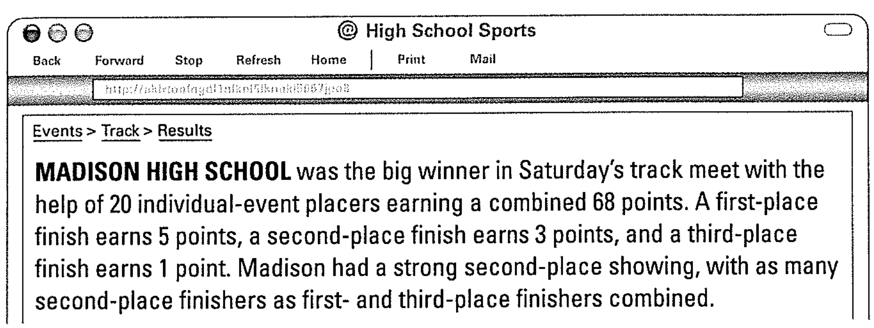
44. **MULTI-STEP PROBLEM** You make a tape of your friend's three favorite TV shows: a comedy, a drama, and a reality show. An episode of the comedy lasts 30 minutes, while an episode of the drama or the reality show lasts 60 minutes. The tape can hold 360 minutes of programming. You completely fill the tape with 7 episodes and include twice as many episodes of the drama as the comedy.

    a. Write a system of equations to represent this situation.

    b. Solve the system from part (a). How many episodes of each show are on the tape?

    c. How would your answer to part (b) change if you completely filled the tape with only 5 episodes but still included twice as many episodes of the drama as the comedy?

45. ★ **SHORT RESPONSE** The following Internet announcement describes the results of a high school track meet.



    a. Write and solve a system of equations to find the number of athletes who finished in first place, in second place, and in third place.

    b. Suppose the announcement had claimed that the Madison athletes scored a total of 70 points instead of 68 points. Show that this claim must be false because the solution of the resulting linear system is not reasonable.

46. FIELD TRIP You and two friends buy snacks for a field trip. You spend a total of \$8, Jeff spends \$9, and Curtis spends \$9. The table shows the amounts of mixed nuts, granola, and dried fruit that each person purchased. What is the price per pound of each type of snack?

	Mixed nuts	Granola	Dried fruit
You	1 lb	0.5 lb	1 lb
Jeff	2 lb	0.5 lb	0.5 lb
Curtis	1 lb	2 lb	0.5 lb

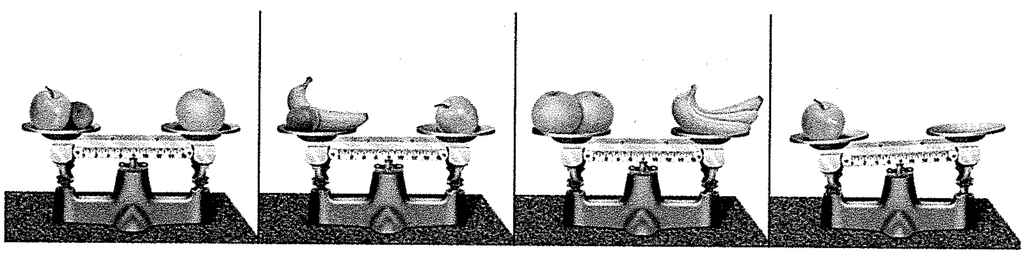
47. ★ EXTENDED RESPONSE A florist must make 5 identical bridesmaid bouquets for a wedding. She has a budget of \$160 and wants 12 flowers for each bouquet. Roses cost \$2.50 each, lilies cost \$4 each, and irises cost \$2 each. She wants twice as many roses as the other two types of flowers combined.

a. **Write** Write a system of equations to represent this situation.

b. **Solve** Solve the system of equations. How many of each type of flower should be in each bouquet?

c. **Analyze** Suppose there is no limitation on the total cost of the bouquets. Does the problem still have a unique solution? If so, state the unique solution. If not, give three possible solutions.

48. CHALLENGE Write a system of equations to represent the first three pictures below. Use the system to determine how many tangerines will balance the apple in the final picture. *Note:* The first picture shows that one tangerine and one apple balance one grapefruit.



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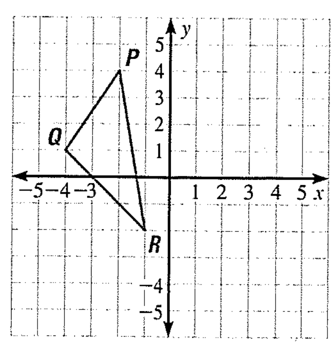
49. What are the vertices of a triangle congruent to $\triangle PQR$ shown at the right?

Ⓐ (3, 1), (1, −2), (4, −5)　　Ⓑ (2, 3), (−1, 1), (2, −2)

Ⓒ (0, 2), (−2, −1), (−3, −4)　　Ⓓ (−2, −3), (−4, 1), (−1, 4)



50. What special type of quadrilateral has the vertices $K(-4, 3)$, $L(-7, 3)$, $M(-9, -1)$, and $N(-2, -1)$?

Ⓐ Square　　Ⓑ Trapezoid

Ⓒ Kite　　Ⓓ Parallelogram

Lessons 3.1–3.4

1. **JEWELRY** Melinda is making jewelry to sell at a craft fair. The cost of materials is \$3.50 to make one necklace and \$2.50 to make one bracelet. She sells the necklaces for \$9.00 each and the bracelets for \$7.50 each. She spends a total of \$121 on materials and sells all of the jewelry for a total of \$324. Which system of equations relates x, the number of necklaces, and y, the number of bracelets?

   A $2.5x - 3.5y = 121$
   $7.5x - 9y = 324$

   B $2.5x + 3.5y = 324$
   $9x + 7.5y = 121$

   C $3.5x - 2.5y = 324$
   $7.5x + 9y = 121$

   D $3.5x + 2.5y = 121$
   $9x + 7.5y = 324$

2. **GIFT BASKETS** Mike is making gift baskets. Each basket will contain three different kinds of candles: tapers, pillars, and jar candles. Tapers cost \$1 each, pillars cost \$4 each, and jar candles cost \$6 each. Mike puts 8 candles costing a total of \$24 in each basket, and he includes as many tapers as pillars and jar candles combined. How many tapers are in a basket?

   A 1 taper

   B 2 tapers

   C 4 tapers

   D 5 tapers

3. **BASEBALL** From 1999 through 2002, the average annual salary s of players on two Major League Baseball teams can be modeled by the equations below, where t is the number of years since 1990.

   Florida Marlins: $s = 320{,}000t - 2{,}300{,}000$
   Kansas City Royals: $s = 440{,}000t - 3{,}500{,}000$

   In what year were the average annual salaries of the two baseball teams equal?

   A 1999 B 2000

   C 2001 D 2002

4. **RESTAURANT SEATING** A restaurant has 20 tables. Each table can seat either 4 people or 6 people. The restaurant can seat a total of 90 people. How many 6 seat tables does the restaurant have?

   A 1 table B 5 tables

   C 7 tables D 15 tables

5. **BUSINESS** A store orders rocking chairs, hand paints them, and sells the chairs for a profit. A small chair costs the store \$51 and sells for \$80. A large chair costs the store \$70 and sells for \$110. The store wants to pay no more than \$2000 for its next order of chairs and wants to sell them all for at least \$2750. What is a possible combination of small and large rocking chairs that the store can buy and sell?

   A 10 small chairs and 25 large chairs

   B 12 small chairs and 20 large chairs

   C 15 small chairs and 20 large chairs

   D 24 small chairs and 8 large chairs

6. **OPEN-ENDED** The table below shows the expected life spans for men and women born in the years 1996–2000.

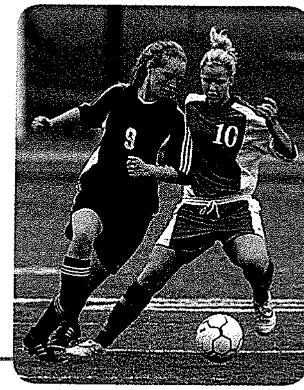
Years since 1996 (x)	Men's Life Span (years) (m)	Women's Life Span (years) (w)
0	73.0	79.0
1	73.6	79.4
2	73.8	79.5
3	73.9	79.4
4	74.3	79.7

   A. Write an equation for the best-fitting line for the data pairs (x, m).

   B. Write an equation for the best-fitting line for the data pairs (x, w).

   C. Assuming that the trend continues, what is the point of intersection of these two lines? *Explain* what both coordinates of this point represent.

3.5 Perform Basic Matrix Operations



Before You performed operations with real numbers.

Now You will perform operations with matrices.

Why? So you can organize sports data, as in Ex. 34.

Key Vocabulary
- **matrix**
- **dimensions**
- **elements**
- **equal matrices**
- **scalar**
- **scalar multiplication**

A **matrix** is a rectangular arrangement of numbers in rows and columns. For example, matrix A below has two rows and three columns. The **dimensions** of a matrix with m rows and n columns are $m \times n$ (read "m by n"). So, the dimensions of matrix A are 2×3. The numbers in a matrix are its **elements**.

The element in the first row and third column is 5.

$$A = \underbrace{\begin{bmatrix} 4 & -1 & 5 \\ 0 & 6 & 3 \end{bmatrix}}_{\textbf{3 columns}} \left.\vphantom{\begin{bmatrix} 4 \\ 0 \end{bmatrix}}\right\} \textbf{2 rows}$$

Two matrices are **equal** if their dimensions are the same and the elements in corresponding positions are equal.

KEY CONCEPT *For Your Notebook*

Adding and Subtracting Matrices

To add or subtract two matrices, simply add or subtract elements in corresponding positions. You can add or subtract matrices only if they have the same dimensions.

Adding Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

EXAMPLE 1 Add and subtract matrices

AVOID ERRORS
Be sure to verify that the dimensions of two matrices are equal before adding or subtracting them.

Perform the indicated operation, if possible.

a. $\begin{bmatrix} 3 & 0 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3+(-1) & 0+4 \\ -5+2 & -1+0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -3 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 7 & 4 \\ 0 & -2 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & -10 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 7-(-2) & 4-5 \\ 0-3 & -2-(-10) \\ -1-(-3) & 6-1 \end{bmatrix} = \begin{bmatrix} 9 & -1 \\ -3 & 8 \\ 2 & 5 \end{bmatrix}$

PROBLEM SOLVING

EXAMPLE 3 on p. 189 for Exs. 31–34

31. SNOWBOARD SALES A sporting goods store sells snowboards in several different styles and lengths. The matrices below show the number of each type of snowboard sold in 2003 and 2004. Write a matrix giving the change in sales for each type of snowboard from 2003 to 2004.

Sales for 2003

$$\begin{array}{l} \\ \text{Freeride} \\ \text{Alpine} \\ \text{Freestyle} \end{array} \begin{array}{c} \begin{array}{cccc} 150\text{ cm} & 155\text{ cm} & 160\text{ cm} & 165\text{ cm} \end{array} \\ \begin{bmatrix} 32 & 42 & 29 & 20 \\ 12 & 17 & 25 & 16 \\ 28 & 40 & 32 & 21 \end{bmatrix} \end{array}$$

Sales for 2004

$$\begin{array}{c} \begin{array}{cccc} 150\text{ cm} & 155\text{ cm} & 160\text{ cm} & 165\text{ cm} \end{array} \\ \begin{bmatrix} 32 & 47 & 30 & 19 \\ 5 & 16 & 20 & 14 \\ 29 & 39 & 36 & 31 \end{bmatrix} \end{array}$$

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32. FUEL ECONOMY A car dealership sells four different models of cars. The fuel economy (in miles per gallon) is shown below for each model. Organize the data using a matrix. Then write a new matrix giving the fuel economy figures for next year's models if each measure of fuel economy increases by 8%.

Economy car: 32 mpg in city driving, 40 mpg in highway driving

Mid-size car: 24 mpg in city driving, 34 mpg in highway driving

Mini-van: 18 mpg in city driving, 25 mpg in highway driving

SUV: 19 mpg in city driving, 22 mpg in highway driving

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33. ★ **EXTENDED RESPONSE** In a certain city, an electronics chain has a downtown store and a store in the mall. Each store carries three models of digital camera. Sales of the cameras for May and June are shown.

May Downtown sales: 31 of model A, 42 of model B, 18 of model C
Mall sales: 22 of model A, 25 of model B, 11 of model C

June Downtown sales: 25 of model A, 36 of model B, 12 of model C
Mall sales: 38 of model A, 32 of model B, 15 of model C

a. Organize the information using two matrices M and J that represent the sales for May and June, respectively.

b. Find $M + J$ and describe what this matrix sum represents.

c. Write a matrix giving the average monthly sales for the two month period.

34. ★ **SHORT RESPONSE** The matrices below show the numbers of female athletes who participated in selected NCAA sports and the average team size for each sport during the 2000–2001 and 2001–2002 seasons. Does the matrix $A + B$ give meaningful information? *Explain.*

2000–2001 (*A*)

$$\begin{array}{l} \\ \text{Basketball} \\ \text{Gymnastics} \\ \text{Skiing} \\ \text{Soccer} \end{array} \begin{array}{c} \begin{array}{cc} \text{Athletes} & \text{Team size} \end{array} \\ \begin{bmatrix} 14{,}439 & 14.5 \\ 1{,}397 & 15.7 \\ 526 & 11.9 \\ 18{,}548 & 22.5 \end{bmatrix} \end{array}$$

2001–2002 (*B*)

$$\begin{array}{l} \\ \text{Basketball} \\ \text{Gymnastics} \\ \text{Skiing} \\ \text{Soccer} \end{array} \begin{array}{c} \begin{array}{cc} \text{Athletes} & \text{Team size} \end{array} \\ \begin{bmatrix} 14{,}524 & 14.3 \\ 1{,}440 & 16.2 \\ 496 & 11.0 \\ 19{,}467 & 22.4 \end{bmatrix} \end{array}$$

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

35. **CHALLENGE** A rectangle has vertices (1, 1), (1, 4), (5, 1), and (5, 4). Write a 2×4 matrix A whose columns are the vertices of the rectangle. Multiply matrix A by 3. In the same coordinate plane, draw the rectangles represented by the matrices A and $3A$. How are the rectangles related?

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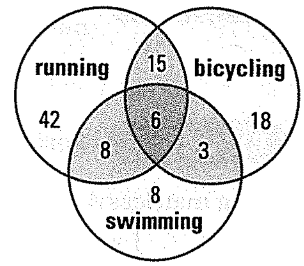
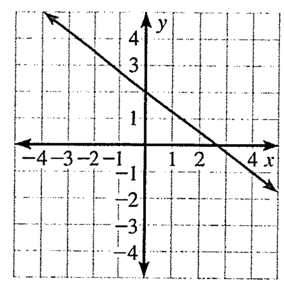
36. A health teacher surveyed 100 students to determine their favorite exercise activity or combination of exercise activities. The results are shown at the right. How many of the students surveyed chose only running as their favorite exercise activity?



Ⓐ 13 Ⓑ 29

Ⓒ 42 Ⓓ 71

37. Which statement best describes the effect on the graph shown when the y-intercept is decreased by 4?



Ⓐ The x-intercept decreases.

Ⓑ The slope decreases.

Ⓒ The x-intercept increases.

Ⓓ The slope increases.

QUIZ for Lessons 3.3–3.5

Graph the system of inequalities. *(p. 168)*

1. $y < 6$
 $x + y > -2$

2. $x \geq -1$
 $-2x + y \leq 5$

3. $x + 3y > 3$
 $x + 3y < -9$

4. $x - y \geq 4$
 $2x + 4y \geq -10$

5. $x + 2y \leq 10$
 $y \geq |x + 2|$

6. $-y < x$
 $2y < 5x + 9$

Solve the system using any algebraic method. *(p. 178)*

7. $2x - y - 3z = 5$
 $x + 2y - 5z = -11$
 $-x - 3y = 10$

8. $x + y + z = -3$
 $2x - 3y + z = 9$
 $4x - 5y + 2z = 16$

9. $2x - 4y + 3z = 1$
 $6x + 2y + 10z = 19$
 $-2x + 5y - 2z = 2$

Use matrices A, B, and C to evaluate the matrix expression, if possible. If not possible, state the reason. *(p. 187)*

$A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 3 \\ 8 & 10 \end{bmatrix}$ $C = \begin{bmatrix} -6 & -2 & 9 \\ 1 & -4 & -1 \end{bmatrix}$

10. $A + B$ 11. $B - 2A$ 12. $3A + C$ 13. $\frac{2}{3}C$

14. **APPLES** You have \$25 to spend on 21 pounds of three types of apples. Empire apples cost \$1.40 per pound, Red Delicious apples cost \$1.10 per pound, and Golden Delicious apples cost \$1.30 per pound. You want twice as many Red Delicious apples as the other two kinds combined. Use a system of equations to find how many pounds of each type you should buy. *(p. 178)*

Graphing Calculator ACTIVITY Use after Lesson 3.5

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Keystrokes

3.5 Use Matrix Operations

QUESTION **How can you use a graphing calculator to perform matrix operations?**

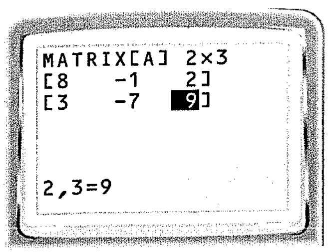
EXAMPLE **Perform operations with matrices**

Using matrices A and B below, find $A + B$ and $3A - 2B$.

$$A = \begin{bmatrix} 8 & -1 & 2 \\ 3 & -7 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -5 \\ -4 & 6 & 10 \end{bmatrix}$$

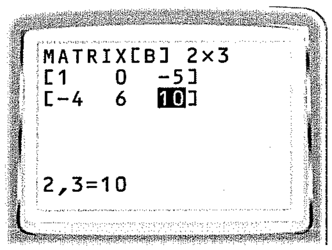
STEP 1 ***Enter matrix A***

Enter the dimensions and elements of matrix A.



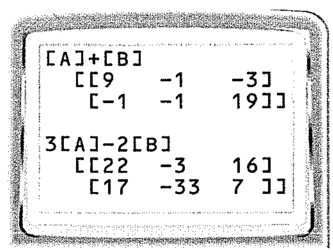
STEP 2 ***Enter matrix B***

Enter the dimensions and elements of matrix B.



STEP 3 ***Perform calculations***

From the home screen, calculate $A + B$ and $3A - 2B$.



PRACTICE

Use a graphing calculator to perform the indicated operation(s).

1. $\begin{bmatrix} 7 & 3 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 12 & -8 \\ 3 & -6 \end{bmatrix}$

2. $2.6\begin{bmatrix} 12.4 & 6.8 & -1.2 \\ -0.8 & 5.6 & -3.2 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 1 & -2 \\ -1 & 5 & 6 \\ 4 & 13 & 0 \end{bmatrix} + \begin{bmatrix} -9 & 10 & -3 \\ 0 & 6 & 1 \\ 14 & 7 & -8 \end{bmatrix}$

4. $3\begin{bmatrix} 4 & -3 \\ 8 & -7 \\ -1 & 2 \end{bmatrix} - 2\begin{bmatrix} -5 & 8 \\ -7 & 9 \\ 4 & -3 \end{bmatrix}$

5. BOOK SALES The matrices below show book sales (in thousands of dollars) at a chain of bookstores for July and August. The book formats are hardcover and paperback. The categories of books are romance (R), mystery (M), science fiction (S), and children's (C). Find the total sales of each format and category for July and August.

$$\begin{array}{l} \\ \\ \textbf{Hardcover} \\ \textbf{Paperback} \end{array} \begin{array}{c} \textbf{July} \\ \begin{array}{cccc} \textbf{R} & \textbf{M} & \textbf{S} & \textbf{C} \end{array} \\ \begin{bmatrix} 18 & 16 & 21 & 13 \\ 36 & 20 & 14 & 30 \end{bmatrix} \end{array} \quad \begin{array}{c} \textbf{August} \\ \begin{array}{cccc} \textbf{R} & \textbf{M} & \textbf{S} & \textbf{C} \end{array} \\ \begin{bmatrix} 26 & 20 & 17 & 8 \\ 40 & 24 & 8 & 20 \end{bmatrix} \end{array}$$

3.6 Multiply Matrices

Before You added and subtracted matrices.

Now You will multiply matrices.

Why? So you can calculate the cost of sports equipment, as in Example 4.

Key Vocabulary
- **matrix,** *p. 187*
- **dimensions,** *p. 187*
- **elements,** *p. 187*

The product of two matrices A and B is defined provided the number of columns in A is equal to the number of rows in B.

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

$$\begin{array}{ccccc} A & \cdot & B & = & AB \\ m \times n & & n \times p & & m \times p \end{array}$$

equal

dimensions of *AB*

EXAMPLE 1 Describe matrix products

State whether the product AB is defined. If so, give the dimensions of AB.

a. A: 4×3, B: 3×2

b. A: 3×4, B: 3×2

Solution

a. Because A is a $\mathbf{4 \times 3}$ matrix and B is a $\mathbf{3 \times 2}$ matrix, the product AB is defined and is a $\mathbf{4 \times 2}$ matrix.

b. Because the number of columns in A (four) does not equal the number of rows in B (three), the product AB is not defined.

✓ GUIDED PRACTICE for Example 1

State whether the product AB is defined. If so, give the dimensions of AB.

1. A: 5×2, B: 2×2

2. A: 3×2, B: 3×2

KEY CONCEPT *For Your Notebook*

Multiplying Matrices

Words To find the element in the ith row and jth column of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, then add the products.

Algebra

$$\begin{array}{ccccc} \textbf{A} & & \textbf{B} & & \textbf{AB} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & \cdot & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \end{array}$$

EXAMPLE 2 Find the product of two matrices

Find AB if $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix}$.

Solution

AVOID ERRORS

Order is important when multiplying matrices. To find AB, write matrix A on the left and matrix B on the right.

Because A is a 2×2 matrix and B is a 2×2 matrix, the product AB is defined and is a 2×2 matrix.

STEP 1 **Multiply** the numbers in the first row of A by the numbers in the first column of B, add the products, and put the result in the first row, first column of AB.

$$\begin{bmatrix} \mathbf{1} & \mathbf{4} \\ 3 & -2 \end{bmatrix}\begin{bmatrix} \mathbf{5} & -7 \\ \mathbf{9} & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & \\ & \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the first row of A by the numbers in the second column of B, add the products, and put the result in the first row, second column of AB.

$$\begin{bmatrix} \mathbf{1} & \mathbf{4} \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 5 & \mathbf{-7} \\ 9 & \mathbf{6} \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ & \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the second row of A by the numbers in the first column of B, add the products, and put the result in the second row, first column of AB.

$$\begin{bmatrix} 1 & 4 \\ \mathbf{3} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} \mathbf{5} & -7 \\ \mathbf{9} & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the second row of A by the numbers in the second column of B, add the products, and put the result in the second row, second column of AB.

$$\begin{bmatrix} 1 & 4 \\ \mathbf{3} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} 5 & \mathbf{-7} \\ 9 & \mathbf{6} \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & 3(-7) + (-2)(6) \end{bmatrix}$$

STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & 3(-7) + (-2)(6) \end{bmatrix} = \begin{bmatrix} 41 & 17 \\ -3 & -33 \end{bmatrix}$$

Animated Algebra at classzone.com

For the matrices A and B in Example 2, notice that the product BA is not the same as the product AB.

$$BA = \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -16 & 34 \\ 27 & 24 \end{bmatrix} \neq AB$$

In general, matrix multiplication is *not* commutative.

✓ GUIDED PRACTICE for Example 2

3. Find AB if $A = \begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix}$.

EXAMPLE 3 Use matrix operations

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 \\ -3 & -1 \end{bmatrix}$$

a. $A(B + C)$

b. $AB + AC$

Solution

a. $$A(B + C) = \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -3 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -14 & 7 \\ 6 & 2 \\ -4 & 8 \end{bmatrix}$$

b. $$AB + AC = \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -6 \\ 1 & 4 \\ -6 & 0 \end{bmatrix} + \begin{bmatrix} -5 & 13 \\ 5 & -2 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} -14 & 7 \\ 6 & 2 \\ -4 & 8 \end{bmatrix}$$

MULTIPLICATION PROPERTIES Notice in Example 3 that $A(B + C) = AB + AC$, which is true in general. This and other properties of matrix multiplication are summarized below.

REVIEW PROPERTIES
For help with properties of real numbers, see p. 2.

CONCEPT SUMMARY — *For Your Notebook*

Properties of Matrix Multiplication

Let A, B, and C be matrices and let k be a scalar.

Associative Property of Matrix Multiplication	$A(BC) = (AB)C$
Left Distributive Property	$A(B + C) = AB + AC$
Right Distributive Property	$(A + B)C = AC + BC$
Associative Property of Scalar Multiplication	$k(AB) = (kA)B = A(kB)$

✓ GUIDED PRACTICE for Example 3

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}, C = \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$$

4. $A(B - C)$

5. $AB - AC$

6. $-\frac{1}{2}(AB)$

EXAMPLE 3
on p. 197
for Exs. 22–29

EVALUATING EXPRESSIONS **Using the given matrices, evaluate the expression.**

$$A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix}, E = \begin{bmatrix} -3 & 1 & 4 \\ 7 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

22. $3AB$

23. $-\frac{1}{2}AC$

24. $AB + AC$

25. $AB - BA$

26. $E(D + E)$

27. $(D + E)D$

28. $-2(BC)$

29. $4AC + 3AB$

SOLVING MATRIX EQUATIONS **Solve for x and y.**

30. $\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ y \end{bmatrix}$

31. $\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} y & -4 \\ -13 & 8 \end{bmatrix}$

FINDING POWERS **Using the given matrix, find $A^2 = AA$ and $A^3 = AAA$.**

32. $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

33. $A = \begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$

34. $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{bmatrix}$

35. ★ **OPEN-ENDED MATH** Find two matrices A and B such that $A \neq B$ and $AB = BA$.

36. **CHALLENGE** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, and let k be a scalar. Prove the associative property of scalar multiplication for 2×2 matrices by showing that $k(AB) = (kA)B = A(kB)$.

PROBLEM SOLVING

EXAMPLE 4
on p. 198
for Exs. 37–42

In Exercises 37 and 38, write an inventory matrix and a cost per item matrix. Then use matrix multiplication to write a total cost matrix.

37. **SOFTBALL** A softball team needs to buy 12 bats, 45 balls, and 15 uniforms. Each bat costs $21, each ball costs $4, and each uniform costs $30.

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38. **ART SUPPLIES** A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes, and 15 canvases. Each tube of paint costs $3.35, each brush costs $1.75, and each canvas costs $4.50.

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39. **MULTI-STEP PROBLEM** Tickets to the senior class play cost $2 for students, $5 for adults, and $4 for senior citizens. At Friday night's performance, there were 120 students, 150 adults, and 40 senior citizens in attendance. At Saturday night's performance, there were 192 students, 215 adults, and 54 senior citizens in attendance. Organize the information using matrices. Then use matrix multiplication to find the income from ticket sales for Friday and Saturday nights' performances.

40. SUMMER OLYMPICS The top three countries in the final medal standings for the 2004 Summer Olympics were the United States, China, and Russia. Each gold medal is worth 3 points, each silver medal is worth 2 points, and each bronze medal is worth 1 point. Organize the information using matrices. How many points did each country score?

Medals Won

	Gold	Silver	Bronze
USA	35	39	29
China	32	17	14
Russia	27	27	38

41. ★ **SHORT RESPONSE** Matrix S gives the numbers of three types of cars sold in February by two car dealers, dealer A and dealer B. Matrix P gives the profit for each type of car sold. Which matrix is defined, SP or PS? Find this matrix and explain what its elements represent.

Matrix S

$$\begin{array}{r c} & \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{array}{r} \text{Compact} \\ \text{Mid-size} \\ \text{Full-size} \end{array} & \begin{bmatrix} 21 & 16 \\ 40 & 33 \\ 15 & 19 \end{bmatrix} \end{array}$$

Matrix P

$$\begin{array}{r c} & \begin{array}{ccc} \text{Compact} & \text{Mid-size} & \text{Full-size} \end{array} \\ \text{Profit} & \begin{bmatrix} \$650 & \$825 & \$1050 \end{bmatrix} \end{array}$$

42. GRADING Your overall grade in math class is a weighted average of three components: homework, quizzes, and tests. Homework counts for 20% of your grade, quizzes count for 30%, and tests count for 50%. The spreadsheet below shows the grades on homework, quizzes, and tests for five students. Organize the information using a matrix, then multiply the matrix by a matrix of weights to find each student's overall grade.

	A	B	C	D
1	Name	Homework	Quizzes	Test
2	Jean	82	88	86
3	Ted	92	88	90
4	Pat	82	73	81
5	Al	74	75	78
6	Matt	88	92	90

43. MULTI-STEP PROBLEM Residents of a certain suburb commute to a nearby city either by driving or by using public transportation. Each year, 20% of those who drive switch to public transportation, and 5% of those who use public transportation switch to driving.

a. The information above can be represented by the *transition matrix*

$$T = \begin{bmatrix} 1-p & q \\ p & 1-q \end{bmatrix}$$

where p is the percent of commuters who switch from driving to public transportation and q is the percent of commuters who switch from public transportation to driving. (Both p and q are expressed as decimals.) Write a transition matrix for the given situation.

b. Suppose 5000 commuters drive and 8000 commuters take public transportation. Let M_0 be the following matrix:

$$M_0 = \begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$$

Find $M_1 = TM_0$. What does this matrix represent?

c. Find $M_2 = TM_1$, $M_3 = TM_2$, and $M_4 = TM_3$. What do these matrices represent?

44. ★ **EXTENDED RESPONSE** Two students have a business selling handmade scarves. The scarves come in four different styles: plain, with the class year, with the school name, and with the school mascot. The costs of making each style of scarf are \$10, \$15, \$20, and \$20, respectively. The prices of each style of scarf are \$15, \$20, \$25, and \$30, respectively.

a. Write a 4×1 matrix C that gives the cost of making each style of scarf and a 4×1 matrix P that gives the price of each style of scarf.

b. The sales for the first three years of the business are shown below.

Year 1: *0 plain, 20 class year, 100 school name, 0 school mascot*

Year 2: *10 plain, 100 class year, 50 school name, 30 school mascot*

Year 3: *20 plain, 300 class year, 100 school name, 50 school mascot*

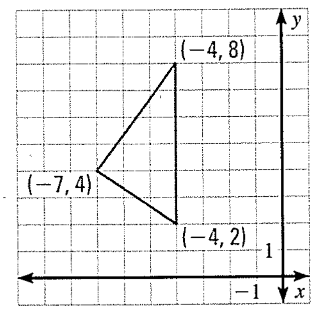
Write a 3×4 matrix S that gives the sales for the first three years.

c. Find SC and SP. What do these matrices represent?

d. Find $SP - SC$. What does this matrix represent?

45. CHALLENGE Matrix A is a 90° rotational matrix. Matrix B contains the coordinates of the vertices of the triangle shown in the graph.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -7 & -4 & -4 \\ 4 & 8 & 2 \end{bmatrix}$$



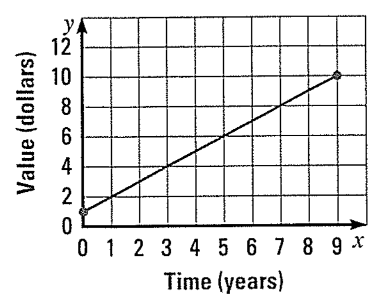
a. Find AB. Draw the triangle whose vertices are given by AB.

b. Find the 180° and 270° rotations of the original triangle by using repeated multiplication of the 90° rotational matrix. What are the coordinates of the vertices of the rotated triangles?

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

46. The graph shows the value of a comic book over a period of 9 years. What is a reasonable conclusion about the value of the comic book during the time shown on the graph?



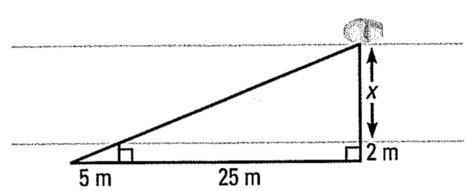
Ⓐ It appreciated \$2 every year.

Ⓑ It appreciated \$3 every 2 years.

Ⓒ Its value at 5 years was twice its value at 2 years.

Ⓓ Its value at 7 years was half its value at 3 years.

47. Use the information in the diagram. What is the distance x across the river?



Ⓐ 10 m Ⓑ 12 m

Ⓒ 22 m Ⓓ 30 m

3.7 Evaluate Determinants and Apply Cramer's Rule

Before You added, subtracted, and multiplied matrices.

Now You will evaluate determinants of matrices.

Why? So you can find areas of habitats, as in Example 2.

Key Vocabulary
- **determinant**
- **Cramer's rule**
- **coefficient matrix**

Associated with each square ($n \times n$) matrix is a real number called its **determinant**. The determinant of a matrix A is denoted by $\det A$ or by $|A|$.

KEY CONCEPT *For Your Notebook*

The Determinant of a Matrix

Determinant of a 2 × 2 Matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 2 × 2 matrix is the difference of the products of the elements on the diagonals.

Determinant of a 3 × 3 Matrix

STEP 1 **Repeat** the first two columns to the right of the determinant.

STEP 2 **Subtract** the sum of the **red products** from the sum of the **blue products**.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right|\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

EXAMPLE 1 Evaluate determinants

Evaluate the determinant of the matrix.

a. $\begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & -1 & -3 \\ 4 & 1 & 0 \\ 3 & -4 & -2 \end{bmatrix}$

Solution

a. $\begin{vmatrix} 5 & 4 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(4) = 5 - 12 = -7$

b. $\left|\begin{matrix} 2 & -1 & -3 \\ 4 & 1 & 0 \\ 3 & -4 & -2 \end{matrix}\right|\begin{matrix} 2 & -1 \\ 4 & 1 \\ 3 & -4 \end{matrix} = (-4 + 0 + 48) - (-9 + 0 + 8) = 44 - (-1) = 45$

AREA OF A TRIANGLE You can use a determinant to find the area of a triangle whose vertices are points in a coordinate plane.

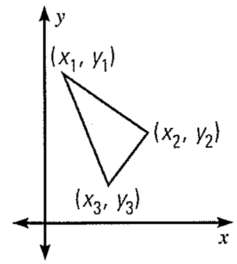
KEY CONCEPT *For Your Notebook*

Area of a Triangle

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

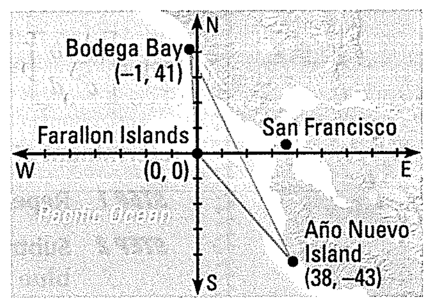
$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.



EXAMPLE 2 Find the area of a triangular region

SEA LIONS Off the coast of California lies a triangular region of the Pacific Ocean where huge populations of sea lions and seals live. The triangle is formed by imaginary lines connecting Bodega Bay, the Farallon Islands, and Año Nuevo Island, as shown. (In the map, the coordinates are measured in miles.) Use a determinant to estimate the area of the region.



Solution

The approximate coordinates of the vertices of the triangular region are $(-1, 41)$, $(38, -43)$, and $(0, 0)$. So, the area of the region is:

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -1 & 41 & 1 \\ 38 & -43 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} -1 & 41 & 1 \\ 38 & -43 & 1 \\ 0 & 0 & 1 \end{vmatrix}\begin{matrix} -1 & 41 \\ 38 & -43 \\ 0 & 0 \end{matrix}$$

$$= \pm\frac{1}{2}[(43 + 0 + 0) - (0 + 0 + 1558)]$$

$$= 757.5$$

▶ The area of the region is about 758 square miles.

✓ **GUIDED PRACTICE** for Examples 1 and 2

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 4 & -1 & 2 \\ -3 & -2 & -1 \\ 0 & 5 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 10 & -2 & 3 \\ 2 & -12 & 4 \\ 0 & -7 & -2 \end{bmatrix}$

4. Find the area of the triangle with vertices $A(5, 11)$, $B(9, 2)$, and $C(1, 3)$.

CRAMER'S RULE You can use determinants to solve a system of linear equations. The method, called **Cramer's rule** and named after the Swiss mathematician Gabriel Cramer (1704–1752), uses the **coefficient matrix** of the linear system.

Linear System	**Coefficient Matrix**
$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

KEY CONCEPT *For Your Notebook*

Cramer's Rule for a 2 × 2 System

Let A be the coefficient matrix of this linear system:

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Notice that the numerators for x and y are the determinants of the matrices formed by replacing the coefficients of x and y, respectively, with the column of constants.

EXAMPLE 3 Use Cramer's rule for a 2 × 2 system

Use Cramer's rule to solve this system: $\begin{aligned} \mathbf{9x + 4y} &= \mathbf{-6} \\ \mathbf{3x - 5y} &= \mathbf{-21} \end{aligned}$

ANOTHER WAY
You can also solve the system in Example 3 using the substitution method or the elimination method you learned in Lesson 3.2.

Solution

STEP 1 **Evaluate** the determinant of the coefficient matrix.

$$\begin{vmatrix} 9 & 4 \\ 3 & -5 \end{vmatrix} = -45 - 12 = -57$$

STEP 2 **Apply** Cramer's rule because the determinant is not 0.

$$x = \frac{\begin{vmatrix} -6 & 4 \\ -21 & -5 \end{vmatrix}}{-57} = \frac{30 - (-84)}{-57} = \frac{114}{-57} = -2$$

$$y = \frac{\begin{vmatrix} 9 & -6 \\ 3 & -21 \end{vmatrix}}{-57} = \frac{-189 - (-18)}{-57} = \frac{-171}{-57} = 3$$

▶ The solution is $(-2, 3)$.

CHECK Check this solution in the original equations.

$$\begin{aligned} 9x + 4y &= -6 \\ 9(-2) + 4(3) &\stackrel{?}{=} -6 \\ -18 + 12 &\stackrel{?}{=} -6 \\ -6 &= -6 \checkmark \end{aligned} \qquad \begin{aligned} 3x - 5y &= -21 \\ 3(-2) - 5(3) &\stackrel{?}{=} -21 \\ -6 - 15 &\stackrel{?}{=} -21 \\ -21 &= -21 \checkmark \end{aligned}$$

KEY CONCEPT — *For Your Notebook*

Cramer's Rule for a 3 × 3 System

Let A be the coefficient matrix of the linear system shown below.

Linear System

$$\begin{aligned} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{aligned}$$

Coefficient Matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, \quad \text{and} \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

SOLVE SYSTEMS

As with Cramer's rule for a 2 × 2 system, the numerators for x, y, and z are the determinants of the matrices formed by replacing the coefficients of x, y, and z respectively with the column of constants.

EXAMPLE 4 Solve a multi-step problem

CHEMISTRY The atomic weights of three compounds are shown. Use a linear system and Cramer's rule to find the atomic weights of carbon (C), hydrogen (H), and oxygen (O).

Compound	Formula	Atomic weight
Glucose	$C_6H_{12}O_6$	180
Carbon dioxide	CO_2	44
Hydrogen peroxide	H_2O_2	34

Solution

STEP 1 **Write** a linear system using the formula for each compound. Let C, H, and O represent the atomic weights of carbon, hydrogen, and oxygen.

$$\begin{aligned} 6C + 12H + 6O &= 180 \\ C \qquad\quad + 2O &= 44 \\ 2H + 2O &= 34 \end{aligned}$$

STEP 2 **Evaluate** the determinant of the coefficient matrix.

$$\left|\begin{array}{ccc|cc} 6 & 12 & 6 & 6 & 12 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 2 & 2 & 0 & 2 \end{array}\right. = (0 + 0 + 12) - (0 + 24 + 24) = -36$$

STEP 3 **Apply** Cramer's rule because the determinant is not 0.

$$C = \frac{\begin{vmatrix} 180 & 12 & 6 \\ 44 & 0 & 2 \\ 34 & 2 & 2 \end{vmatrix}}{-36} \qquad H = \frac{\begin{vmatrix} 6 & 180 & 6 \\ 1 & 44 & 2 \\ 0 & 34 & 2 \end{vmatrix}}{-36} \qquad O = \frac{\begin{vmatrix} 6 & 12 & 180 \\ 1 & 0 & 44 \\ 0 & 2 & 34 \end{vmatrix}}{-36}$$

$$= \frac{-432}{-36} \qquad\qquad = \frac{-36}{-36} \qquad\qquad = \frac{-576}{-36}$$

$$= 12 \qquad\qquad = 1 \qquad\qquad = 16$$

▶ The atomic weights of carbon, hydrogen, and oxygen are 12, 1, and 16, respectively.

✓ GUIDED PRACTICE for Examples 3 and 4

Use Cramer's rule to solve the linear system.

5. $3x - 4y = -15$
   $2x + 5y = 13$

6. $4x + 7y = 2$
   $-3x - 2y = -8$

7. $3x - 4y + 2z = 18$
   $4x + y - 5z = -13$
   $2x - 3y + z = 11$

3.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS6 for Exs. 11, 23, and 43

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 38, 42, and 45

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The _?_ of a 2×2 matrix is the difference of the products of the elements on the diagonals.

2. ★ **WRITING** *Explain* Cramer's rule and how it is used.

EXAMPLE 1 on p. 203 for Exs. 3–21

2 × 2 DETERMINANTS **Evaluate the determinant of the matrix.**

3. $\begin{bmatrix} 2 & -1 \\ 4 & -5 \end{bmatrix}$

4. $\begin{bmatrix} 7 & 1 \\ 0 & 3 \end{bmatrix}$

5. $\begin{bmatrix} -4 & 3 \\ 1 & -7 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -3 \\ 2 & 6 \end{bmatrix}$

7. $\begin{bmatrix} 10 & -6 \\ -7 & 5 \end{bmatrix}$

8. $\begin{bmatrix} 0 & 3 \\ 5 & -3 \end{bmatrix}$

9. $\begin{bmatrix} 9 & -3 \\ 7 & 2 \end{bmatrix}$

10. $\begin{bmatrix} -5 & 12 \\ 4 & 6 \end{bmatrix}$

3 × 3 DETERMINANTS **Evaluate the determinant of the matrix.**

11. $\begin{bmatrix} -1 & 12 & 4 \\ 0 & 2 & -5 \\ 3 & 0 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 2 & 3 \\ 5 & -8 & 1 \\ 2 & 4 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 5 & 0 & 2 \\ -3 & 9 & -2 \\ 1 & -4 & 0 \end{bmatrix}$

14. $\begin{bmatrix} -7 & 4 & 5 \\ 1 & 2 & -4 \\ -10 & 1 & 6 \end{bmatrix}$

15. $\begin{bmatrix} 12 & 5 & 8 \\ 0 & 6 & -8 \\ 1 & 10 & 4 \end{bmatrix}$

16. $\begin{bmatrix} -4 & 3 & -9 \\ 12 & 6 & 0 \\ 8 & -12 & 0 \end{bmatrix}$

17. $\begin{bmatrix} -2 & 6 & 0 \\ 8 & 15 & 3 \\ 4 & -1 & 7 \end{bmatrix}$

18. $\begin{bmatrix} 5 & 7 & 6 \\ -4 & 0 & 8 \\ 1 & 8 & 7 \end{bmatrix}$

ERROR ANALYSIS *Describe* **and correct the error in evaluating the determinant.**

19.
$$\left|\begin{array}{ccc|cc} 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 6 & 4 & 1 \\ -3 & 2 & 5 & -3 & 2 \end{array}\right|$$
$$= 10 + 0 + (-8) + (3 + 24 + 0)$$
$$= 2 + 27 = 29$$

20.
$$\left|\begin{array}{cc|ccc} 3 & 0 & 3 & 0 & 1 \\ 2 & 2 & 2 & 2 & -3 \\ -3 & 5 & -3 & 5 & 0 \end{array}\right|$$
$$= -18 + 0 + 0 - (-18 + 0 - 6)$$
$$= -18 - (-24) = 6$$

21. ★ **MULTIPLE CHOICE** Which matrix has the greatest determinant?

Ⓐ $\begin{bmatrix} -4 & 1 \\ 6 & 3 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1 & 6 \\ 3 & 8 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 5 & -3 \\ 7 & -1 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}$

EXAMPLE 2
on p. 204
for Exs. 22–28

AREA OF A TRIANGLE **Find the area of the triangle with the given vertices.**

22. $A(1, 5)$, $B(4, 6)$, $C(7, 3)$

23. $A(4, 2)$, $B(4, 8)$, $C(8, 5)$

24. $A(-4, 6)$, $B(0, 3)$, $C(6, 6)$

25. $A(-4, -4)$, $B(-1, 2)$, $C(2, -6)$

26. $A(5, -4)$, $B(6, 3)$, $C(8, -1)$

27. $A(-6, 1)$, $B(-2, -6)$, $C(0, 3)$

28. ★ **MULTIPLE CHOICE** What is the area of the triangle with vertices $(-3, 4)$, $(6, 3)$, and $(2, -1)$?

(A) 20 (B) 26 (C) 30 (D) 40

EXAMPLES 3 and 4
on pp. 205–206
for Exs. 29–37

USING CRAMER'S RULE **Use Cramer's rule to solve the linear system.**

29. $3x + 5y = 3$
$-x + 2y = 10$

30. $2x - y = -2$
$x + 2y = 14$

31. $5x + y = -40$
$2x - 5y = 11$

32. $-x + y + z = -3$
$4x - y + 4z = -14$
$x + 2y - z = 9$

33. $-x - 2y + 4z = -28$
$x + y + 2z = -11$
$2x + y - 3z = 30$

34. $4x + y + 3z = 7$
$2x - 5y + 4z = -19$
$x - y + 2z = -2$

35. $5x - y - 2z = -6$
$x + 3y + 4z = 16$
$2x - 4y + z = -15$

36. $x + y + z = -8$
$3x - 3y + 2z = -21$
$-x + 2y - 2z = 11$

37. $3x - y + z = 25$
$-x + 2y - 3z = -17$
$x + y + z = 21$

38. ★ **OPEN-ENDED MATH** Write a 2×2 matrix that has a determinant of 5.

39. **CHALLENGE** Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$.

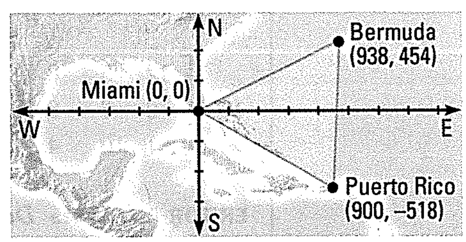
a. How is det AB related to det A and det B?

b. How is det kA related to det A if k is a scalar? Give an algebraic justification for your answer.

PROBLEM SOLVING

EXAMPLE 2
on p. 204
for Exs. 40–41

40. **BERMUDA TRIANGLE** The Bermuda Triangle is a large triangular region in the Atlantic Ocean. The triangle is formed by imaginary lines connecting Bermuda, Puerto Rico, and Miami, Florida. (In the map, the coordinates are measured in miles.) Use a determinant to estimate the area of the Bermuda Triangle.



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41. **GARDENING** You are planning to turn a triangular region of your yard into a garden. The vertices of the triangle are $(0, 0)$, $(5, 2)$, and $(3, 6)$ where the coordinates are measured in feet. Find the area of the triangular region.

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EXAMPLES 3 and 4
on pp. 205–206
for Exs. 42–44

42. ★ **SHORT RESPONSE** The attendance at a rock concert was 6700 people. The tickets for the concert cost \$40 for floor seats and \$25 for all other seats. The total income of ticket sales was \$185,500. Write a linear system that models this situation. Solve the system in three ways: using Cramer's rule, using the substitution method, and using the elimination method. *Compare* the methods, and explain which one you prefer in this situation.

43. **MULTI-STEP PROBLEM** An ice cream shop sells the following sizes of ice cream cones: single scoop for \$.90, double scoop for \$1.20, and triple scoop for \$1.60. One day, a total of 120 cones are sold for \$134, as many single-scoop cones are sold as double-scoop and triple-scoop cones combined.

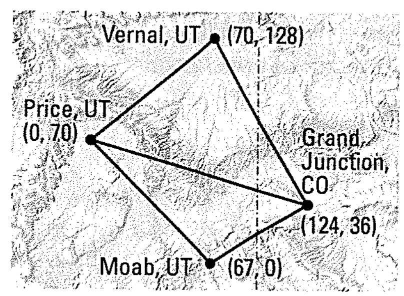
a. Use a linear system and Cramer's rule to find how many of each size of cone are sold.

b. The next day, the shop raises prices by 10%. As a result, the number of each size of cone sold falls by 5%. What is the revenue from cone sales?

44. **SCIENCE** The atomic weights of three compounds are shown in the table. Use a linear system and Cramer's rule to find the atomic weights of fluorine (F), sodium (Na), and chlorine (Cl).

Compound	Formula	Atomic weight
Sodium fluoride	FNa	42
Sodium chloride	NaCl	58.5
Chlorine pentafluoride	ClF_5	130.5

45. ★ **EXTENDED RESPONSE** In Utah and Colorado, an area called the Dinosaur Diamond is known for containing many dinosaur fossils. The map at the right shows the towns at the four vertices of the diamond. The coordinates given are measured in miles.



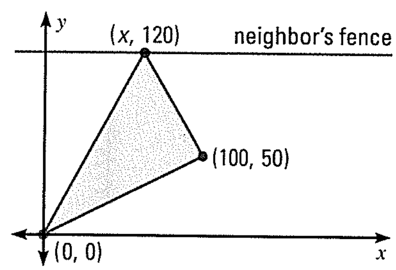
a. Find the area of the top triangular region.

b. Find the area of the bottom triangular region.

c. What is the total area of the Dinosaur Diamond?

d. *Describe* another way in which you can divide the Dinosaur Diamond into two triangles in order to find its area.

46. **CHALLENGE** A farmer is fencing off a triangular region of a pasture, as shown. The area of the region should be 5000 square feet. The farmer has planted the first two fence posts at (0, 0) and (100, 50). He wants to plant the final post along his neighbor's fence, which lies on the horizontal line $y = 120$. At which *two* points could the farmer plant the final post so that the triangular region has the desired area?

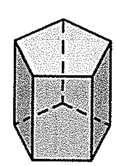


PA **PENNSYLVANIA MIXED REVIEW** **TEST PRACTICE** at classzone.com

47. Nadia's weekly salary is \$390, and she receives a \$5 bonus for each new customer she brings in. Which inequality represents the number of new customers, c, she needs to bring in per week to earn at least \$450 per week?

(A) $c < 60$ (B) $c < 12$ (C) $c \geq 12$ (D) $c \geq 60$

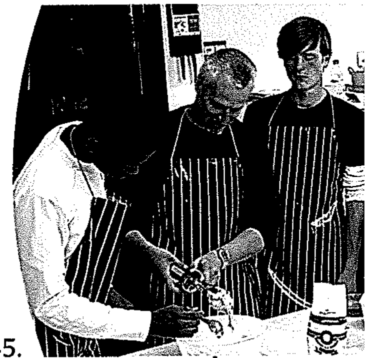
48. How many edges does the pentagonal prism have?



(A) 7 (B) 10

(C) 15 (D) 17

3.8 Use Inverse Matrices to Solve Linear Systems



Before You solved linear systems using Cramer's rule.

Now You will solve linear systems using inverse matrices.

Why? So you can find how many batches of a recipe to make, as in Ex. 45.

Key Vocabulary
- **identity matrix**
- **inverse matrices**
- **matrix of variables**
- **matrix of constants**

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

2 × 2 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix. That is, $AB = I$ and $BA = I$. An $n \times n$ matrix A has an inverse if and only if $\det A \neq 0$. The symbol for the inverse of A is A^{-1}.

KEY CONCEPT *For Your Notebook*

The Inverse of a 2 × 2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

EXAMPLE 1 Find the inverse of a 2 × 2 matrix

CHECK INVERSES
In Example 1, you can check the inverse by showing that $AA^{-1} = I = A^{-1}A$.

Find the inverse of $A = \begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$.

$$A^{-1} = \frac{1}{15 - 16}\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix} = -1\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$$

✓ **GUIDED PRACTICE** for Example 1

Find the inverse of the matrix.

1. $\begin{bmatrix} 6 & 1 \\ 2 & 4 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 5 \\ -4 & 8 \end{bmatrix}$

3. $\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

EXAMPLE 2 Solve a matrix equation

Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$$\overbrace{\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}}^{A} X = \overbrace{\begin{bmatrix} -21 & 3 \\ 12 & -2 \end{bmatrix}}^{B}$$

Solution

Begin by finding the inverse of A.

$$A^{-1} = \frac{1}{8-7}\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$$

To solve the equation for X, multiply both sides of the equation by A^{-1} *on the left.*

$$\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} X = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -21 & 3 \\ 12 & -2 \end{bmatrix} \qquad \boldsymbol{A^{-1}AX = A^{-1}B}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} \qquad \boldsymbol{IX = A^{-1}B}$$

$$X = \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} \qquad \boldsymbol{X = A^{-1}B}$$

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✓ GUIDED PRACTICE for Example 2

4. Solve the matrix equation $\begin{bmatrix} -4 & 1 \\ 0 & 6 \end{bmatrix} X = \begin{bmatrix} 8 & 9 \\ 24 & 6 \end{bmatrix}$.

INVERSE OF A 3 × 3 MATRIX The inverse of a 3×3 matrix is difficult to compute by hand. A calculator that will compute inverse matrices is useful in this case.

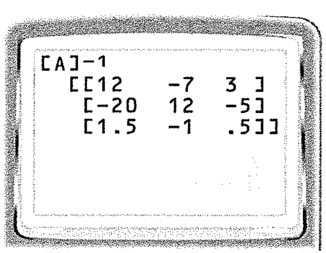
EXAMPLE 3 Find the inverse of a 3 × 3 matrix

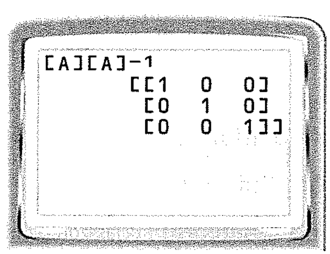
Use a graphing calculator to find the inverse of A. Then use the calculator to verify your result.

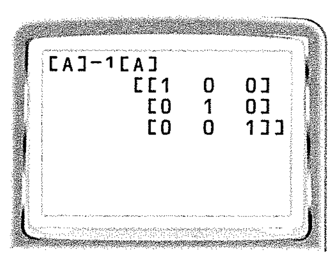
$$A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$$

Solution

Enter matrix A into a graphing calculator and calculate A^{-1}. Then compute AA^{-1} and $A^{-1}A$ to verify that you obtain the 3×3 identity matrix.







PROBLEM SOLVING WORKSHOP
LESSON 3.8

Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 213

MULTIPLE REPRESENTATIONS In Example 5 on page 213, you solved a linear system using an inverse matrix. You can also solve systems using *augmented matrices*. An **augmented matrix** for a system contains the system's coefficient matrix and matrix of constants.

Linear System		**Augmented Matrix**
$x - 4y = 9$ $-6x + 7y = -2$	$\Rightarrow$	$\left[\begin{array}{rr\|r} 1 & -4 & 9 \\ -6 & 7 & -2 \end{array}\right]$

Recall from Lesson 3.2 that an equation in a system can be multiplied by a constant, or a multiple of one equation can be added to another equation. Similar operations can be performed on the rows of an augmented matrix to solve the corresponding system.

KEY CONCEPT *For Your Notebook*

Elementary Row Operations for Augmented Matrices

Two augmented matrices are *row-equivalent* if their corresponding systems have the same solution(s). Any of these row operations performed on an augmented matrix will produce a matrix that is row-equivalent to the original:

- Interchange two rows.
- Multiply a row by a nonzero constant.
- Add a multiple of one row to another row.

PROBLEM

GIFTS A company sells three types of movie gift baskets. A basic basket with 2 movie passes and 1 package of microwave popcorn costs \$15.50. A medium basket with 2 movie passes, 2 packages of popcorn, and 1 DVD costs \$37. A super basket with 4 movie passes, 3 packages of popcorn, and 2 DVDs costs \$72.50. Find the cost of each item in the gift baskets.

METHOD

Using an Augmented Matrix You need to write a linear system, write the corresponding augmented matrix, and use row operations to transform the augmented matrix into a matrix with 1's along the main diagonal and 0's below the main diagonal. Such a matrix is in *triangular form* and can be used to solve for the variables in the system.

Let m be the cost of a movie pass, p be the cost of a package of popcorn, and d be the cost of a DVD.

STEP 1 **Write** a linear system and then write an augmented matrix.

$$\begin{aligned} 2m + p &= 15.5 \\ 2m + 2p + d &= 37 \\ 4m + 3p + 2d &= 72.5 \end{aligned} \qquad \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 2 & 2 & 1 & 37 \\ 4 & 3 & 2 & 72.5 \end{array}\right]$$

STEP 2 **Add** −2 times the first row to the third row.

$$\begin{array}{r} \\ \\ (-2)R_1 + R_3 \longrightarrow \end{array} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 2 & 2 & 1 & 37 \\ 0 & 1 & 2 & 41.5 \end{array}\right]$$

STEP 3 **Add** −1 times the first row to the second row.

$$\begin{array}{r} \\ (-1)R_1 + R_2 \longrightarrow \\ \\ \end{array} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 0 & 1 & 1 & 21.5 \\ 0 & 1 & 2 & 41.5 \end{array}\right]$$

STEP 4 **Add** −1 times the second row to the third row.

$$\begin{array}{r} \\ \\ (-1)R_2 + R_3 \longrightarrow \end{array} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 0 & 1 & 1 & 21.5 \\ 0 & 0 & 1 & 20 \end{array}\right]$$

STEP 5 **Multiply** the first row by 0.5.

$$\begin{array}{r} 0.5R_1 \longrightarrow \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0.5 & 0 & 7.75 \\ 0 & 1 & 1 & 21.5 \\ 0 & 0 & 1 & 20 \end{array}\right]$$

The third row of the matrix tells you that $d = 20$. Substitute 20 for d in the equation for the second row, $p + d = 21.5$, to obtain $p + 20 = 21.5$, or $p = 1.5$. Then substitute 1.5 for p in the equation for the first row, $m + 0.5p = 7.75$, to obtain $m + 0.5(1.5) = 7.75$, or $m = 7$.

▶ A movie pass costs $7, a package of popcorn costs $1.50, and a DVD costs $20.

PRACTICE

1. **WHAT IF?** In the problem on page 218, suppose a basic basket costs $17.75, a medium basket costs $34.50, and a super basket costs $67.25. Use an augmented matrix to find the cost of each item.

2. **FINANCE** You have $18,000 to invest. You want an overall annual return of 8%. The expected annual returns are 10% for a stock fund, 7% for a bond fund, and 5% for a money market fund. You want to invest as much in stocks as in bonds and the money market combined. Use an augmented matrix to find how much to invest in each fund.

3. **BIRDSEED** A pet store sells 20 pounds of birdseed for $10.85. The birdseed is made from two kinds of seeds, sunflower seeds and thistle seeds. Sunflower seeds cost $.34 per pound and thistle seeds cost $.79 per pound. Use an augmented matrix to find how many pounds of each variety are in the mixture.

4. **REASONING** Solve the given system using an augmented matrix. What can you say about the system's solution(s)?

$$\begin{aligned} x - 2y + 4z &= -10 \\ 5x + y - z &= 24 \\ 3x - 6y + 12z &= -30 \end{aligned}$$

3 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- system of two linear equations in two variables, *p. 153*
- solution of a system of linear equations, *p. 153*
- consistent, inconsistent, independent, dependent, *p. 154*
- substitution method, *p. 160*
- elimination method, *p. 161*
- system of linear inequalities in two variables, *p. 168*
- solution, graph of a system of inequalities, *p. 168*
- linear equation in three variables, *p. 178*
- system of three linear equations in three variables, *p. 178*
- solution of a system of three linear equations, *p. 178*
- ordered triple, *p. 178*
- matrix, *p. 187*
- dimensions, elements of a matrix, *p. 187*
- equal matrices, *p. 187*
- scalar, *p. 188*
- scalar multiplication, *p. 188*
- determinant, *p. 203*
- Cramer's rule, *p. 205*
- coefficient matrix, *p. 205*
- identity matrix, inverse matrices, *p. 210*
- matrix of variables, *p. 212*
- matrix of constants, *p. 212*

VOCABULARY EXERCISES

1. Copy and complete: A system of linear equations with at least one solution is __?__, while a system with no solution is __?__.
2. Copy and complete: A solution (x, y, z) of a system of linear equations in three variables is called a(n) __?__.
3. **WRITING** *Explain* when the product of two matrices is defined.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 3.

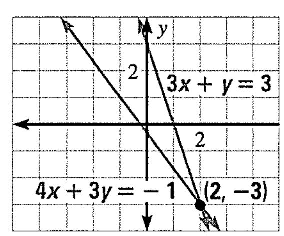
3.1 Solve Linear Systems by Graphing

pp. 153–158

EXAMPLE

Graph the system and estimate the solution. Check the solution algebraically.

$3x + y = 3$ **Equation 1**
$4x + 3y = -1$ **Equation 2**



Graph both equations. From the graph, the lines appear to intersect at (2, −3). You can check this algebraically.

$3(2) + (-3) = 3$ ✓ **Equation 1 checks.**

$4(2) + 3(-3) = -1$ ✓ **Equation 2 checks.**

EXERCISES

EXAMPLE 1
on p. 153
for Exs. 4–6

Graph the system and estimate the solution. Check the solution algebraically.

4. $2x - y = 9$
   $x + 3y = 8$
5. $2x - 3y = -2$
   $x + y = -6$
6. $3x + y = 6$
   $-x + 2y = 12$

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Chapter Review Practice

3.2 Solve Linear Systems Algebraically

pp. 160–167

EXAMPLE

Solve the system using the elimination method.

$2x + 5y = 8$ **Equation 1**
$4x + 3y = -12$ **Equation 2**

Multiply Equation 1 by -2 so that the coefficients of x differ only in sign.

$2x + 5y = 8$ $\xrightarrow{\times -2}$ $-4x - 10y = -16$

$4x + 3y = -12$ $\longrightarrow$ $4x + 3y = -12$

Add the revised equations and solve for y. $-7y = -28$

$y = 4$

Substitute the value of y into one of the original equations and solve for x.

$2x + 5(4) = 8$ **Substitute 4 for y in Equation 1.**

$2x = -12$ **Subtract $5(4) = 20$ from each side.**

$x = -6$ **Divide each side by 2.**

▶ The solution is $(-6, 4)$.

EXERCISES

EXAMPLES 2 and 3 on pp. 161–162 for Exs. 7–10

Solve the system using the elimination method.

7. $3x + 2y = 5$
   $-2x + 3y = 27$

8. $3x + 5y = 5$
   $2x - 3y = 16$

9. $2x + 3y = 9$
   $-3x + y = 25$

10. **FUEL COSTS** The cost of 14 gallons of regular gasoline and 10 gallons of premium gasoline is \$46.68. Premium costs \$.30 more per gallon than regular. What is the cost per gallon of each type of gasoline?

3.3 Graph Systems of Linear Inequalities

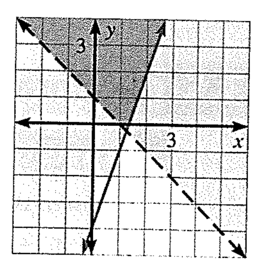
pp. 168–173

EXAMPLE

Graph the system of linear inequalities.

$3x - y \leq 4$ **Inequality 1**
$x + y > 1$ **Inequality 2**

Graph each inequality in the system. Use a different color for each half-plane. Then identify the region that is common to both graphs. It is the region that is shaded purple.



EXERCISES

EXAMPLE 1 on p. 168 for Exs. 11–13

Graph the system of linear inequalities.

11. $4x + y < 1$
    $-x + 2y \leq 5$

12. $2x + 3y > 6$
    $2x - y \leq 8$

13. $x + 3y \geq 5$
    $-x + 2y < 4$

3 CHAPTER REVIEW

3.4 Solve Systems of Linear Equations in Three Variables

pp. 178–185

EXAMPLE

Solve the system.

$$\begin{aligned} 2x + y + 3z &= 5 && \textbf{Equation 1} \\ -x + 3y + z &= -14 && \textbf{Equation 2} \\ 3x - y - 2z &= 11 && \textbf{Equation 3} \end{aligned}$$

Rewrite the system as a linear system in two variables. Add -3 times Equation 1 to Equation 2. Then add Equation 1 and Equation 3.

$$\begin{array}{rcl} -6x - 3y - 9z &=& -15 \\ -x + 3y + z &=& -14 \\ \hline -7x - 8z &=& -29 \end{array} \qquad \begin{array}{rcl} 2x + y + 3z &=& 5 \\ 3x - y - 2z &=& 11 \\ \hline 5x + z &=& 16 \end{array}$$

Solve the new linear system for both of its variables.

$$\begin{array}{rcll} -7x - 8z &=& -29 & \textbf{Add new Equation 1 to} \\ 40x + 8z &=& 128 & \textbf{8 times new Equation 2.} \\ \hline 33x &=& 99 & \\ x &=& 3 & \textbf{Solve for } x. \\ z &=& 1 & \textbf{Substitute into new Equation 1 or 2 to find } z. \end{array}$$

Substituting $x = 3$ and $z = 1$ into one of the original equations and solving for y gives $y = -4$. The solution is $(3, -4, 1)$.

EXERCISES

EXAMPLES 1 and 4 on pp. 179–181 for Exs. 14–17

Solve the system.

14. $x - y + z = 10$
    $4x + y - 2z = 15$
    $-3x + 5y - z = -18$

15. $6x - y + 4z = 6$
    $-x - 3y + z = 31$
    $2x + 2y - 5z = -42$

16. $5x + y - z = 40$
    $x + 7y + 4z = 44$
    $-x + 3y + z = 16$

17. **MUSIC** Fifteen band members from a school were selected to play in the state orchestra. Twice as many students who play a wind instrument were selected as students who play a string or percussion instrument combined. Of the students selected, one fifth play a string instrument. How many of the students selected play each type of instrument?

3.5 Perform Basic Matrix Operations

pp. 187–193

EXAMPLE

Perform the indicated operation.

a. $\begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4 + (-5) & -1 + 2 \\ 2 + (-3) & 5 + 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 6 \end{bmatrix}$

b. $4\begin{bmatrix} -2 & 0 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 4(-2) & 4(0) \\ 4(3) & 4(5) \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 12 & 20 \end{bmatrix}$

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Chapter Review Practice

EXAMPLES 2 and 3 on pp. 188–189 for Exs. 18–23

EXERCISES

Perform the indicated operation.

18. $\begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ -7 & 4 \end{bmatrix}$

19. $\begin{bmatrix} -1 & 8 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 7 & -4 \\ 6 & -1 \end{bmatrix}$

20. $\begin{bmatrix} 10 & -4 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 2 & 7 \end{bmatrix}$

21. $\begin{bmatrix} -2 & 3 & 5 \\ -1 & 6 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 7 & 5 \\ -8 & 0 & -9 \end{bmatrix}$

22. $-3\begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}$

23. $8\begin{bmatrix} 8 & 4 & 5 \\ -1 & 6 & -2 \end{bmatrix}$

3.6 Multiply Matrices

pp. 195–202

EXAMPLE

Find AB if $A = \begin{bmatrix} 2 & -3 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 2 & -3 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2(-2) + (-3)(3) & 2(3) + (-3)(1) \\ -1(-2) + 0(3) & -1(3) + 0(1) \\ 4(-2) + 5(3) & 4(3) + 5(1) \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 3 \\ 2 & -3 \\ 7 & 17 \end{bmatrix}$$

EXAMPLES 2 and 4 on pp. 196–198 for Exs. 24–28

EXERCISES

Find the product.

24. $\begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 2 \\ -6 & -9 \end{bmatrix}$

25. $\begin{bmatrix} 11 & 7 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 0 & -5 \\ 4 & -3 \end{bmatrix}$

26. $\begin{bmatrix} 4 & -1 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 5 & -2 & 4 \\ 3 & 12 & 6 \end{bmatrix}$

27. $\begin{bmatrix} -2 & 5 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 6 & -3 & 5 \\ 2 & 0 & -1 \end{bmatrix}$

28. MANUFACTURING A company manufactures three models of flat-screen color TVs: a 19 inch model, a 27 inch model, and a 32 inch model. The TVs are shipped to two warehouses. The numbers of units shipped to each warehouse are given in matrix A, and the prices of the models are given in matrix B. Write a matrix that gives the total value of the TVs in each warehouse.

Matrix *A*

$$\begin{array}{r} \\ \textbf{Warehouse 1} \\ \textbf{Warehouse 2} \end{array}\begin{array}{c} \begin{array}{ccc} \textbf{19 in.} & \textbf{27 in.} & \textbf{32 in.} \end{array} \\ \begin{bmatrix} 5{,}000 & 6{,}000 & 8{,}000 \\ 4{,}000 & 10{,}000 & 5{,}000 \end{bmatrix} \end{array}$$

Matrix *B*

$$\begin{array}{r} \\ \textbf{19 inch} \\ \textbf{27 inch} \\ \textbf{32 inch} \end{array}\begin{array}{c} \textbf{Price} \\ \begin{bmatrix} \$109.99 \\ \$319.99 \\ \$549.99 \end{bmatrix} \end{array}$$

CUMULATIVE REVIEW

Simplify the expression. *(p. 10)*

1. $3x^2 - 8x + 12x - 5x^2 + 3x$ **2.** $15x - 6x + 10y - 3y + 4x$ **3.** $3(x + 2) - 4x^2 + 3x + 9$

Solve the equation. Check your solution.

4. $6x - 7 = -2x + 9$ *(p. 18)* **5.** $4(x - 3) = 16x + 18$ *(p. 18)* **6.** $\frac{1}{3}x + 3 = -\frac{7}{2}x - \frac{3}{2}$ *(p. 18)*

7. $|x + 3| = 5$ *(p. 51)* **8.** $|4x - 1| = 27$ *(p. 51)* **9.** $|9 - 2x| = 41$ *(p. 51)*

Solve the inequality. Then graph the solution.

10. $6(x - 4) > 2x + 8$ *(p. 41)* **11.** $3 \le x - 2 \le 8$ *(p. 41)* **12.** $2x < -6$ or $x + 2 > 5$ *(p. 41)*

13. $|x - 4| < 5$ *(p. 51)* **14.** $|x + 3| \ge 15$ *(p. 51)* **15.** $|6x + 1| < 23$ *(p. 51)*

Find the slope of the line passing through the given points. Then tell whether the line *rises, falls, is horizontal,* or *is vertical.* *(p. 82)*

16. $(3, 2), (-1, -5)$ **17.** $(-7, 4), (5, -3)$ **18.** $(-4, -6), (-4, 4)$ **19.** $\left(-\frac{5}{4}, 3\right), \left(\frac{2}{3}, 3\right)$

Graph the equation or inequality.

20. $y = 3x + 5$ *(p. 89)* **21.** $x = -6$ *(p. 89)* **22.** $-x + 4y = 16$ *(p. 89)*

23. $y = 2|x|$ *(p. 123)* **24.** $y = |x - 3|$ *(p. 123)* **25.** $y = -4|x| + 5$ *(p. 123)*

26. $y \le x - 7$ *(p. 132)* **27.** $2x + y > 1$ *(p. 132)* **28.** $2x - 5y \ge -15$ *(p. 132)*

Graph the relation. Then tell whether the relation is a function. *(p. 72)*

29.

x	−4	−2	0	2	4
y	−1	0	1	2	3

30.

x	4	−2	1	1	−3
y	−2	0	1	4	3

Solve the system using any algebraic method.

31. $4x - 3y = 32$
$-2x + y = -14$ *(p. 160)*

32. $5x - 2y = -4$
$3x + 6y = 36$ *(p. 160)*

33. $x - y + 2z = -4$
$3x + y - 4z = -6$
$2x + 3y + z = 9$ *(p. 178)*

Use the given matrices to evaluate the expression. *(p. 195)*

$$A = \begin{bmatrix} -2 & 6 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}, C = \begin{bmatrix} -4 & 8 \\ -7 & 12 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & -4 \\ -2 & 3 & -1 \end{bmatrix}$$

34. $B - 3A$ **35.** $2(A + B) - C$ **36.** $(C - A)B$ **37.** $(B + C)D$

Find the inverse of the matrix. *(p. 210)*

38. $\begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$ **39.** $\begin{bmatrix} 6 & 9 \\ -3 & -4 \end{bmatrix}$ **40.** $\begin{bmatrix} -2 & 2 \\ 4 & 1 \end{bmatrix}$ **41.** $\begin{bmatrix} -5 & 8 \\ 2 & -8 \end{bmatrix}$

42. CITY PARK A triangular section of a city park is being turned into a playground. The triangle's vertices are (0, 0), (15, 10), and (8, 25) where the coordinates are measured in yards. Find the area of the playground. *(p. 203)*

43. **BASEBALL** The Pythagorean Theorem of Baseball is a formula for approximating a team's ratio of wins to games played. Let R be the number of runs the team scores during the season, A be the number of runs allowed to opponents, W be the number of wins, and T be the total number of games played. Then the formula below approximates the team's ratio of wins to games played. *(p. 26)*

$$\frac{W}{T} = \frac{R^2}{R^2 + A^2}$$

   a. Solve the formula for W.

   b. In 2004 the Boston Red Sox scored 949 runs and allowed 768 runs. How many of its 162 games would you estimate the team won? *Compare* your answer to the team's actual number of wins, which was 98.

44. **HIGHWAY DRIVING** A sport utility vehicle has a 21 gallon gas tank. On a long highway trip, gas is used at a rate of approximately 4 gallons per hour. Assume the gas tank is full at the start of the trip. *(p. 72)*

   a. Write a function giving the number of gallons g of gasoline in the tank after traveling for t hours.

   b. Graph the function from part (a).

   c. Identify the domain and range of the function from part (a).

45. **COMMISSION** A real estate agent's commission c varies directly with the selling price p of a house. An agent made \$3900 in commission after selling a \$78,000 house. Write an equation that gives c as a function of p. Predict the agent's commission if the selling price of a house is \$125,000. *(p. 107)*

46. **WASTE RECOVERY** The table shows the amount of material (in millions of tons) recovered from solid waste in the United States from 1994 to 2001. Make a scatter plot of the data and approximate the best-fitting line. Predict the amount of material that will be recovered from solid waste in 2010. *(p. 113)*

Years since 1994, t	0	1	2	3	4	5	6	7
Recovered material, m	50.6	54.9	57.3	59.4	61.1	64.8	67.7	68.0

47. **WEIGHTLIFTING RECORDS** The men's world weightlifting records for the 105-kg-and-over weight category are shown in the table. The combined lift is the sum of the snatch lift and the clean and jerk lift. Let s be the weight lifted in the snatch and let j be the weight lifted in the clean and jerk. Write and graph a system of inequalities to describe the weights an athlete could lift to break the records for both the snatch and combined lifts, but *not* the clean and jerk lift. *(p. 168)*

Men's 105+ kg World Weightlifting Records		
Snatch	**Clean and Jerk**	**Combined**
213.0	263.0	472.5

4 Quadratic Functions and Factoring

PA

M11.D.2.2.1

M11.D.2.1.5

M11.D.2.1.5

M11.A.1.1.3

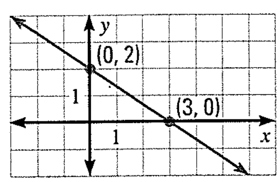
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 4: evaluating expressions, graphing functions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.



1. The ***x*-intercept** of the line shown is _?_.
2. The ***y*-intercept** of the line shown is _?_.

SKILLS CHECK

Evaluate the expression when $x = -3$. ***(Review p. 10 for 4.1, 4.7.)***

3. $-5x^2 + 1$
4. $x^2 - x - 8$
5. $(x + 4)^2$
6. $-3(x - 7)^2 + 2$

Graph the function and label the vertex. ***(Review p. 123 for 4.2.)***

7. $y = |x| + 2$
8. $y = |x - 3|$
9. $y = -2|x|$
10. $y = |x - 5| + 4$

Solve the equation. ***(Review p. 18 for 4.3, 4.4.)***

11. $x + 8 = 0$
12. $3x - 5 = 0$
13. $2x + 1 = x$
14. $4(x - 3) = x + 9$