4 Quadratic Functions and Factoring

PA

M11.D.2.2.1
M11.D.2.1.5
M11.D.2.1.5
M11.A.1.1.3

Before

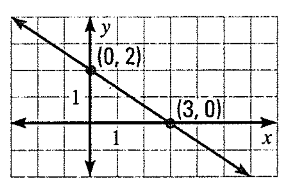
In previous chapters, you learned the following skills, which you'll use in Chapter 4: evaluating expressions, graphing functions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The ***x*-intercept** of the line shown is ___?___.
2. The ***y*-intercept** of the line shown is ___?___.



SKILLS CHECK

Evaluate the expression when $x = -3$. ***(Review p. 10 for 4.1, 4.7.)***

3. $-5x^2 + 1$ 4. $x^2 - x - 8$ 5. $(x + 4)^2$ 6. $-3(x - 7)^2 + 2$

Graph the function and label the vertex. ***(Review p. 123 for 4.2.)***

7. $y = |x| + 2$ 8. $y = |x - 3|$ 9. $y = -2|x|$ 10. $y = |x - 5| + 4$

Solve the equation. ***(Review p. 18 for 4.3, 4.4.)***

11. $x + 8 = 0$ 12. $3x - 5 = 0$ 13. $2x + 1 = x$ 14. $4(x - 3) = x + 9$

43. **BASEBALL** The Pythagorean Theorem of Baseball is a formula for approximating a team's ratio of wins to games played. Let R be the number of runs the team scores during the season, A be the number of runs allowed to opponents, W be the number of wins, and T be the total number of games played. Then the formula below approximates the team's ratio of wins to games played. ***(p. 26)***

$$\frac{W}{T} = \frac{R^2}{R^2 + A^2}$$

a. Solve the formula for W.

b. In 2004 the Boston Red Sox scored 949 runs and allowed 768 runs. How many of its 162 games would you estimate the team won? *Compare* your answer to the team's actual number of wins, which was 98.

44. **HIGHWAY DRIVING** A sport utility vehicle has a 21 gallon gas tank. On a long highway trip, gas is used at a rate of approximately 4 gallons per hour. Assume the gas tank is full at the start of the trip. ***(p. 72)***

a. Write a function giving the number of gallons g of gasoline in the tank after traveling for t hours.

b. Graph the function from part (a).

c. Identify the domain and range of the function from part (a).

45. **COMMISSION** A real estate agent's commission c varies directly with the selling price p of a house. An agent made \$3900 in commission after selling a \$78,000 house. Write an equation that gives c as a function of p. Predict the agent's commission if the selling price of a house is \$125,000. ***(p. 107)***

46. **WASTE RECOVERY** The table shows the amount of material (in millions of tons) recovered from solid waste in the United States from 1994 to 2001. Make a scatter plot of the data and approximate the best-fitting line. Predict the amount of material that will be recovered from solid waste in 2010. ***(p. 113)***

Years since 1994, t	0	1	2	3	4	5	6	7
Recovered material, m	50.6	54.9	57.3	59.4	61.1	64.8	67.7	68.0

47. **WEIGHTLIFTING RECORDS** The men's world weightlifting records for the 105-kg-and-over weight category are shown in the table. The combined lift is the sum of the snatch lift and the clean and jerk lift. Let s be the weight lifted in the snatch and let j be the weight lifted in the clean and jerk. Write and graph a system of inequalities to describe the weights an athlete could lift to break the records for both the snatch and combined lifts, but *not* the clean and jerk lift. ***(p. 168)***

Men's 105+ kg World Weightlifting Records		
Snatch	**Clean and Jerk**	**Combined**
213.0	263.0	472.5

Now

In Chapter 4, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 317. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing and writing quadratic functions in several forms**
2. **Solving quadratic equations using a variety of methods**
3. **Performing operations with square roots and complex numbers**

KEY VOCABULARY

- standard form of a quadratic function, *p. 236*
- parabola, *p. 236*
- vertex form, *p. 245*
- intercept form, *p. 246*
- quadratic equation, *p. 253*
- root of an equation, *p. 253*
- zero of a function, *p. 254*
- square root, *p. 266*
- complex number, *p. 276*
- imaginary number, *p. 276*
- completing the square, *p. 284*
- quadratic formula, *p. 292*
- discriminant, *p. 294*
- best-fitting quadratic model, *p. 311*

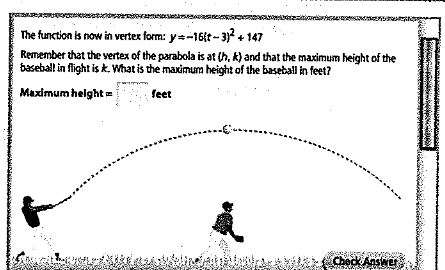
Why?

You can use quadratic functions to model the heights of projectiles. For example, the height of a baseball hit by a batter can be modeled by a quadratic function.

Animated Algebra

The animation illustrated below for Example 7 on page 287 helps you answer this question: How does changing the ball speed and hitting angle affect the maximum height of a baseball?

A quadratic function models the height of a baseball in flight.



Rewrite the function in vertex form to find the maximum height of the ball.

Animated Algebra at classzone.com

Other animations for Chapter 4: pages 238, 247, 269, 279, 300, and 317

4.1 Graph Quadratic Functions in Standard Form

Before You graphed linear functions.

Now You will graph quadratic functions.

Why? So you can model sports revenue, as in Example 5.

Key Vocabulary
- **quadratic function**
- **parabola**
- **vertex**
- **axis of symmetry**
- **minimum value**
- **maximum value**

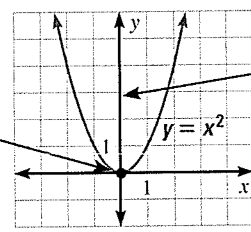
A **quadratic function** is a function that can be written in the **standard form** $y = ax^2 + bx + c$ where $a \neq 0$. The graph of a quadratic function is a **parabola**.

KEY CONCEPT *For Your Notebook*

Parent Function for Quadratic Functions

The parent function for the family of all quadratic functions is $f(x) = x^2$. The graph of $f(x) = x^2$ is the parabola shown below.

The lowest or highest point on a parabola is the vertex. The vertex for $f(x) = x^2$ is (0, 0).



The axis of symmetry divides the parabola into mirror images and passes through the vertex.

For $f(x) = x^2$, and for any quadratic function $g(x) = ax^2 + bx + c$ where $b = 0$, the vertex lies on the y-axis and the axis of symmetry is $x = 0$.

EXAMPLE 1 Graph a function of the form $y = ax^2$

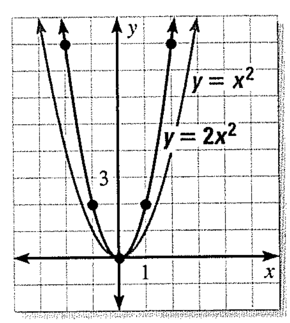
Graph $y = 2x^2$. Compare the graph with the graph of $y = x^2$.

Solution

STEP 1 **Make** a table of values for $y = 2x^2$.

SKETCH A GRAPH
Choose values of x on *both* sides of the axis of symmetry $x = 0$.

x	−2	−1	0	1	2
y	8	2	0	2	8



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = 2x^2$ and $y = x^2$. Both open up and have the same vertex and axis of symmetry. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$.

EXAMPLE 2 Graph a function of the form $y = ax^2 + c$

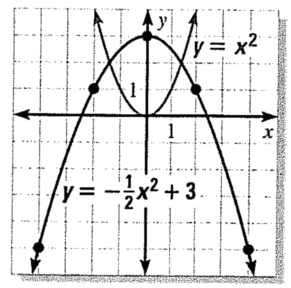
Graph $y = -\frac{1}{2}x^2 + 3$. Compare the graph with the graph of $y = x^2$.

Solution

SKETCH A GRAPH
Choose values of x that are multiples of 2 so that the values of y will be integers.

STEP 1 **Make** a table of values for $y = -\frac{1}{2}x^2 + 3$.

x	−4	−2	0	2	4
y	−5	1	3	1	−5



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = -\frac{1}{2}x^2 + 3$ and $y = x^2$. Both graphs have the same axis of symmetry. However, the graph of $y = -\frac{1}{2}x^2 + 3$ opens down and is wider than the graph of $y = x^2$. Also, its vertex is 3 units higher.

✓ GUIDED PRACTICE for Examples 1 and 2

Graph the function. Compare the graph with the graph of $y = x^2$.

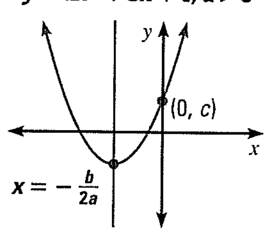
1. $y = -4x^2$
2. $y = -x^2 - 5$
3. $f(x) = \frac{1}{4}x^2 + 2$

GRAPHING ANY QUADRATIC FUNCTION You can use the following properties to graph *any* quadratic function $y = ax^2 + bx + c$, including a function where $b \neq 0$.

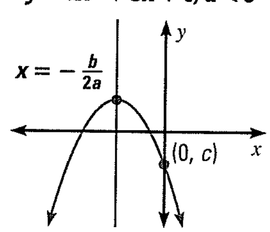
KEY CONCEPT *For Your Notebook*

Properties of the Graph of $y = ax^2 + bx + c$

$y = ax^2 + bx + c, a > 0$



$y = ax^2 + bx + c, a < 0$



Characteristics of the graph of $y = ax^2 + bx + c$:

- The graph opens up if $a > 0$ and opens down if $a < 0$.
- The graph is narrower than the graph of $y = x^2$ if $|a| > 1$ and wider if $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x-coordinate $-\frac{b}{2a}$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

EXAMPLE 3 Graph a function of the form $y = ax^2 + bx + c$

Graph $y = 2x^2 - 8x + 6$.

Solution

STEP 1 **Identify** the coefficients of the function. The coefficients are $a = 2$, $b = -8$, and $c = 6$. Because $a > 0$, the parabola opens up.

STEP 2 **Find** the vertex. Calculate the x-coordinate.

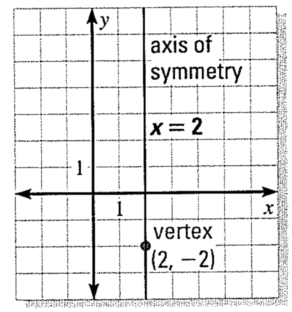
> **AVOID ERRORS**
> Be sure to include the negative sign before the fraction when calculating the x-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-8)}{2(2)} = 2$$

Then find the y-coordinate of the vertex.

$$y = 2(\mathbf{2})^2 - 8(\mathbf{2}) + 6 = -2$$

So, the vertex is $(2, -2)$. Plot this point.



STEP 3 **Draw** the axis of symmetry $x = 2$.

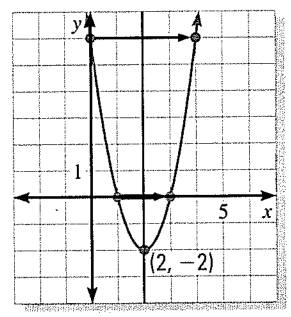
STEP 4 **Identify** the y-intercept c, which is 6. Plot the point $(0, 6)$. Then reflect this point in the axis of symmetry to plot another point, $(4, 6)$.

STEP 5 **Evaluate** the function for another value of x, such as $x = 1$.

$$y = 2(1)^2 - 8(1) + 6 = 0$$

Plot the point $(1, 0)$ and its reflection $(3, 0)$ in the axis of symmetry.

STEP 6 **Draw** a parabola through the plotted points.



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✓ **GUIDED PRACTICE** for Example 3

Graph the function. Label the vertex and axis of symmetry.

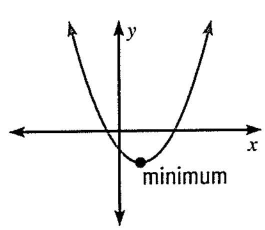
4. $y = x^2 - 2x - 1$
5. $y = 2x^2 + 6x + 3$
6. $f(x) = -\frac{1}{3}x^2 - 5x + 2$

KEY CONCEPT *For Your Notebook*

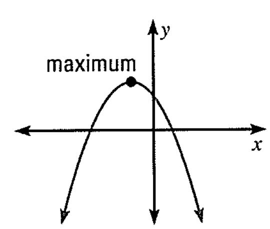
Minimum and Maximum Values

Words For $y = ax^2 + bx + c$, the vertex's y-coordinate is the **minimum value** of the function if $a > 0$ and the **maximum value** if $a < 0$.

Graphs



a **is positive**



a **is negative**

EXAMPLE 4 Find the minimum or maximum value

Tell whether the function $y = 3x^2 - 18x + 20$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

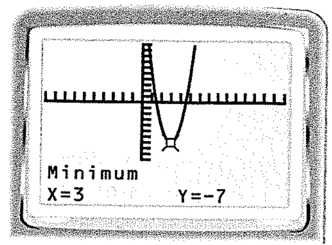
Solution

Because $a > 0$, the function has a minimum value. To find it, calculate the coordinates of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-18)}{2(3)} = \mathbf{3}$$

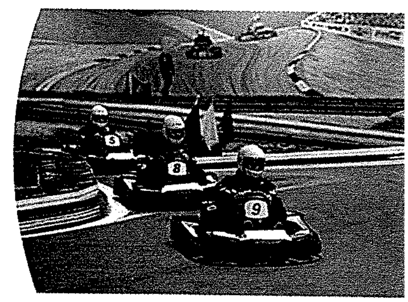
$$y = 3(\mathbf{3})^2 - 18(\mathbf{3}) + 20 = -7$$

▶ The minimum value is $y = -7$. You can check the answer on a graphing calculator.



EXAMPLE 5 Solve a multi-step problem

GO-CARTS A go-cart track has about 380 racers per week and charges each racer $35 to race. The owner estimates that there will be 20 more racers per week for every $1 reduction in the price per racer. How can the owner of the go-cart track maximize weekly revenue?



Solution

STEP 1 **Define** the variables. Let x represent the price reduction and $R(x)$ represent the weekly revenue.

STEP 2 **Write** a verbal model. Then write and simplify a quadratic function.

Revenue (dollars)	=	Price (dollars/racer)	•	Attendance (racers)
↓		↓		↓
$\mathbf{R(x)}$	=	$\mathbf{(35 - x)}$	•	$\mathbf{(380 + 20x)}$
$R(x)$	=	$13{,}300 + 700x - 380x - 20x^2$		
$R(x)$	=	$-20x^2 + 320x + 13{,}300$		

INTERPRET FUNCTIONS
Notice that $a = -20 < 0$, so the revenue function has a maximum value.

STEP 3 **Find** the coordinates $(x, R(x))$ of the vertex.

$$x = -\frac{b}{2a} = -\frac{320}{2(-20)} = \mathbf{8}$$ **Find x-coordinate.**

$$R(\mathbf{8}) = -20(\mathbf{8})^2 + 320(\mathbf{8}) + 13{,}300 = 14{,}580$$ **Evaluate $R(8)$.**

▶ The vertex is (8, 14,580), which means the owner should reduce the price per racer by $8 to increase the weekly revenue to $14,580.

✓ GUIDED PRACTICE for Examples 4 and 5

7. Find the minimum value of $y = 4x^2 + 16x - 3$.

8. **WHAT IF?** In Example 5, suppose each $1 reduction in the price per racer brings in 40 more racers per week. How can weekly revenue be maximized?

4.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS7 for Exs. 15, 37, and 57

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 39, 40, 43, 53, 58, and 60

◆ = **MULTIPLE REPRESENTATIONS** Ex. 59

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The graph of a quadratic function is called a(n) _?_.

2. ★ **WRITING** *Describe* how to determine whether a quadratic function has a minimum value or a maximum value.

EXAMPLE 1 on p. 236 for Exs. 3–12

USING A TABLE Copy and complete the table of values for the function.

3. $y = 4x^2$

x	−2	−1	0	1	2
y	?	?	?	?	?

4. $y = -3x^2$

x	−2	−1	0	1	2
y	?	?	?	?	?

5. $y = \frac{1}{2}x^2$

x	−4	−2	0	2	4
y	?	?	?	?	?

6. $y = -\frac{1}{3}x^2$

x	−6	−3	0	3	6
y	?	?	?	?	?

MAKING A GRAPH Graph the function. *Compare* the graph with the graph of $y = x^2$.

7. $y = 3x^2$
8. $y = 5x^2$
9. $y = -2x^2$
10. $y = -x^2$
11. $f(x) = \frac{1}{3}x^2$
12. $g(x) = -\frac{1}{4}x^2$

EXAMPLE 2 on p. 237 for Exs. 13–18

13. $y = 5x^2 + 1$
14. $y = 4x^2 + 1$
15. $f(x) = -x^2 + 2$
16. $g(x) = -2x^2 - 5$
17. $f(x) = \frac{3}{4}x^2 - 5$
18. $g(x) = -\frac{1}{5}x^2 - 2$

ERROR ANALYSIS *Describe* and correct the error in analyzing the graph of $y = 4x^2 + 24x - 7$.

19. The x-coordinate of the vertex is: $x = \frac{b}{2a} = \frac{24}{2(4)} = 3$ ✗

20. The y-intercept of the graph is the value of c, which is 7. ✗

EXAMPLE 3 on p. 238 for Exs. 21–32

MAKING A GRAPH Graph the function. Label the vertex and axis of symmetry.

21. $y = x^2 + 2x + 1$
22. $y = 3x^2 - 6x + 4$
23. $y = -4x^2 + 8x + 2$
24. $y = -2x^2 - 6x + 3$
25. $g(x) = -x^2 - 2x - 1$
26. $f(x) = -6x^2 - 4x - 5$
27. $y = \frac{2}{3}x^2 - 3x + 6$
28. $y = -\frac{3}{4}x^2 - 4x - 1$
29. $g(x) = -\frac{3}{5}x^2 + 2x + 2$
30. $f(x) = \frac{1}{2}x^2 + x - 3$
31. $y = \frac{8}{5}x^2 - 4x + 5$
32. $y = -\frac{5}{3}x^2 - x - 4$

EXAMPLE 4
on p. 239
for Exs. 33–38

MINIMUMS OR MAXIMUMS **Tell whether the function has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.**

33. $y = -6x^2 - 1$
34. $y = 9x^2 + 7$
35. $f(x) = 2x^2 + 8x + 7$
36. $g(x) = -3x^2 + 18x - 5$
37. $f(x) = \frac{3}{2}x^2 + 6x + 4$
38. $y = -\frac{1}{4}x^2 - 7x + 2$

39. ★ **MULTIPLE CHOICE** What is the effect on the graph of the function $y = x^2 + 2$ when it is changed to $y = x^2 - 3$?

Ⓐ The graph widens.
Ⓑ The graph narrows.
Ⓒ The graph opens down.
Ⓓ The vertex moves down the y-axis.

40. ★ **MULTIPLE CHOICE** Which function has the widest graph?

Ⓐ $y = 2x^2$
Ⓑ $y = x^2$
Ⓒ $y = 0.5x^2$
Ⓓ $y = -x^2$

IDENTIFYING COEFFICIENTS **In Exercises 41 and 42, identify the values of *a*, *b*, and *c* for the quadratic function.**

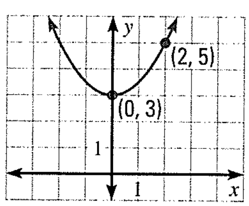
41. The path of a basketball thrown at an angle of 45° can be modeled by $y = -0.02x^2 + x + 6$.

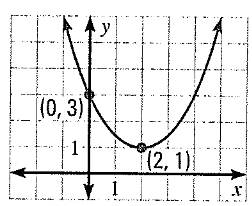
42. The path of a shot put released at an angle of 35° can be modeled by $y = -0.01x^2 + 0.7x + 6$.

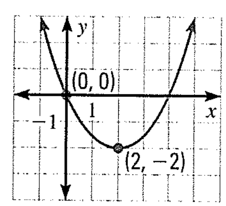
43. ★ **OPEN-ENDED MATH** Write three different quadratic functions whose graphs have the line $x = 4$ as an axis of symmetry but have different y-intercepts.

MATCHING **In Exercises 44–46, match the equation with its graph.**

44. $y = 0.5x^2 - 2x$
45. $y = 0.5x^2 + 3$
46. $y = 0.5x^2 - 2x + 3$

A.


B.


C.


MAKING A GRAPH **Graph the function. Label the vertex and axis of symmetry.**

47. $f(x) = 0.1x^2 + 2$
48. $g(x) = -0.5x^2 - 5$
49. $y = 0.3x^2 + 3x - 1$
50. $y = 0.25x^2 - 1.5x + 3$
51. $f(x) = 4.2x^2 + 6x - 1$
52. $g(x) = 1.75x^2 - 2.5$

53. ★ **SHORT RESPONSE** The points (2, 3) and (−4, 3) lie on the graph of a quadratic function. *Explain* how these points can be used to find an equation of the axis of symmetry. Then write an equation of the axis of symmetry.

54. **CHALLENGE** For the graph of $y = ax^2 + bx + c$, show that the y-coordinate of the vertex is $-\frac{b^2}{4a} + c$.

PROBLEM SOLVING

EXAMPLE 5
on p. 239
for Exs. 55–58

55. **ONLINE MUSIC** An online music store sells about 4000 songs each day when it charges $1 per song. For each $.05 increase in price, about 80 fewer songs per day are sold. Use the verbal model and quadratic function to find how the store can maximize daily revenue.

Revenue (dollars)	=	Price (dollars/song)	•	Sales (songs)
$R(x)$	=	$(1 + 0.05x)$	•	$(4000 - 80x)$

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56. **DIGITAL CAMERAS** An electronics store sells about 70 of a new model of digital camera per month at a price of $320 each. For each $20 decrease in price, about 5 more cameras per month are sold. Write a function that models the situation. Then tell how the store can maximize monthly revenue from sales of the camera.

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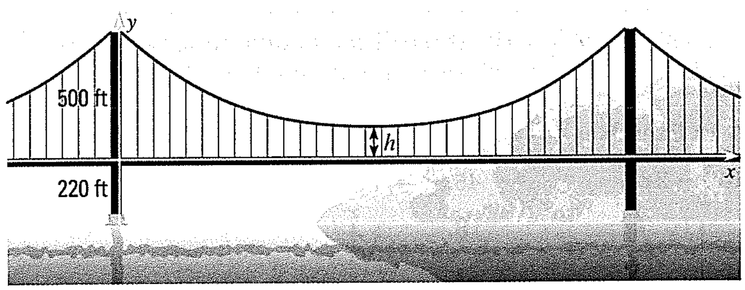
57. **GOLDEN GATE BRIDGE** Each cable joining the two towers on the Golden Gate Bridge can be modeled by the function

$$y = \frac{1}{9000}x^2 - \frac{7}{15}x + 500$$

where x and y are measured in feet. What is the height h above the road of a cable at its lowest point?



58. ★ **SHORT RESPONSE** A woodland jumping mouse hops along a parabolic path given by $y = -0.2x^2 + 1.3x$ where x is the mouse's horizontal position (in feet) and y is the corresponding height (in feet). Can the mouse jump over a fence that is 3 feet high? *Explain.*

59. ❖ **MULTIPLE REPRESENTATIONS** A community theater sells about 150 tickets to a play each week when it charges $20 per ticket. For each $1 decrease in price, about 10 more tickets per week are sold. The theater has fixed expenses of $1500 per week.

   a. **Writing a Model** Write a verbal model and a quadratic function to represent the theater's weekly profit.

   b. **Making a Table** Make a table of values for the quadratic function.

   c. **Drawing a Graph** Use the table to graph the quadratic function. Then use the graph to find how the theater can maximize weekly profit.

○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE** ❖ = **MULTIPLE REPRESENTATIONS**

60. ★ **EXTENDED RESPONSE** In 1971, astronaut Alan Shepard hit a golf ball on the moon. The path of a golf ball hit at an angle of 45° and with a speed of 100 feet per second can be modeled by

$$y = -\frac{g}{10{,}000}x^2 + x$$

where x is the ball's horizontal position (in feet), y is the corresponding height (in feet), and g is the acceleration due to gravity (in feet per second squared).

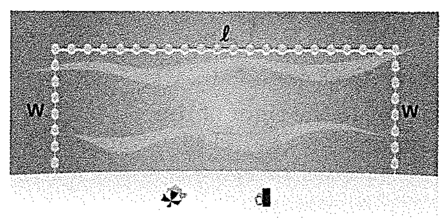
a. **Model** Use the information in the diagram to write functions for the paths of a golf ball hit on Earth and a golf ball hit on the moon.

GRAPHING CALCULATOR In part (b), use the calculator's *zero* feature to answer the questions.

b. **Graphing Calculator** Graph the functions from part (a) on a graphing calculator. How far does the golf ball travel on Earth? on the moon?

c. **Interpret** *Compare* the distances traveled by a golf ball on Earth and on the moon. Your answer should include the following:
- a calculation of the ratio of the distances traveled
- a discussion of how the distances and values of g are related

61. **CHALLENGE** Lifeguards at a beach want to rope off a rectangular swimming section. They have P feet of rope with buoys. In terms of P, what is the maximum area that the swimming section can have?



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62. Liz's high score in a video game is 1200 points less than three times her friend's high score. Let x represent her friend's high score. Which expression can be used to determine Liz's high score?

(A) $1200 - 3x$ (B) $\frac{x - 1200}{3}$ (C) $\frac{x}{3} - 1200$ (D) $3x - 1200$

63. The total cost, c, of a school banquet is given by $c = 25n + 1400$, where n is the total number of students attending the banquet. The total cost of the banquet was $9900. How many students attended the banquet?

(A) 177 (B) 340 (C) 396 (D) 452

Graphing Calculator ACTIVITY Use after Lesson 4.1

@HomeTutor
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Keystrokes

4.1 Find Maximum and Minimum Values

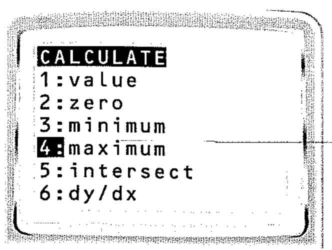
QUESTION **How can you use a graphing calculator to find the maximum or minimum value of a function?**

EXAMPLE **Find the maximum value of a function**

Find the maximum value of $y = -2x^2 - 10x - 5$ and the value of x where it occurs.

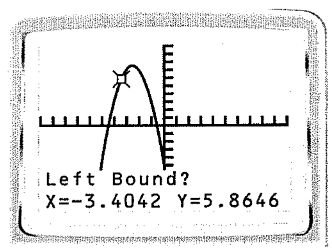
STEP 1 ***Graph function***

Graph the given function and select the *maximum* feature.



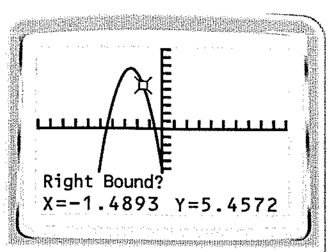
STEP 2 ***Choose left bound***

Move the cursor to the left of the maximum point. Press ENTER.



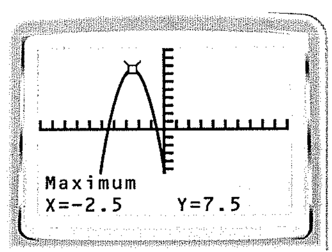
STEP 3 ***Choose right bound***

Move the cursor to the right of the maximum point. Press ENTER.



STEP 4 ***Find maximum***

Put the cursor approximately on the maximum point. Press ENTER.



▸ The maximum value of the function is $y = 7.5$ and occurs at $x = -2.5$.

PRACTICE

Tell whether the function has a *maximum value* or a *minimum value*. Then find the maximum or minimum value and the value of x where it occurs.

1. $y = x^2 - 6x + 4$

2. $f(x) = x^2 - 3x + 3$

3. $y = -3x^2 + 9x + 2$

4. $y = 0.5x^2 + 0.8x - 2$

5. $h(x) = \frac{1}{2}x^2 - 3x + 2$

6. $y = -\frac{3}{8}x^2 + 6x - 5$

4.2 Graph Quadratic Functions in Vertex or Intercept Form

PA M11.D.2.2.1 Add, subtract and/or multiply polynomial expressions (express answers in simplest form. . .)

Before You graphed quadratic functions in standard form.

Now You will graph quadratic functions in vertex form or intercept form.

Why? So you can find the height of a jump, as in Ex. 51.

Key Vocabulary
- **vertex form**
- **intercept form**

In Lesson 4.1, you learned that the standard form of a quadratic function is $y = ax^2 + bx + c$ where $a \neq 0$. Another useful form of a quadratic function is the **vertex form,** $y = a(x - h)^2 + k$.

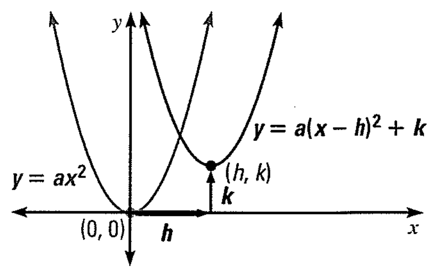
KEY CONCEPT *For Your Notebook*

Graph of Vertex Form $y = a(x - h)^2 + k$

The graph of $y = a(x - h)^2 + k$ is the parabola $y = ax^2$ translated horizontally h units and vertically k units.

Characteristics of the graph of $y = a(x - h)^2 + k$:

- The vertex is (h, k).
- The axis of symmetry is $x = h$.
- The graph opens up if $a > 0$ and down if $a < 0$.



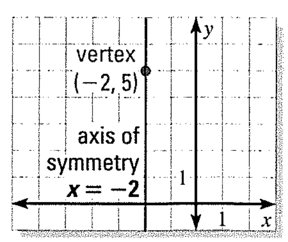
EXAMPLE 1 Graph a quadratic function in vertex form

Graph $y = -\frac{1}{4}(x + 2)^2 + 5$.

Solution

STEP 1 **Identify** the constants $a = -\frac{1}{4}$, $h = -2$, and $k = 5$. Because $a < 0$, the parabola opens down.

STEP 2 **Plot** the vertex $(h, k) = (-2, 5)$ and draw the axis of symmetry $x = -2$.



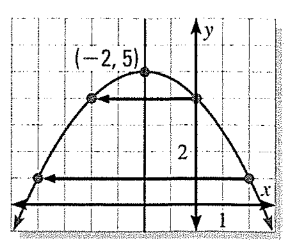
STEP 3 **Evaluate** the function for two values of x.

$$x = 0: \; y = -\frac{1}{4}(0 + 2)^2 + 5 = 4$$

$$x = 2: \; y = -\frac{1}{4}(2 + 2)^2 + 5 = 1$$

Plot the points (0, 4) and (2, 1) and their reflections in the axis of symmetry.

STEP 4 **Draw** a parabola through the plotted points.

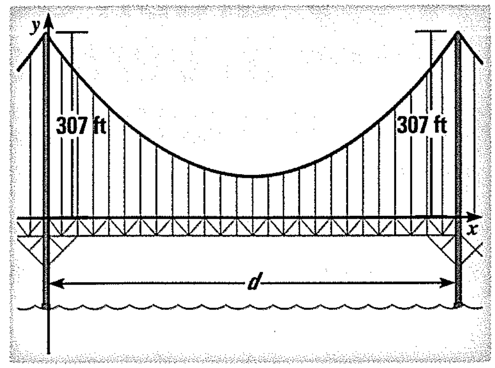


EXAMPLE 2 Use a quadratic model in vertex form

CIVIL ENGINEERING The Tacoma Narrows Bridge in Washington has two towers that each rise 307 feet above the roadway and are connected by suspension cables as shown. Each cable can be modeled by the function

$$y = \frac{1}{7000}(x - 1400)^2 + 27$$

where x and y are measured in feet. What is the distance d between the two towers?



Not drawn to scale

Solution

The vertex of the parabola is (1400, 27). So, a cable's lowest point is 1400 feet from the left tower shown above. Because the heights of the two towers are the same, the symmetry of the parabola implies that the vertex is also 1400 feet from the right tower. So, the distance between the two towers is $d = 2(1400) = 2800$ feet.

GUIDED PRACTICE for Examples 1 and 2

Graph the function. Label the vertex and axis of symmetry.

1. $y = (x + 2)^2 - 3$
2. $y = -(x - 1)^2 + 5$
3. $f(x) = \frac{1}{2}(x - 3)^2 - 4$

4. **WHAT IF?** Suppose an architect designs a bridge with cables that can be modeled by $y = \frac{1}{6500}(x - 1400)^2 + 27$ where x and y are measured in feet. Compare this function's graph to the graph of the function in Example 2.

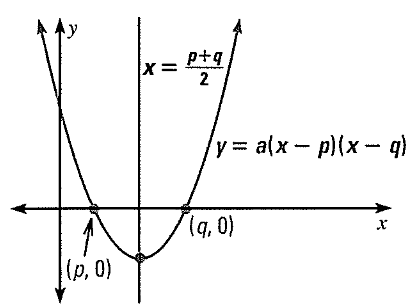
INTERCEPT FORM If the graph of a quadratic function has at least one x-intercept, then the function can be represented in **intercept form**, $y = a(x - p)(x - q)$.

KEY CONCEPT *For Your Notebook*

Graph of Intercept Form $y = a(x - p)(x - q)$

Characteristics of the graph of $y = a(x - p)(x - q)$:

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. It has equation $x = \frac{p + q}{2}$.
- The graph opens up if $a > 0$ and opens down if $a < 0$.



EXAMPLE 3 Graph a quadratic function in intercept form

Graph $y = 2(x + 3)(x - 1)$.

AVOID ERRORS
Remember that the x-intercepts for a quadratic function written in the form $y = a(x - p)(x - q)$ are p and q, not $-p$ and $-q$.

Solution

STEP 1 **Identify** the x-intercepts. Because $p = -3$ and $q = 1$, the x-intercepts occur at the points $(-3, 0)$ and $(1, 0)$.

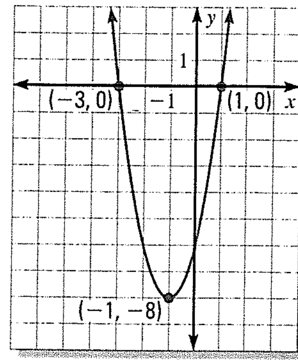
STEP 2 **Find** the coordinates of the vertex.

$$x = \frac{p + q}{2} = \frac{-3 + 1}{2} = \mathbf{-1}$$

$$y = 2(\mathbf{-1} + 3)(\mathbf{-1} - 1) = -8$$

So, the vertex is $(-1, -8)$.

STEP 3 **Draw** a parabola through the vertex and the points where the x-intercepts occur.



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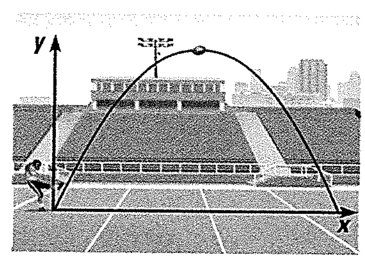
EXAMPLE 4 Use a quadratic function in intercept form

FOOTBALL The path of a placekicked football can be modeled by the function $y = -0.026x(x - 46)$ where x is the horizontal distance (in yards) and y is the corresponding height (in yards).

a. How far is the football kicked?

b. What is the football's maximum height?



Solution

a. Rewrite the function as $y = -0.026(x - 0)(x - 46)$. Because $p = 0$ and $q = 46$, you know the x-intercepts are 0 and 46. So, you can conclude that the football is kicked a distance of 46 yards.

b. To find the football's maximum height, calculate the coordinates of the vertex.

$$x = \frac{p + q}{2} = \frac{0 + 46}{2} = \mathbf{23}$$

$$y = -0.026(\mathbf{23})(\mathbf{23} - 46) \approx 13.8$$

The maximum height is the y-coordinate of the vertex, or about 13.8 yards.

✓ GUIDED PRACTICE for Examples 3 and 4

Graph the function. Label the vertex, axis of symmetry, and x-intercepts.

5. $y = (x - 3)(x - 7)$ **6.** $f(x) = 2(x - 4)(x + 1)$ **7.** $y = -(x + 1)(x - 5)$

8. WHAT IF? In Example 4, what is the maximum height of the football if the football's path can be modeled by the function $y = -0.025x(x - 50)$?

FOIL METHOD You can change quadratic functions from intercept form or vertex form to standard form by multiplying algebraic expressions. One method for multiplying two expressions each containing two terms is *FOIL*.

KEY CONCEPT — *For Your Notebook*

FOIL Method

Words To multiply two expressions that each contain two terms, add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

Example

$$(x + 4)(x + 7) = \overset{F}{x^2} + \overset{O}{7x} + \overset{I}{4x} + \overset{L}{28} = x^2 + 11x + 28$$

EXAMPLE 5 Change from intercept form to standard form

REVIEW FOIL
For help with using the FOIL method, see p. 985.

Write $y = -2(x + 5)(x - 8)$ in standard form.

$y = -2(x + 5)(x - 8)$	Write original function.
$= -2(x^2 - 8x + 5x - 40)$	Multiply using FOIL.
$= -2(x^2 - 3x - 40)$	Combine like terms.
$= -2x^2 + 6x + 80$	Distributive property

EXAMPLE 6 Change from vertex form to standard form

Write $f(x) = 4(x - 1)^2 + 9$ in standard form.

$f(x) = 4(x - 1)^2 + 9$	Write original function.
$= 4(x - 1)(x - 1) + 9$	Rewrite $(x - 1)^2$.
$= 4(x^2 - x - x + 1) + 9$	Multiply using FOIL.
$= 4(x^2 - 2x + 1) + 9$	Combine like terms.
$= 4x^2 - 8x + 4 + 9$	Distributive property
$= 4x^2 - 8x + 13$	Combine like terms.

✓ GUIDED PRACTICE for Examples 5 and 6

Write the quadratic function in standard form.

9. $y = -(x - 2)(x - 7)$

10. $y = -4(x - 1)(x + 3)$

11. $f(x) = 2(x + 5)(x + 4)$

12. $y = -7(x - 6)(x + 1)$

13. $y = -3(x + 5)^2 - 1$

14. $g(x) = 6(x - 4)^2 - 10$

15. $f(x) = -(x + 2)^2 + 4$

16. $y = 2(x - 3)^2 + 9$

4.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 19, 29, and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 22, 49, 54, and 55

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A quadratic function in the form $y = a(x - h)^2 + k$ is in _?_ form.

2. ★ **WRITING** *Explain* how to find a quadratic function's maximum value or minimum value when the function is given in intercept form.

EXAMPLE 1 on p. 245 for Exs. 3–12

GRAPHING WITH VERTEX FORM **Graph the function. Label the vertex and axis of symmetry.**

3. $y = (x - 3)^2$
4. $y = (x + 4)^2$
5. $f(x) = -(x + 3)^2 + 5$
6. $y = 3(x - 7)^2 - 1$
7. $g(x) = -4(x - 2)^2 + 4$
8. $y = 2(x + 1)^2 - 3$
9. $f(x) = -2(x - 1)^2 - 5$
10. $y = -\frac{1}{4}(x + 2)^2 + 1$
11. $y = \frac{1}{2}(x - 3)^2 + 2$

12. ★ **MULTIPLE CHOICE** What is the vertex of the graph of the function $y = 3(x + 2)^2 - 5$?

Ⓐ $(2, -5)$ Ⓑ $(-2, -5)$ Ⓒ $(-5, 2)$ Ⓓ $(5, -2)$

EXAMPLE 3 on p. 247 for Exs. 13–23

GRAPHING WITH INTERCEPT FORM **Graph the function. Label the vertex, axis of symmetry, and x-intercepts.**

13. $y = (x + 3)(x - 3)$
14. $y = (x + 1)(x - 3)$
15. $y = 3(x + 2)(x + 6)$
16. $f(x) = 2(x - 5)(x - 1)$
17. $y = -(x - 4)(x + 6)$
18. $g(x) = -4(x + 3)(x + 7)$
19. $y = (x + 1)(x + 2)$
20. $f(x) = -2(x - 3)(x + 4)$
21. $y = 4(x - 7)(x + 2)$

22. ★ **MULTIPLE CHOICE** What is the vertex of the graph of the function $y = -(x - 6)(x + 4)$?

Ⓐ $(1, 25)$ Ⓑ $(-1, 21)$ Ⓒ $(-6, 4)$ Ⓓ $(6, -4)$

23. **ERROR ANALYSIS** *Describe* and correct the error in analyzing the graph of the function $y = 5(x - 2)(x + 3)$.

> The x-intercepts of the graph are −2 and 3.

EXAMPLES 5 and 6 on p. 248 for Exs. 24–32

WRITING IN STANDARD FORM **Write the quadratic function in standard form.**

24. $y = (x + 4)(x + 3)$
25. $y = (x - 5)(x + 3)$
26. $h(x) = 4(x + 1)(x - 6)$
27. $y = -3(x - 2)(x - 4)$
28. $f(x) = (x + 5)^2 - 2$
29. $y = (x - 3)^2 + 6$
30. $g(x) = -(x + 6)^2 + 10$
31. $y = 5(x + 3)^2 - 4$
32. $f(x) = 12(x - 1)^2 + 4$

MINIMUM OR MAXIMUM VALUES **Find the minimum value or the maximum value of the function.**

33. $y = 3(x - 3)^2 - 4$
34. $g(x) = -4(x + 6)^2 - 12$
35. $y = 15(x - 25)^2 + 130$
36. $f(x) = 3(x + 10)(x - 8)$
37. $y = -(x - 36)(x + 18)$
38. $y = -12x(x - 9)$
39. $y = 8x(x + 15)$
40. $y = 2(x - 3)(x - 6)$
41. $g(x) = -5(x + 9)(x - 4)$

42. **GRAPHING CALCULATOR** Consider the function $y = a(x - h)^2 + k$ where $a = 1$, $h = 3$, and $k = -2$. Predict the effect of each change in a, h, or k described in parts (a)–(c). Use a graphing calculator to check your prediction by graphing the original and revised functions in the same coordinate plane.

a. a changes to -3 **b.** h changes to -1 **c.** k changes to 2

MAKING A GRAPH **Graph the function. Label the vertex and axis of symmetry.**

43. $y = 5(x - 2.25)^2 - 2.75$
44. $g(x) = -8(x + 3.2)^2 + 6.4$
45. $y = -0.25(x - 5.2)^2 + 8.5$
46. $y = -\frac{2}{3}\left(x - \frac{1}{2}\right)^2 + \frac{4}{5}$
47. $f(x) = -\frac{3}{4}(x + 5)(x + 8)$
48. $g(x) = \frac{5}{2}\left(x - \frac{4}{3}\right)\left(x - \frac{2}{5}\right)$

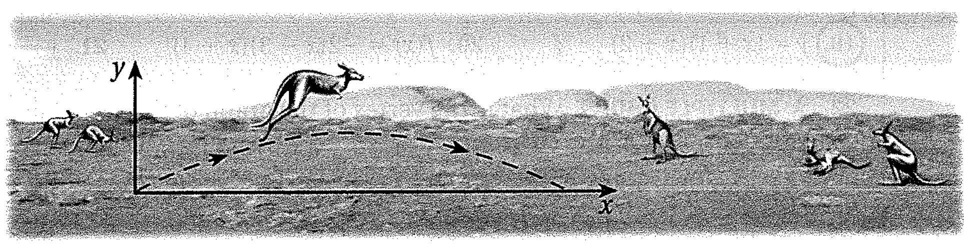
49. ★ **OPEN-ENDED MATH** Write two different quadratic functions in intercept form whose graphs have axis of symmetry $x = 3$.

50. **CHALLENGE** Write $y = a(x - h)^2 + k$ and $y = a(x - p)(x - q)$ in standard form. Knowing the vertex of the graph of $y = ax^2 + bx + c$ occurs at $x = -\frac{b}{2a}$, show that the vertex of the graph of $y = a(x - h)^2 + k$ occurs at $x = h$ and that the vertex of the graph of $y = a(x - p)(x - q)$ occurs at $x = \frac{p + q}{2}$.

PROBLEM SOLVING

EXAMPLES 2 and 4 on pp. 246–247 for Exs. 51–54

51. **BIOLOGY** The function $y = -0.03(x - 14)^2 + 6$ models the jump of a red kangaroo where x is the horizontal distance (in feet) and y is the corresponding height (in feet). What is the kangaroo's maximum height? How long is the kangaroo's jump?



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52. **CIVIL ENGINEERING** The arch of the Gateshead Millennium Bridge forms a parabola with equation $y = -0.016(x - 52.5)^2 + 45$ where x is the horizontal distance (in meters) from the arch's left end and y is the distance (in meters) from the base of the arch. What is the width of the arch?

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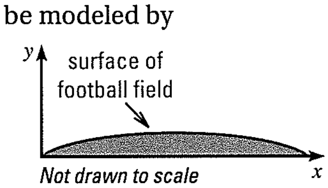
53. **MULTI-STEP PROBLEM** Although a football field appears to be flat, its surface is actually shaped like a parabola so that rain runs off to both sides. The cross section of a field with synthetic turf can be modeled by

$$y = -0.000234x(x - 160)$$

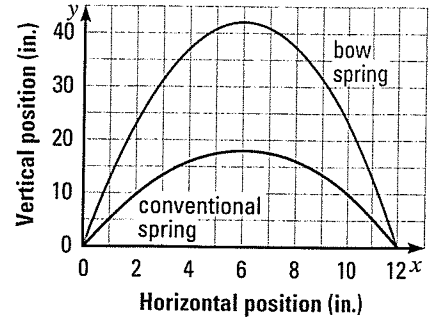
where x and y are measured in feet.

a. What is the field's width?

b. What is the maximum height of the field's surface?



54. ★ **SHORT RESPONSE** A jump on a pogo stick with a conventional spring can be modeled by $y = -0.5(x - 6)^2 + 18$, and a jump on a pogo stick with a bow spring can be modeled by $y = -1.17(x - 6)^2 + 42$, where x and y are measured in inches. *Compare* the maximum heights of the jumps on the two pogo sticks. Which constants in the functions affect the maximum heights of the jumps? Which do not?



55. ★ **EXTENDED RESPONSE** A kernel of popcorn contains water that expands when the kernel is heated, causing it to pop. The equations below give the "popping volume" y (in cubic centimeters per gram) of popcorn with moisture content x (as a percent of the popcorn's weight).

Hot-air popping: $y = -0.761(x - 5.52)(x - 22.6)$

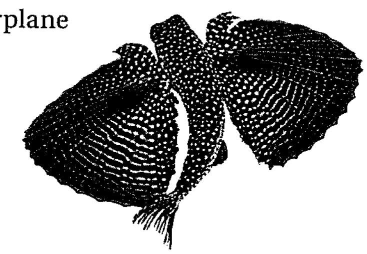
Hot-oil popping: $y = -0.652(x - 5.35)(x - 21.8)$

a. **Interpret** For hot-air popping, what moisture content maximizes popping volume? What is the maximum volume?

b. **Interpret** For hot-oil popping, what moisture content maximizes popping volume? What is the maximum volume?

c. **Graphing Calculator** Graph the functions in the same coordinate plane. What are the domain and range of each function in this situation? *Explain* how you determined the domain and range.

56. **CHALLENGE** Flying fish use their pectoral fins like airplane wings to glide through the air. Suppose a flying fish reaches a maximum height of 5 feet after flying a horizontal distance of 33 feet. Write a quadratic function $y = a(x - h)^2 + k$ that models the flight path, assuming the fish leaves the water at (0, 0). *Describe* how changing the value of a, h, or k affects the flight path.



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57. A salesperson wants to analyze the time he spends driving to visit clients. In a typical week, the salesperson drives 870 miles during a period of 22 hours. His average speed is 65 miles per hour on the highway and 30 miles per hour in the city. About how many hours a week does the salesperson spend driving in the city?

(A) 6 h (B) 8.2 h (C) 13.9 h (D) 16 h

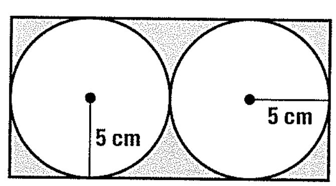
58. What is the approximate area of the shaded region?

(A) 21.5 cm^2

(B) 42.9 cm^2

(C) 121.4 cm^2

(D) 150 cm^2



4.3 Solve $x^2 + bx + c = 0$ by Factoring

PA M11.D.2.1.5 Solve quadratic equations using factoring (integers only—not including completing the square or the Quadratic Formula).

Before You graphed quadratic functions.

Now You will solve quadratic equations.

Why? So you can double the area of a picnic site, as in Ex. 42.

Key Vocabulary
- **monomial**
- **binomial**
- **trinomial**
- **quadratic equation**
- **root of an equation**
- **zero of a function**

A **monomial** is an expression that is either a number, a variable, or the product of a number and one or more variables. A **binomial**, such as $x + 4$, is the sum of two monomials. A **trinomial**, such as $x^2 + 11x + 28$, is the sum of three monomials.

You know how to use FOIL to write $(x + 4)(x + 7)$ as $x^2 + 11x + 28$. You can use factoring to write a trinomial as a product of binomials. To factor $x^2 + bx + c$, find integers m and n such that:

$$\begin{aligned} x^2 + \mathbf{b}x + c &= (x + m)(x + n) \\ &= x^2 + (\mathbf{m + n})x + mn \end{aligned}$$

So, the *sum* of m and n must equal b and the *product* of m and n must equal c.

EXAMPLE 1 Factor trinomials of the form $x^2 + bx + c$

Factor the expression.

a. $x^2 - 9x + 20$ **b.** $x^2 + 3x - 12$

Solution

a. You want $x^2 - 9x + 20 = (x + m)(x + n)$ where $mn = 20$ and $m + n = -9$.

AVOID ERRORS
When factoring $x^2 + bx + c$ where $c > 0$, you must choose factors $x + m$ and $x + n$ such that m and n have the same sign.

Factors of 20: m, n	1, 20	−1, −20	2, 10	−2, −10	4, 5	**−4, −5**
Sum of factors: $m + n$	21	−21	12	−12	9	**−9**

▶ Notice that $m = -4$ and $n = -5$. So, $x^2 - 9x + 20 = (x - 4)(x - 5)$.

b. You want $x^2 + 3x - 12 = (x + m)(x + n)$ where $mn = -12$ and $m + n = 3$.

Factors of −12: m, n	−1, 12	1, −12	−2, 6	2, −6	−3, 4	3, −4
Sum of factors: $m + n$	11	−11	4	−4	1	−1

▶ Notice that there are no factors m and n such that $m + n = 3$.
So, $x^2 + 3x - 12$ cannot be factored.

✓ **GUIDED PRACTICE** for Example 1

Factor the expression. If the expression cannot be factored, say so.

1. $x^2 - 3x - 18$ **2.** $n^2 - 3n + 9$ **3.** $r^2 + 2r - 63$

FACTORING SPECIAL PRODUCTS Factoring quadratic expressions often involves trial and error. However, some expressions are easy to factor because they follow special patterns.

KEY CONCEPT *For Your Notebook*

Special Factoring Patterns

Pattern Name	Pattern	Example
Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 4 = (x + 2)(x - 2)$
Perfect Square Trinomial	$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)^2$
	$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 4x + 4 = (x - 2)^2$

EXAMPLE 2 Factor with special patterns

Factor the expression.

a. $x^2 - 49 = x^2 - 7^2$ **Difference of two squares**

$= (x + 7)(x - 7)$

b. $d^2 + 12d + 36 = d^2 + 2(d)(6) + 6^2$ **Perfect square trinomial**

$= (d + 6)^2$

c. $z^2 - 26z + 169 = z^2 - 2(z)(13) + 13^2$ **Perfect square trinomial**

$= (z - 13)^2$

✓ **GUIDED PRACTICE** for Example 2

Factor the expression.

4. $x^2 - 9$ **5.** $q^2 - 100$ **6.** $y^2 + 16y + 64$ **7.** $w^2 - 18w + 81$

SOLVING QUADRATIC EQUATIONS You can use factoring to solve certain *quadratic equations.* A **quadratic equation** in one variable can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$. This is called the **standard form** of the equation. The solutions of a quadratic equation are called the **roots** of the equation. If the left side of $ax^2 + bx + c = 0$ can be factored, then the equation can be solved using the *zero product property.*

KEY CONCEPT *For Your Notebook*

Zero Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Example If $(x + 5)(x + 2) = 0$, then $x + 5 = 0$ or $x + 2 = 0$. That is, $x = -5$ or $x = -2$.

EXAMPLE 2
on p. 253
for Exs. 15–23

FACTORING WITH SPECIAL PATTERNS **Factor the expression.**

15. $x^2 - 36$
16. $b^2 - 81$
17. $x^2 - 24x + 144$
18. $t^2 - 16t + 64$
19. $x^2 + 8x + 16$
20. $c^2 + 28c + 196$
21. $n^2 + 14n + 49$
22. $s^2 - 26s + 169$
23. $z^2 - 121$

EXAMPLE 3
on p. 254
for Exs. 24–41

SOLVING EQUATIONS **Solve the equation.**

24. $x^2 - 8x + 12 = 0$
25. $x^2 - 11x + 30 = 0$
26. $x^2 + 2x - 35 = 0$
27. $a^2 - 49 = 0$
28. $b^2 - 6b + 9 = 0$
29. $c^2 + 5c + 4 = 0$
30. $n^2 - 6n = 0$
31. $t^2 + 10t + 25 = 0$
32. $w^2 - 16w + 48 = 0$
33. $z^2 - 3z = 54$
34. $r^2 + 2r = 80$
35. $u^2 = -9u$
36. $m^2 = 7m$
37. $14x - 49 = x^2$
38. $-3y + 28 = y^2$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

39.

$$x^2 - x - 6 = 0$$
$$(x - 2)(x + 3) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 2 \quad \text{or} \quad x = -3$$

40.

$$x^2 + 7x + 6 = 14$$
$$(x + 6)(x + 1) = 14$$
$$x + 6 = 14 \quad \text{or} \quad x + 1 = 14$$
$$x = 8 \quad \text{or} \quad x = 13$$

41. ★ **MULTIPLE CHOICE** What are the roots of the equation $x^2 + 2x - 63 = 0$?

(A) 7, −9 (B) −7, −9 (C) −7, 9 (D) 7, 9

EXAMPLE 4
on p. 254
for Exs. 42–43

WRITING EQUATIONS **Write an equation that you can solve to find the value of x.**

42. A rectangular picnic site measures 24 feet by 10 feet. You want to double the site's area by adding the same distance x to the length and the width.

43. A rectangular performing platform in a park measures 10 feet by 12 feet. You want to triple the platform's area by adding the same distance x to the length and the width.

EXAMPLE 5
on p. 255
for Exs. 44–55

FINDING ZEROS **Find the zeros of the function by rewriting the function in intercept form.**

44. $y = x^2 + 6x + 8$
45. $y = x^2 - 8x + 16$
46. $y = x^2 - 4x - 32$
47. $y = x^2 + 7x - 30$
48. $f(x) = x^2 + 11x$
49. $g(x) = x^2 - 8x$
50. $y = x^2 - 64$
51. $y = x^2 - 25$
52. $f(x) = x^2 - 12x - 45$
53. $g(x) = x^2 + 19x + 84$
54. $y = x^2 + 22x + 121$
55. $y = x^2 + 2x + 1$

56. ★ **MULTIPLE CHOICE** What are the zeros of $f(x) = x^2 + 6x - 55$?

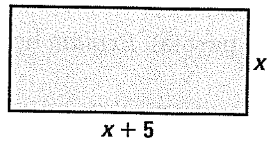
(A) −11, −5 (B) −11, 5 (C) −5, 11 (D) 5, 11

57. **REASONING** Write a quadratic equation of the form $x^2 + bx + c = 0$ that has roots 8 and 11.

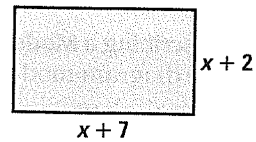
58. ★ **SHORT RESPONSE** For what integers b can the expression $x^2 + bx + 7$ be factored? *Explain.*

GEOMETRY **Find the value of x.**

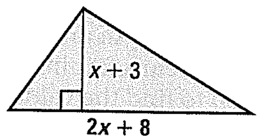
59. Area of rectangle = 36



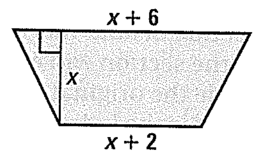
60. Area of rectangle = 84



61. Area of triangle = 42



62. Area of trapezoid = 32



63. ★ **OPEN-ENDED MATH** Write a quadratic function with zeros that are equidistant from 10 on a number line.

64. **CHALLENGE** Is there a formula for factoring the *sum* of two squares? You will investigate this question in parts (a) and (b).

a. Consider the sum of two squares $x^2 + 16$. If this sum can be factored, then there are integers m and n such that $x^2 + 16 = (x + m)(x + n)$. Write two equations that m and n must satisfy.

b. Show that there are no integers m and n that satisfy both equations you wrote in part (a). What can you conclude?

PROBLEM SOLVING

EXAMPLE 4 on p. 254 for Exs. 65–67

65. **SKATE PARK** A city's skate park is a rectangle 100 feet long by 50 feet wide. The city wants to triple the area of the skate park by adding the same distance x to the length and the width. Write and solve an equation to find the value of x. What are the new dimensions of the skate park?

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66. **ZOO** A rectangular enclosure at a zoo is 35 feet long by 18 feet wide. The zoo wants to double the area of the enclosure by adding the same distance x to the length and the width. Write and solve an equation to find the value of x. What are the new dimensions of the enclosure?

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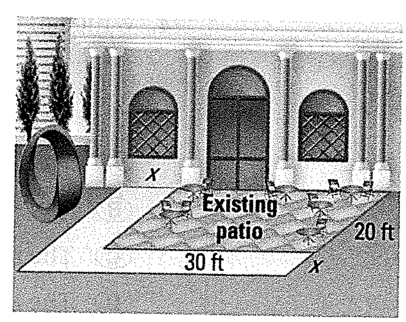
67. **MULTI-STEP PROBLEM** A museum has a café with a rectangular patio. The museum wants to add 464 square feet to the area of the patio by expanding the existing patio as shown.

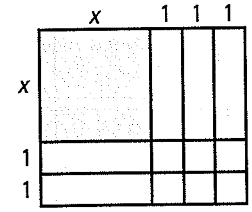


a. Find the area of the existing patio.

b. Write a verbal model and an equation that you can use to find the value of x.

c. Solve your equation. By what distance x should the length and the width of the patio be expanded?

68. **MULTIPLE REPRESENTATIONS** Use the diagram shown.



a. **Writing an Expression** Write a quadratic trinomial that represents the area of the diagram.

b. **Describing a Model** Factor the expression from part (a). *Explain* how the diagram models the factorization.

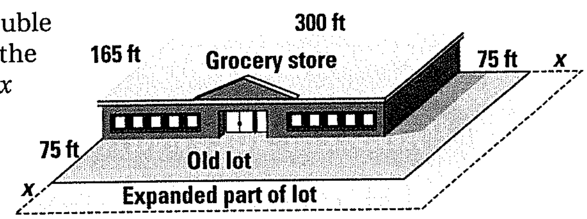
c. **Drawing a Diagram** Draw a diagram that models the factorization $x^2 + 8x + 15 = (x + 5)(x + 3)$.

69. **SCHOOL FAIR** At last year's school fair, an 18 foot by 15 foot rectangular section of land was roped off for a dunking booth. The length and width of the section will each be increased by x feet for this year's fair in order to triple the original area. Write and solve an equation to find the value of x. What is the length of rope needed to enclose the new section?

70. **RECREATION CENTER** A rectangular deck for a recreation center is 21 feet long by 20 feet wide. Its area is to be halved by subtracting the same distance x from the length and the width. Write and solve an equation to find the value of x. What are the deck's new dimensions?

71. ★ **SHORT RESPONSE** A square garden has sides that are 10 feet long. A gardener wants to double the area of the garden by adding the same distance x to the length and the width. Write an equation that x must satisfy. Can you solve the equation you wrote by factoring? *Explain* why or why not.

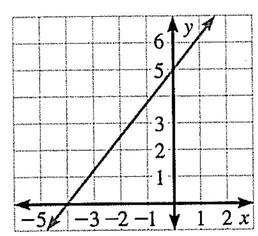
72. **CHALLENGE** A grocery store wants to double the area of its parking lot by expanding the existing lot as shown. By what distance x should the lot be expanded?



PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

73. What is the slope of the line shown?



Ⓐ $-\frac{5}{4}$ Ⓑ $-\frac{4}{5}$

Ⓒ $\frac{4}{5}$ Ⓓ $\frac{5}{4}$

74. Which of the following best describes the graphs of the equations below?

$$y = 3x - 2$$
$$-4y = x + 8$$

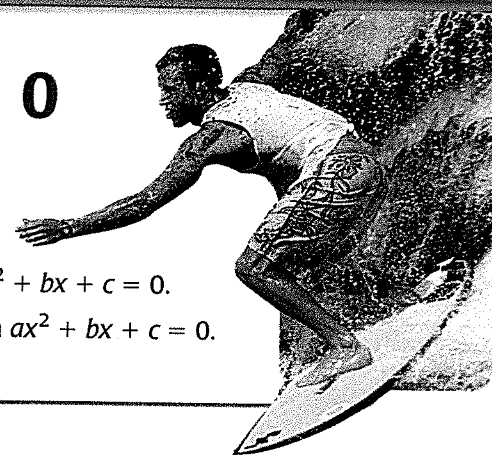
Ⓐ The lines have the same x-intercept.

Ⓑ The lines have the same y-intercept.

Ⓒ The lines are perpendicular to each other.

Ⓓ The lines are parallel to each other.

4.4 Solve $ax^2 + bx + c = 0$ by Factoring



PA M11.D.2.1.5 Solve quadratic equations using factoring (integers only—not including completing the square or the Quadratic Formula).

Before You used factoring to solve equations of the form $x^2 + bx + c = 0$.

Now You will use factoring to solve equations of the form $ax^2 + bx + c = 0$.

Why? So you can maximize a shop's revenue, as in Ex. 64.

Key Vocabulary
- **monomial,** *p. 252*

To factor $ax^2 + bx + c$ when $a \neq 1$, find integers k, l, m, and n such that:

$$ax^2 + bx + c = (kx + m)(lx + n) = klx^2 + (kn + lm)x + mn$$

So, k and l must be factors of a, and m and n must be factors of c.

EXAMPLE 1 Factor $ax^2 + bx + c$ where $c > 0$

Factor $5x^2 - 17x + 6$.

Solution

You want $5x^2 - 17x + 6 = (kx + m)(lx + n)$ where k and l are factors of 5 and m and n are factors of 6. You can assume that k and l are positive and $k \geq l$. Because $mn > 0$, m and n have the same sign. So, m and n must both be negative because the coefficient of x, -17, is negative.

FACTOR EXPRESSIONS
When factoring $ax^2 + bx + c$ where $a > 0$, it is customary to choose factors $kx + m$ and $lx + n$ such that k and l are positive.

k, l	5, 1	5, 1	5, 1	**5, 1**
m, n	−6, −1	−1, −6	−3, −2	**−2, −3**
(kx + m)(lx + n)	$(5x - 6)(x - 1)$	$(5x - 1)(x - 6)$	$(5x - 3)(x - 2)$	$\mathbf{(5x - 2)(x - 3)}$
$ax^2 + bx + c$	$5x^2 - 11x + 6$	$5x^2 - 31x + 6$	$5x^2 - 13x + 6$	$\mathbf{5x^2 - 17x + 6}$

▶ The correct factorization is $5x^2 - 17x + 6 = (5x - 2)(x - 3)$.

EXAMPLE 2 Factor $ax^2 + bx + c$ where $c < 0$

Factor $3x^2 + 20x - 7$.

Solution

You want $3x^2 + 20x - 7 = (kx + m)(lx + n)$ where k and l are factors of 3 and m and n are factors of -7. Because $mn < 0$, m and n have opposite signs.

k, l	3, 1	**3, 1**	3, 1	3, 1
m, n	7, −1	**−1, 7**	−7, 1	1, −7
(kx + m)(lx + n)	$(3x + 7)(x - 1)$	$\mathbf{(3x - 1)(x + 7)}$	$(3x - 7)(x + 1)$	$(3x + 1)(x - 7)$
$ax^2 + bx + c$	$3x^2 + 4x - 7$	$\mathbf{3x^2 + 20x - 7}$	$3x^2 - 4x - 7$	$3x^2 - 20x - 7$

▶ The correct factorization is $3x^2 + 20x - 7 = (3x - 1)(x + 7)$.

GUIDED PRACTICE for Examples 1 and 2

Factor the expression. If the expression cannot be factored, say so.

1. $7x^2 - 20x - 3$
2. $5z^2 + 16z + 3$
3. $2w^2 + w + 3$
4. $3x^2 + 5x - 12$
5. $4u^2 + 12u + 5$
6. $4x^2 - 9x + 2$

FACTORING SPECIAL PRODUCTS If the values of a and c in $ax^2 + bx + c$ are perfect squares, check to see whether you can use one of the special factoring patterns from Lesson 4.3 to factor the expression.

EXAMPLE 3 Factor with special patterns

Factor the expression.

a. $9x^2 - 64 = (3x)^2 - 8^2$ — **Difference of two squares**

$= (3x + 8)(3x - 8)$

b. $4y^2 + 20y + 25 = (2y)^2 + 2(2y)(5) + 5^2$ — **Perfect square trinomial**

$= (2y + 5)^2$

c. $36w^2 - 12w + 1 = (6w)^2 - 2(6w)(1) + 1^2$ — **Perfect square trinomial**

$= (6w - 1)^2$

GUIDED PRACTICE for Example 3

Factor the expression.

7. $16x^2 - 1$
8. $9y^2 + 12y + 4$
9. $4r^2 - 28r + 49$
10. $25s^2 - 80s + 64$
11. $49z^2 + 42z + 9$
12. $36n^2 - 9$

FACTORING OUT MONOMIALS When factoring an expression, first check to see whether the terms have a common monomial factor.

EXAMPLE 4 Factor out monomials first

AVOID ERRORS
Be sure to factor out the common monomial from all of the terms of the expression, not just the first term.

Factor the expression.

a. $5x^2 - 45 = 5(x^2 - 9)$

$= 5(x + 3)(x - 3)$

b. $6q^2 - 14q + 8 = 2(3q^2 - 7q + 4)$

$= 2(3q - 4)(q - 1)$

c. $-5z^2 + 20z = -5z(z - 4)$

d. $12p^2 - 21p + 3 = 3(4p^2 - 7p + 1)$

GUIDED PRACTICE for Example 4

Factor the expression.

13. $3s^2 - 24$
14. $8t^2 + 38t - 10$
15. $6x^2 + 24x + 15$
16. $12x^2 - 28x - 24$
17. $-16n^2 + 12n$
18. $6z^2 + 33z + 36$

SOLVING QUADRATIC EQUATIONS As you saw in Lesson 4.3, if the left side of the quadratic equation $ax^2 + bx + c = 0$ can be factored, then the equation can be solved using the zero product property.

EXAMPLE 5 Solve quadratic equations

Solve (a) $3x^2 + 10x - 8 = 0$ and (b) $5p^2 - 16p + 15 = 4p - 5$.

a.

$3x^2 + 10x - 8 = 0$	**Write original equation.**
$(3x - 2)(x + 4) = 0$	**Factor.**
$3x - 2 = 0$ or $x + 4 = 0$	**Zero product property**
$x = \frac{2}{3}$ or $x = -4$	**Solve for *x*.**

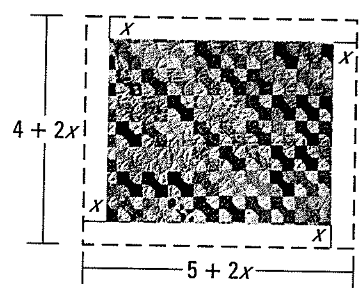
b.

$5p^2 - 16p + 15 = 4p - 5$	**Write original equation.**
$5p^2 - 20p + 20 = 0$	**Write in standard form.**
$p^2 - 4p + 4 = 0$	**Divide each side by 5.**
$(p - 2)^2 = 0$	**Factor.**
$p - 2 = 0$	**Zero product property**
$p = 2$	**Solve for *p*.**

INTERPRET EQUATIONS
If the square of an expression is zero, then the expression itself must be zero.

EXAMPLE 6 Use a quadratic equation as a model

QUILTS You have made a rectangular quilt that is 5 feet by 4 feet. You want to use the remaining 10 square feet of fabric to add a decorative border of uniform width to the quilt. What should the width of the quilt's border be?



Solution

Write a verbal model. Then write an equation.

Area of border (square feet)	=	Area of quilt and border (square feet)	−	Area of quilt (square feet)
10	=	$\mathbf{(5 + 2x)(4 + 2x)}$	−	$\mathbf{(5)(4)}$

$10 = 20 + 18x + 4x^2 - 20$	**Multiply using FOIL.**
$0 = 4x^2 + 18x - 10$	**Write in standard form.**
$0 = 2x^2 + 9x - 5$	**Divide each side by 2.**
$0 = (2x - 1)(x + 5)$	**Factor.**
$2x - 1 = 0$ or $x + 5 = 0$	**Zero product property**
$x = \frac{1}{2}$ or $x = -5$	**Solve for *x*.**

▶ Reject the negative value, −5. The border's width should be $\frac{1}{2}$ ft, or 6 in.

FACTORING AND ZEROS To find the maximum or minimum value of a quadratic function, you can first use factoring to write the function in intercept form $y = a(x - p)(x - q)$. Because the function's vertex lies on the axis of symmetry $x = \frac{p + q}{2}$, the maximum or minimum occurs at the *average* of the zeros p and q.

EXAMPLE 7 Solve a multi-step problem

MAGAZINES A monthly teen magazine has 28,000 subscribers when it charges $10 per annual subscription. For each $1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

Solution

STEP 1 **Define** the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

STEP 2 **Write** a verbal model. Then write and simplify a quadratic function.

Annual revenue (dollars)	=	Number of subscribers (people)	•	Subscription price (dollars/person)
$R(x)$	=	$(28{,}000 - 2000x)$	•	$(10 + x)$

$$R(x) = (-2000x + 28{,}000)(x + 10)$$

$$R(x) = -2000(x - 14)(x + 10)$$

STEP 3 **Identify** the zeros and find their average. Find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 14 and −10. The average of the zeros is $\frac{14 + (-10)}{2} = \mathbf{2}$. To maximize revenue, each subscription should cost $10 + **$2** = $12.

STEP 4 **Find** the maximum annual revenue.

$R(2) = -2000(2 - 14)(2 + 10) = \$288{,}000$

▶ The magazine should charge $12 per subscription to maximize annual revenue. The maximum annual revenue is $288,000.

✓ GUIDED PRACTICE for Examples 5, 6, and 7

Solve the equation.

19. $6x^2 - 3x - 63 = 0$ **20.** $12x^2 + 7x + 2 = x + 8$ **21.** $7x^2 + 70x + 175 = 0$

22. WHAT IF? In Example 7, suppose the magazine initially charges $11 per annual subscription. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

4.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 27, 39, and 63

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 64, 65, and 67

SKILL PRACTICE

1. **VOCABULARY** What is the greatest common monomial factor of the terms of the expression $12x^2 + 8x + 20$?

2. ★ **WRITING** *Explain* how the values of a and c in $ax^2 + bx + c$ help you determine whether you can use a perfect square trinomial factoring pattern.

EXAMPLES 1 and 2 on p. 259 for Exs. 3–12

FACTORING **Factor the expression. If the expression cannot be factored, say so.**

3. $2x^2 + 5x + 3$
4. $3n^2 + 7n + 4$
5. $4r^2 + 5r + 1$
6. $6p^2 + 5p + 1$
7. $11z^2 + 2z - 9$
8. $15x^2 - 2x - 8$
9. $4y^2 - 5y - 4$
10. $14m^2 + m - 3$
11. $9d^2 - 13d - 10$

12. ★ **MULTIPLE CHOICE** Which factorization of $5x^2 + 14x - 3$ is correct?

Ⓐ $(5x - 3)(x + 1)$ Ⓑ $(5x + 1)(x - 3)$

Ⓒ $5(x - 1)(x + 3)$ Ⓓ $(5x - 1)(x + 3)$

EXAMPLE 3 on p. 260 for Exs. 13–21

FACTORING WITH SPECIAL PATTERNS **Factor the expression.**

13. $9x^2 - 1$
14. $4r^2 - 25$
15. $49n^2 - 16$
16. $16s^2 + 8s + 1$
17. $49x^2 + 70x + 25$
18. $64w^2 + 144w + 81$
19. $9p^2 - 12p + 4$
20. $25t^2 - 30t + 9$
21. $36x^2 - 84x + 49$

EXAMPLE 4 on p. 260 for Exs. 22–31

FACTORING MONOMIALS FIRST **Factor the expression.**

22. $12x^2 - 4x - 40$
23. $18z^2 + 36z + 16$
24. $32v^2 - 2$
25. $6u^2 - 24u$
26. $12m^2 - 36m + 27$
27. $20x^2 + 124x + 24$
28. $21x^2 - 77x - 28$
29. $-36n^2 + 48n - 15$
30. $-8y^2 + 28y - 60$

31. **ERROR ANALYSIS** *Describe* and correct the error in factoring the expression.

$$4x^2 - 36 = 4(x^2 - 36)$$
$$= 4(x + 6)(x - 6)$$

EXAMPLE 5 on p. 261 for Exs. 32–40

SOLVING EQUATIONS **Solve the equation.**

32. $16x^2 - 1 = 0$
33. $11q^2 - 44 = 0$
34. $14s^2 - 21s = 0$
35. $45n^2 + 10n = 0$
36. $4x^2 - 20x + 25 = 0$
37. $4p^2 + 12p + 9 = 0$
38. $15x^2 + 7x - 2 = 0$
39. $6r^2 - 7r - 5 = 0$
40. $36z^2 + 96z + 15 = 0$

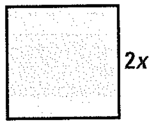
EXAMPLE 7 on p. 262 for Exs. 41–49

FINDING ZEROS **Find the zeros of the function by rewriting the function in intercept form.**

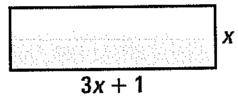
41. $y = 4x^2 - 19x - 5$
42. $g(x) = 3x^2 - 8x + 5$
43. $y = 5x^2 - 27x - 18$
44. $f(x) = 3x^2 - 3x$
45. $y = 11x^2 - 19x - 6$
46. $y = 16x^2 - 2x - 5$
47. $y = 15x^2 - 5x - 20$
48. $y = 18x^2 - 6x - 4$
49. $g(x) = 12x^2 + 5x - 7$

GEOMETRY **Find the value of x.**

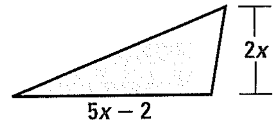
50. Area of square = 36



51. Area of rectangle = 30



52. Area of triangle = 115



SOLVING EQUATIONS **Solve the equation.**

53. $2x^2 - 4x - 8 = -x^2 + x$

54. $24x^2 + 8x + 2 = 5 - 6x$

55. $18x^2 - 22x = 28$

56. $13x^2 + 21x = -5x^2 + 22$

57. $x = 4x^2 - 15x$

58. $(x + 8)^2 = 16 - x^2 + 9x$

CHALLENGE **Factor the expression.**

59. $2x^3 - 5x^2 + 3x$

60. $8x^4 - 8x^3 - 6x^2$

61. $9x^3 - 4x$

PROBLEM SOLVING

EXAMPLE 6
on p. 261
for Exs. 62–63

62. ARTS AND CRAFTS You have a rectangular stained glass window that measures 2 feet by 1 foot. You have 4 square feet of glass with which to make a border of uniform width around the window. What should the width of the border be?

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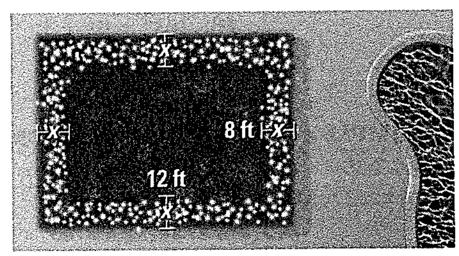
63. URBAN PLANNING You have just planted a rectangular flower bed of red roses in a city park. You want to plant a border of yellow roses around the flower bed as shown. Because you bought the same number of red and yellow roses, the areas of the border and flower bed will be equal. What should the width of the border of yellow roses be?



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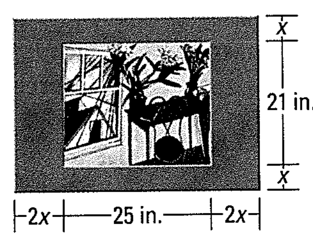
EXAMPLE 7
on p. 262
for Exs. 64–65

64. ★ MULTIPLE CHOICE A surfboard shop sells 45 surfboards per month when it charges $500 per surfboard. For each $20 decrease in price, the store sells 5 more surfboards per month. How much should the shop charge per surfboard in order to maximize monthly revenue?

(A) $340 (B) $492 (C) $508 (D) $660

65. ★ SHORT RESPONSE A restaurant sells about 330 sandwiches each day at a price of $6 each. For each $.25 decrease in price, 15 more sandwiches are sold per day. How much should the restaurant charge to maximize daily revenue? *Explain* each step of your solution. What is the maximum daily revenue?

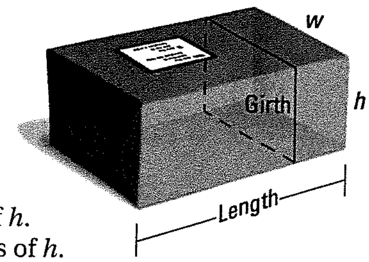
66. PAINTINGS You place a mat around a 25 inch by 21 inch painting as shown. The mat is twice as wide at the left and right of the painting as it is at the top and bottom of the painting. The area of the mat is 714 square inches. How wide is the mat at the left and right of the painting? at the top and bottom of the painting?



○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

67. ★ **EXTENDED RESPONSE** A U.S. Postal Service guideline states that for a rectangular package like the one shown, the sum of the length and the girth cannot exceed 108 inches. Suppose that for one such package, the length is 36 inches and the girth is as large as possible.

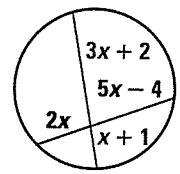


a. What is the girth of the package?

b. Write an expression for the package's width w in terms of h. Write an equation giving the package's volume V in terms of h.

c. What height and width maximize the volume of the package? What is the maximum volume? *Explain* how you found it.

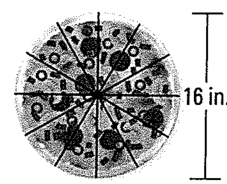
68. **CHALLENGE** Recall from geometry the theorem about the products of the lengths of segments of two chords that intersect in the interior of a circle. Use this theorem to find the value of x in the diagram.



PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

69. A pizza is divided into 12 equal slices as shown. The diameter of the pizza is 16 inches. What is the approximate area of one slice of pizza?



Ⓐ 15.47 in.^{2} Ⓑ 16.76 in.^{2}

Ⓒ 21.21 in.^{2} Ⓓ 67.02 in.^{2}

70. While shopping at Store A, Sam finds a television on sale for \$210. His friend tells him that the same television at Store B is on sale for \$161. About what percent of the cost of the television at Store A does Sam save by buying the television at Store B?

Ⓐ 20% Ⓑ 23% Ⓒ 30% Ⓓ 77%

QUIZ *for Lessons 4.1–4.4*

Graph the function. Label the vertex and axis of symmetry. *(p. 236)*

1. $y = x^2 - 6x + 14$ 2. $y = 2x^2 + 8x + 15$ 3. $f(x) = -3x^2 + 6x - 5$

Write the quadratic function in standard form. *(p. 245)*

4. $y = (x - 4)(x - 8)$ 5. $g(x) = -2(x + 3)(x - 7)$ 6. $y = 5(x + 6)^2 - 2$

Solve the equation.

7. $x^2 + 9x + 20 = 0$ *(p. 252)* 8. $n^2 - 11n + 24 = 0$ *(p. 252)* 9. $z^2 - 3z - 40 = 0$ *(p. 252)*

10. $5s^2 - 14s - 3 = 0$ *(p. 259)* 11. $7a^2 - 30a + 8 = 0$ *(p. 259)* 12. $4x^2 + 20x + 25 = 0$ *(p. 259)*

13. **DVD PLAYERS** A store sells about 50 of a new model of DVD player per month at a price of \$140 each. For each \$10 decrease in price, about 5 more DVD players per month are sold. How much should the store charge in order to maximize monthly revenue? What is the maximum monthly revenue? *(p. 259)*

PROBLEM SOLVING WORKSHOP
LESSON 4.5

Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 269

MULTIPLE REPRESENTATIONS In Example 5 on page 269, you solved a quadratic equation by finding square roots. You can also solve a quadratic equation using a table or a graph.

PROBLEM

SCIENCE COMPETITION For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet. How long does the container take to hit the ground?

METHOD 1

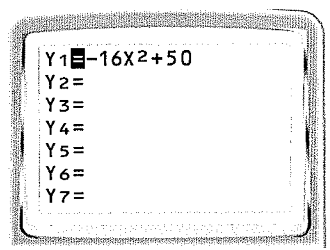
Using a Table One alternative approach is to write a quadratic equation and then use a table of values to solve the equation. You can use a graphing calculator to make the table.

STEP 1 **Write** an equation that models the situation using the height function $h = -16t^2 + h_0$.

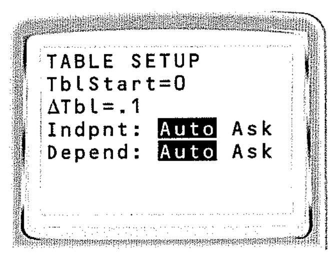
$h = -16t^2 + h_0$ **Write height function.**

$0 = -16t^2 + 50$ **Substitute 0 for *h* and 50 for h_0.**

STEP 2 **Enter** the function $y = -16x^2 + 50$ into a graphing calculator. Note that time is now represented by x and height is now represented by y.

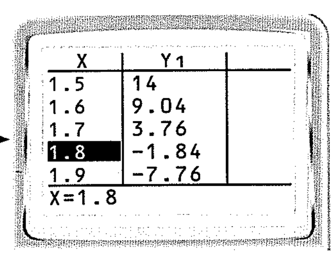


STEP 3 **Make** a table of values for the function. Set the table so that the x-values start at 0 and increase in increments of 0.1.



STEP 4 **Scroll** through the table to find the time x at which the height y of the container is 0 feet.

The table shows that $y = 0$ between $x = 1.7$ and $x = 1.8$ because y has a change of sign.



▸ The container hits the ground between 1.7 and 1.8 seconds after it is dropped.

METHOD 2

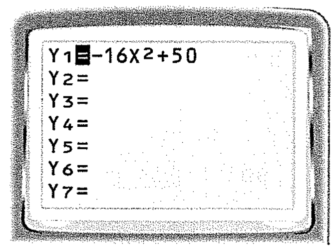
Using a Graph Another approach is to write a quadratic equation and then use a graph to solve the equation. You can use a graphing calculator to make the graph.

STEP 1 **Write** an equation that models the situation using the height function $h = -16t^2 + h_0$.

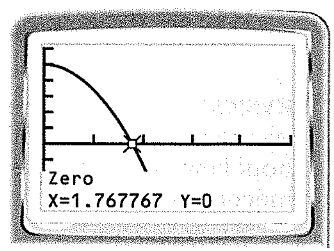
$h = -16t^2 + h_0$ **Write height function.**

$0 = -16t^2 + 50$ **Substitute 0 for *h* and 50 for h_0.**

STEP 2 **Enter** the function $y = -16x^2 + 50$ into a graphing calculator. Note that time is now represented by x and height is now represented by y.



STEP 3 **Graph** the height function. Adjust the viewing window so that you can see the point where the graph crosses the positive x-axis. Find the positive x-value for which $y = 0$ using the *zero* feature. The graph shows that $y = 0$ when $x \approx 1.8$.



▶ The container hits the ground about 1.8 seconds after it is dropped.

PRACTICE

SOLVING EQUATIONS Solve the quadratic equation using a table and using a graph.

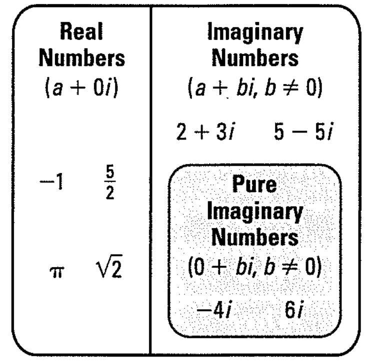
1. $2x^2 - 12x + 10 = 0$
2. $x^2 + 7x + 12 = 0$
3. $9x^2 - 30x + 25 = 0$
4. $7x^2 - 3 = 0$
5. $x^2 + 3x - 6 = 0$
6. **WHAT IF?** How long does it take for an egg container to hit the ground when dropped from a height of 100 feet? Find the answer using a table and using a graph.
7. **WIND PRESSURE** The pressure P (in pounds per square foot) from wind blowing at s miles per hour is given by $P = 0.00256s^2$. What wind speed produces a pressure of 30 lb/ft^2? Solve this problem using a table and using a graph.
8. **BIRDS** A bird flying at a height of 30 feet carries a shellfish. The bird drops the shellfish to break it and get the food inside. How long does it take for the shellfish to hit the ground? Find the answer using a table and using a graph.
9. **DROPPED OBJECT** You are dropping a ball from a window 29 feet above the ground to your friend who will catch it 4 feet above the ground. How long is the ball in the air before your friend catches it? Solve this problem using a table and using a graph.
10. **REASONING** *Explain* how to use the *table* feature of a graphing calculator to approximate the solution of the problem on page 272 to the nearest hundredth of a second. Use this procedure to find the approximate solution.

COMPLEX NUMBERS A **complex number** written in **standard form** is a number $a + bi$ where a and b are real numbers. The number a is the *real part* of the complex number, and the number bi is the *imaginary part.*

If $b \neq 0$, then $a + bi$ is an **imaginary number**. If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number**. The diagram shows how different types of complex numbers are related.

Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$. For example, if $x + yi = 5 - 3i$, then $x = 5$ and $y = -3$.

Complex Numbers ($a + bi$)



KEY CONCEPT *For Your Notebook*

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

EXAMPLE 2 Add and subtract complex numbers

Write the expression as a complex number in standard form.

a. $(8 - i) + (5 + 4i)$ **b.** $(7 - 6i) - (3 - 6i)$ **c.** $10 - (6 + 7i) + 4i$

Solution

a. $(8 - i) + (5 + 4i) = (8 + 5) + (-1 + 4)i$ — **Definition of complex addition**

$= 13 + 3i$ — **Write in standard form.**

b. $(7 - 6i) - (3 - 6i) = (7 - 3) + (-6 + 6)i$ — **Definition of complex subtraction**

$= 4 + 0i$ — **Simplify.**

$= 4$ — **Write in standard form.**

c. $10 - (6 + 7i) + 4i = [(10 - 6) - 7i] + 4i$ — **Definition of complex subtraction**

$= (4 - 7i) + 4i$ — **Simplify.**

$= 4 + (-7 + 4)i$ — **Definition of complex addition**

$= 4 - 3i$ — **Write in standard form.**

✓ GUIDED PRACTICE for Example 2

Write the expression as a complex number in standard form.

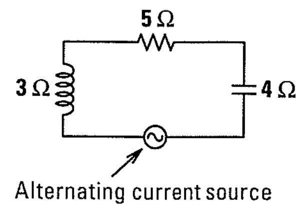
7. $(9 - i) + (-6 + 7i)$ **8.** $(3 + 7i) - (8 - 2i)$ **9.** $-4 - (1 + i) - (5 + 9i)$

EXAMPLE 3 Use addition of complex numbers in real life

ELECTRICITY Circuit components such as resistors, inductors, and capacitors all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. A circuit's total opposition to current flow is *impedance*. All of these quantities are measured in ohms (Ω).

READING Note that while a component's resistance or reactance is a real number, its impedance is a complex number.

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance	R	L	C
Impedance	R	Li	$-Ci$



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled.

The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit shown above.

Solution

The resistor has a resistance of 5 ohms, so its impedance is 5 ohms. The inductor has a reactance of 3 ohms, so its impedance is $3i$ ohms. The capacitor has a reactance of 4 ohms, so its impedance is $-4i$ ohms.

Impedance of circuit $= 5 + 3i + (-4i)$ **Add the individual impedances.**

$= 5 - i$ **Simplify.**

▶ The impedance of the circuit is $5 - i$ ohms.

MULTIPLYING COMPLEX NUMBERS To multiply two complex numbers, use the distributive property or the FOIL method just as you do when multiplying real numbers or algebraic expressions.

EXAMPLE 4 Multiply complex numbers

Write the expression as a complex number in standard form.

a. $4i(-6 + i)$ **b.** $(9 - 2i)(-4 + 7i)$

Solution

AVOID ERRORS When simplifying an expression that involves complex numbers, be sure to simplify i^2 to -1.

a. $4i(-6 + i) = -24i + 4i^2$ **Distributive property**

$= -24i + 4(-1)$ **Use $i^2 = -1$.**

$= -24i - 4$ **Simplify.**

$= -4 - 24i$ **Write in standard form.**

b. $(9 - 2i)(-4 + 7i) = -36 + 63i + 8i - 14i^2$ **Multiply using FOIL.**

$= -36 + 71i - 14(-1)$ **Simplify and use $i^2 = -1$.**

$= -36 + 71i + 14$ **Simplify.**

$= -22 + 71i$ **Write in standard form.**

COMPLEX CONJUGATES Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. For example, $(2 + 4i)(2 - 4i) = 4 - 8i + 8i + 16 = 20$. You can use this fact to write the quotient of two complex numbers in standard form.

EXAMPLE 5 Divide complex numbers

REWRITE QUOTIENTS
When a quotient has an imaginary number in the denominator, rewrite the denominator as a real number so you can express the quotient in standard form.

Write the quotient $\frac{7 + 5i}{1 - 4i}$ in standard form.

$\frac{7 + 5i}{1 - 4i} = \frac{7 + 5i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i}$	**Multiply numerator and denominator by $1 + 4i$, the complex conjugate of $1 - 4i$.**
$= \frac{7 + 28i + 5i + 20i^2}{1 + 4i - 4i - 16i^2}$	**Multiply using FOIL.**
$= \frac{7 + 33i + 20(-1)}{1 - 16(-1)}$	**Simplify and use $i^2 = 1$.**
$= \frac{-13 + 33i}{17}$	**Simplify.**
$= -\frac{13}{17} + \frac{33}{17}i$	**Write in standard form.**

✓ GUIDED PRACTICE for Examples 3, 4, and 5

10. **WHAT IF?** In Example 3, what is the impedance of the circuit if the given capacitor is replaced with one having a reactance of 7 ohms?

Write the expression as a complex number in standard form.

11. $i(9 - i)$
12. $(3 + i)(5 - i)$
13. $\frac{5}{1 + i}$
14. $\frac{5 + 2i}{3 - 2i}$

COMPLEX PLANE Just as every real number corresponds to a point on the real number line, every complex number corresponds to a point in the **complex plane**. As shown in the next example, the complex plane has a horizontal axis called the *real axis* and a vertical axis called the *imaginary axis.*

EXAMPLE 6 Plot complex numbers

Plot the complex numbers in the same complex plane.

a. $3 - 2i$ **b.** $-2 + 4i$ **c.** $3i$ **d.** $-4 - 3i$

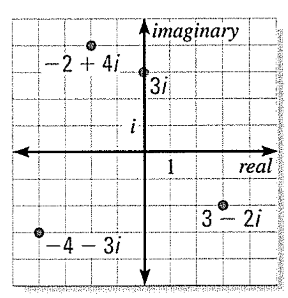
Solution

a. To plot $3 - 2i$, start at the origin, move 3 units to the right, and then move 2 units down.

b. To plot $-2 + 4i$, start at the origin, move 2 units to the left, and then move 4 units up.

c. To plot $3i$, start at the origin and move 3 units up.

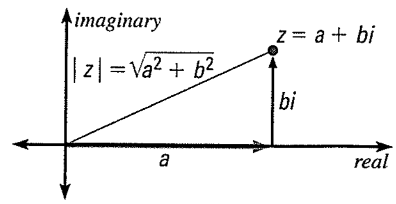
d. To plot $-4 - 3i$, start at the origin, move 4 units to the left, and then move 3 units down.



KEY CONCEPT — *For Your Notebook*

Absolute Value of a Complex Number

The **absolute value** of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance between z and the the origin in the complex plane.



EXAMPLE 7 Find absolute values of complex numbers

Find the absolute value of (a) $-4 + 3i$ and (b) $-3i$.

a. $|-4 + 3i| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

b. $|-3i| = |0 + (-3i)| = \sqrt{0^2 + (-3)^2} = \sqrt{9} = 3$

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✓ GUIDED PRACTICE for Examples 6 and 7

Plot the complex numbers in the same complex plane. Then find the absolute value of each complex number.

15. $4 - i$ **16.** $-3 - 4i$ **17.** $2 + 5i$ **18.** $-4i$

4.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 11, 29, and 67

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 50, 60, 69, and 74

SKILL PRACTICE

1. VOCABULARY What is the complex conjugate of $a - bi$?

2. ★ WRITING Is every complex number an imaginary number? *Explain.*

EXAMPLE 1 on p. 275 for Exs. 3–11

SOLVING QUADRATIC EQUATIONS **Solve the equation.**

3. $x^2 = -28$ **4.** $r^2 = -624$ **5.** $z^2 + 8 = 4$

6. $s^2 - 22 = -112$ **7.** $2x^2 + 31 = 9$ **8.** $9 - 4y^2 = 57$

9. $6t^2 + 5 = 2t^2 + 1$ **10.** $3p^2 + 7 = -9p^2 + 4$ **11.** $-5(n - 3)^2 = 10$

EXAMPLE 2 on p. 276 for Exs. 12–21

ADDING AND SUBTRACTING **Write the expression as a complex number in standard form.**

12. $(6 - 3i) + (5 + 4i)$ **13.** $(9 + 8i) + (8 - 9i)$ **14.** $(-2 - 6i) - (4 - 6i)$

15. $(-1 + i) - (7 - 5i)$ **16.** $(8 + 20i) - (-8 + 12i)$ **17.** $(8 - 5i) - (-11 + 4i)$

18. $(10 - 2i) + (-11 - 7i)$ **19.** $(14 + 3i) + (7 + 6i)$ **20.** $(-1 + 4i) + (-9 - 2i)$

21. ★ **MULTIPLE CHOICE** What is the standard form of the expression $(2 + 3i) - (7 + 4i)$?

Ⓐ -4 Ⓑ $-5 + 7i$ Ⓒ $-5 - i$ Ⓓ $5 + i$

EXAMPLES 4 and 5 on pp. 277–278 for Exs. 22–33

MULTIPLYING AND DIVIDING Write the expression as a complex number in standard form.

22. $6i(3 + 2i)$
23. $-i(4 - 8i)$
24. $(5 - 7i)(-4 - 3i)$
25. $(-2 + 5i)(-1 + 4i)$
26. $(-1 - 5i)(-1 + 5i)$
27. $(8 - 3i)(8 + 3i)$
28. $\dfrac{7i}{8 + i}$
29. $\dfrac{6i}{3 - i}$
30. $\dfrac{-2 - 5i}{3i}$
31. $\dfrac{4 + 9i}{12i}$
32. $\dfrac{7 + 4i}{2 - 3i}$
33. $\dfrac{-1 - 6i}{5 + 9i}$

EXAMPLE 6 on p. 278 for Exs. 34–41

PLOTTING COMPLEX NUMBERS Plot the numbers in the same complex plane.

34. $1 + 2i$
35. $-5 + 3i$
36. $-6i$
37. $4i$
38. $-7 - i$
39. $5 - 5i$
40. 7
41. -2

EXAMPLE 7 on p. 279 for Exs. 42–50

FINDING ABSOLUTE VALUE Find the absolute value of the complex number.

42. $4 + 3i$
43. $-3 + 10i$
44. $10 - 7i$
45. $-1 - 6i$
46. $-8i$
47. $4i$
48. $-4 + i$
49. $7 + 7i$

50. ★ **MULTIPLE CHOICE** What is the absolute value of $9 + 12i$?

Ⓐ 7 Ⓑ 15 Ⓒ 108 Ⓓ 225

STANDARD FORM Write the expression as a complex number in standard form.

51. $-8 - (3 + 2i) - (9 - 4i)$
52. $(3 + 2i) + (5 - i) + 6i$
53. $5i(3 + 2i)(8 + 3i)$
54. $(1 - 9i)(1 - 4i)(4 - 3i)$
55. $\dfrac{(5 - 2i) + (5 + 3i)}{(1 + i) - (2 - 4i)}$
56. $\dfrac{(10 + 4i) - (3 - 2i)}{(6 - 7i)(1 - 2i)}$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

57.
$$\begin{aligned}&(1 + 2i)(4 - i)\\ &= 4 - i + 8i - 2i^2\\ &= -2i^2 + 7i + 4\end{aligned}$$

58.
$$\begin{aligned}|2 - 3i| &= \sqrt{2^2 - 3^2}\\ &= \sqrt{-5}\\ &= i\sqrt{5}\end{aligned}$$

59. **ADDITIVE AND MULTIPLICATIVE INVERSES** The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. The multiplicative inverse of z is a complex number z_m such that $z \cdot z_m = 1$. Find the additive and multiplicative inverses of each complex number.

a. $z = 2 + i$ b. $z = 5 - i$ c. $z = -1 + 3i$

60. ★ **OPEN-ENDED MATH** Find two imaginary numbers whose sum is a real number. How are the imaginary numbers related?

CHALLENGE Write the expression as a complex number in standard form.

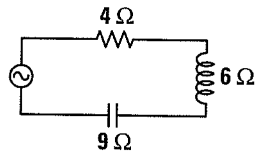
61. $\dfrac{a + bi}{c + di}$
62. $\dfrac{a - bi}{c - di}$
63. $\dfrac{a + bi}{c - di}$
64. $\dfrac{a - bi}{c + di}$

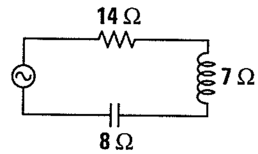
○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

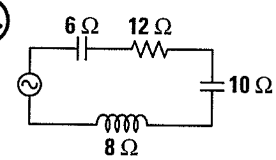
PROBLEM SOLVING

EXAMPLE 3 on p. 277 for Exs. 65–67

CIRCUITS In Exercises 65–67, each component of the circuit has been labeled with its resistance or reactance. Find the impedance of the circuit.

65. 

66. 

67. 

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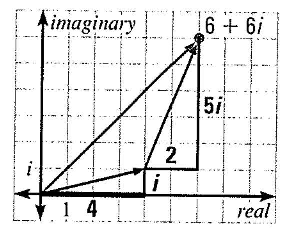
68. **VISUAL THINKING** The graph shows how you can geometrically add two complex numbers (in this case, $4 + i$ and $2 + 5i$) to find their sum (in this case, $6 + 6i$). Find each of the following sums by drawing a graph.



a. $(5 + i) + (1 + 4i)$

b. $(-7 + 3i) + (2 - 2i)$

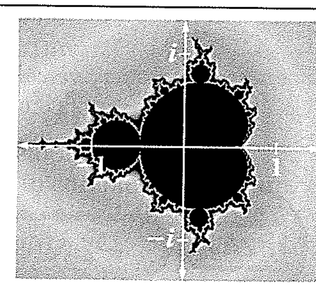
c. $(3 - 2i) + (-1 - i)$

d. $(4 + 2i) + (-5 - 3i)$

69. ★ **SHORT RESPONSE** Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. *Describe* the pattern you observe in the table. Verify that the pattern continues by evaluating the next four powers of i.

In Exercises 70–73, use the example below to determine whether the complex number c belongs to the Mandelbrot set. *Justify* your answer.

EXAMPLE Investigate the Mandelbrot set



The Mandelbrot set is the black region in the complex plane above.

Consider the function $f(z) = z^2 + c$ and this infinite list of complex numbers: $z_0 = 0$, $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$, ... If the absolute values of $z_0, z_1, z_2, z_3, \ldots$ are all less than some fixed number N, then c belongs to the *Mandelbrot set*. If the absolute values become infinitely large, then c does not belong to the Mandelbrot set.

Tell whether $c = 1 + i$ belongs to the Mandelbrot set.

Solution

Let $f(z) = z^2 + (1 + i)$.

$z_0 = \mathbf{0}$ $\qquad |z_0| = \mathbf{0}$

$z_1 = f(\mathbf{0}) = 0^2 + (1 + i) = \mathbf{1 + i}$ $\qquad |z_1| \approx \mathbf{1.41}$

$z_2 = f(\mathbf{1 + i}) = (1 + i)^2 + (1 + i) = \mathbf{1 + 3i}$ $\qquad |z_2| \approx \mathbf{3.16}$

$z_3 = f(\mathbf{1 + 3i}) = (1 + 3i)^2 + (1 + i) = \mathbf{-7 + 7i}$ $\qquad |z_3| \approx \mathbf{9.90}$

$z_4 = f(\mathbf{-7 + 7i}) = (-7 + 7i)^2 + (1 + i) = 1 - 97i$ $\qquad |z_4| \approx \mathbf{97.0}$

▶ Because the absolute values are becoming infinitely large, $c = 1 + i$ does not belong to the Mandelbrot set.

70. $c = i$

71. $c = -1 + i$

72. $c = -1$

73. $c = -0.5i$

4.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 27, 45, and 65

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 34, 58, 59, and 67

◆ = **MULTIPLE REPRESENTATIONS** Ex. 66

SKILL PRACTICE

1. **VOCABULARY** What is the difference between a binomial and a trinomial?

2. ★ **WRITING** *Describe* what completing the square means for an expression of the form $x^2 + bx$.

EXAMPLE 1 on p. 284 for Exs. 3–12

SOLVING BY SQUARE ROOTS Solve the equation by finding square roots.

3. $x^2 + 4x + 4 = 9$
4. $x^2 + 10x + 25 = 64$
5. $n^2 + 16n + 64 = 36$
6. $m^2 - 2m + 1 = 144$
7. $x^2 - 22x + 121 = 13$
8. $x^2 - 18x + 81 = 5$
9. $t^2 + 8t + 16 = 45$
10. $4u^2 + 4u + 1 = 75$
11. $9x^2 - 12x + 4 = -3$

12. ★ **MULTIPLE CHOICE** What are the solutions of $x^2 - 4x + 4 = -1$?

Ⓐ $2 \pm i$ Ⓑ $-2 \pm i$ Ⓒ $-3, -1$ Ⓓ $1, 3$

EXAMPLE 2 on p. 285 for Exs. 13–21

FINDING *C* Find the value of *c* that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

13. $x^2 + 6x + c$
14. $x^2 + 12x + c$
15. $x^2 - 24x + c$
16. $x^2 - 30x + c$
17. $x^2 - 2x + c$
18. $x^2 + 50x + c$
19. $x^2 + 7x + c$
20. $x^2 - 13x + c$
21. $x^2 - x + c$

EXAMPLES 3 and 4 on pp. 285–286 for Exs. 22–34

COMPLETING THE SQUARE Solve the equation by completing the square.

22. $x^2 + 4x = 10$
23. $x^2 + 8x = -1$
24. $x^2 + 6x - 3 = 0$
25. $x^2 + 12x + 18 = 0$
26. $x^2 - 18x + 86 = 0$
27. $x^2 - 2x + 25 = 0$
28. $2k^2 + 16k = -12$
29. $3x^2 + 42x = -24$
30. $4x^2 - 40x - 12 = 0$
31. $3s^2 + 6s + 9 = 0$
32. $7t^2 + 28t + 56 = 0$
33. $6r^2 + 6r + 12 = 0$

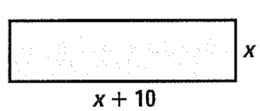
34. ★ **MULTIPLE CHOICE** What are the solutions of $x^2 + 10x + 8 = -5$?

Ⓐ $5 \pm 2\sqrt{3}$ Ⓑ $5 \pm 4\sqrt{3}$ Ⓒ $-5 \pm 2\sqrt{3}$ Ⓓ $-5 \pm 4\sqrt{3}$

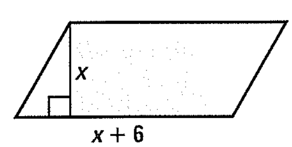
EXAMPLE 5 on p. 286 for Exs. 35–38

GEOMETRY Find the value of *x*.

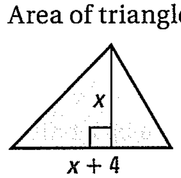
35. Area of rectangle = 50



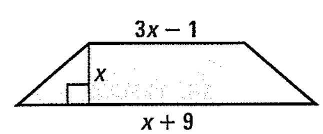
36. Area of parallelogram = 48



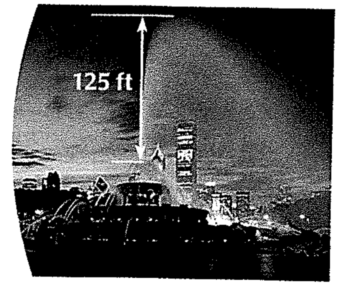
37. Area of triangle = 40



38. Area of trapezoid = 20



FINDING THE VERTEX **In Exercises 39 and 40, use completing the square to find the vertex of the given function's graph. Then tell what the vertex represents.**



Buckingham Fountain

39. At Buckingham Fountain in Chicago, the water's height h (in feet) above the main nozzle can be modeled by $h = -16t^2 + 89.6t$ where t is the time (in seconds) since the water has left the nozzle.

40. When you walk x meters per minute, your rate y of energy use (in calories per minute) can be modeled by $y = 0.0085x^2 - 1.5x + 120$.

EXAMPLES 6 and 7 on p. 287 for Exs. 41–49

WRITING IN VERTEX FORM **Write the quadratic function in vertex form. Then identify the vertex.**

41. $y = x^2 - 8x + 19$ **42.** $y = x^2 - 4x - 1$ **43.** $y = x^2 + 12x + 37$

44. $y = x^2 + 20x + 90$ **45.** $f(x) = x^2 - 3x + 4$ **46.** $g(x) = x^2 + 7x + 2$

47. $y = 2x^2 + 24x + 25$ **48.** $y = 5x^2 + 10x + 7$ **49.** $y = 2x^2 - 28x + 99$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

50.

$$x^2 + 10x + 13 = 0$$
$$x^2 + 10x = -13$$
$$x^2 + 10x + 25 = -13 + 25$$
$$(x + 5)^2 = 12$$
$$x + 5 = \pm\sqrt{12}$$
$$x = -5 \pm \sqrt{12}$$
$$x = -5 \pm 4\sqrt{3}$$

51.

$$4x^2 + 24x - 11 = 0$$
$$4(x^2 + 6x) = 11$$
$$4(x^2 + 6x + 9) = 11 + 9$$
$$4(x + 3)^2 = 20$$
$$(x + 3)^2 = 5$$
$$x + 3 = \pm\sqrt{5}$$
$$x = -3 \pm \sqrt{5}$$

COMPLETING THE SQUARE **Solve the equation by completing the square.**

52. $x^2 + 9x + 20 = 0$ **53.** $x^2 + 3x + 14 = 0$ **54.** $7q^2 + 10q = 2q^2 + 155$

55. $3x^2 + x = 2x - 6$ **56.** $0.1x^2 - x + 9 = 0.2x$ **57.** $0.4v^2 + 0.7v = 0.3v - 2$

58. ★ **OPEN-ENDED MATH** Write a quadratic equation with real-number solutions that can be solved by completing the square but not by factoring.

59. ★ **SHORT RESPONSE** In this exercise, you will investigate the graphical effect of completing the square.

a. Graph each pair of functions in the same coordinate plane.

$y = x^2 + 2x$	$y = x^2 + 4x$	$y = x^2 - 6x$
$y = (x + 1)^2$	$y = (x + 2)^2$	$y = (x - 3)^2$

b. *Compare* the graphs of $y = x^2 + bx$ and $y = \left(x + \frac{b}{2}\right)^2$. What happens to the graph of $y = x^2 + bx$ when you complete the square?

60. **REASONING** For what value(s) of k does $x^2 + bx + \left(\frac{b}{2}\right)^2 = k$ have exactly 1 real solution? 2 real solutions? 2 imaginary solutions?

61. **CHALLENGE** Solve $x^2 + bx + c = 0$ by completing the square. Your answer will be an expression for x in terms of b and c.

PROBLEM SOLVING

EXAMPLE 7
on p. 287
for Exs. 62–65

62. **DRUM MAJOR** While marching, a drum major tosses a baton into the air and catches it. The height h (in feet) of the baton after t seconds can be modeled by $h = -16t^2 + 32t + 6$. Find the maximum height of the baton.

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63. **VOLLEYBALL** The height h (in feet) of a volleyball t seconds after it is hit can be modeled by $h = -16t^2 + 48t + 4$. Find the volleyball's maximum height.

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64. **SKATEBOARD REVENUE** A skateboard shop sells about 50 skateboards per week for the price advertised. For each \$1 decrease in price, about 1 more skateboard per week is sold. The shop's revenue can be modeled by $y = (70 - x)(50 + x)$. Use vertex form to find how the shop can maximize weekly revenue.

65. **VIDEO GAME REVENUE** A store sells about 40 video game systems each month when it charges \$200 per system. For each \$10 increase in price, about 1 less system per month is sold. The store's revenue can be modeled by $y = (200 + 10x)(40 - x)$. Use vertex form to find how the store can maximize monthly revenue.

66. **MULTIPLE REPRESENTATIONS** The path of a ball thrown by a softball player can be modeled by the function

$$y = -0.0110x^2 + 1.23x + 5.50$$

where x is the softball's horizontal position (in feet) and y is the corresponding height (in feet).

   a. **Rewriting a Function** Write the given function in vertex form.

   b. **Making a Table** Make a table of values for the function. Include values of x from 0 to 120 in increments of 10.

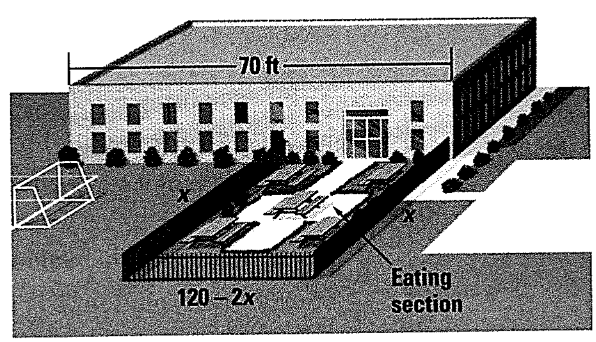
   c. **Drawing a Graph** Use your table to graph the function. What is the maximum height of the softball? How far does it travel?

67. ★ **EXTENDED RESPONSE** Your school is adding a rectangular outdoor eating section along part of a 70 foot side of the school. The eating section will be enclosed by a fence along its three open sides. The school has 120 feet of fencing and plans to use 1500 square feet of land for the eating section.

   a. Write an equation for the area of the eating section.

   b. Solve the equation. *Explain* why you must reject one of the solutions.

   c. What are the dimensions of the eating section?

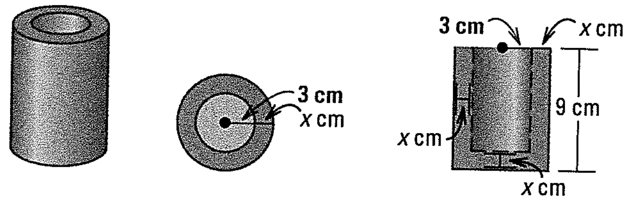


○ = **WORKED-OUT SOLUTIONS** on p. WS1   ★ = **STANDARDIZED TEST PRACTICE**   = **MULTIPLE REPRESENTATIONS**

GEOMETRY REVIEW

The volume of clay equals the difference of the volumes of two cylinders.

68. **CHALLENGE** In your pottery class, you are given a lump of clay with a volume of 200 cubic centimeters and are asked to make a cylindrical pencil holder. The pencil holder should be 9 centimeters high and have an inner radius of 3 centimeters. What thickness x should your pencil holder have if you want to use all of the clay?

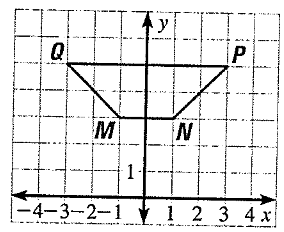


Top view **Side view**

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

69. If quadrilateral $MNPQ$ is reflected in the line $y = 3$, in which quadrant will the image of point N appear?



Ⓐ Quadrant I Ⓑ Quadrant II

Ⓒ Quadrant III Ⓓ Quadrant IV

70. A hose adds 120 gallons of water to a swimming pool in 1.5 hours. How many hours will it take for the hose to fill a different swimming pool that holds 600 gallons of water?

Ⓐ 5 h Ⓑ 6.25 h Ⓒ 7.5 h Ⓓ 8 h

QUIZ for Lessons 4.5–4.7

Solve the equation.

1. $4x^2 = 64$ *(p. 266)* **2.** $3(p-1)^2 = 15$ *(p. 266)* **3.** $16(m+5)^2 = 8$ *(p. 266)*

4. $-2z^2 = 424$ *(p. 275)* **5.** $s^2 + 12 = 9$ *(p. 275)* **6.** $7x^2 - 4 = -6$ *(p. 275)*

Write the expression as a complex number in standard form. *(p. 275)*

7. $(5 - 3i) + (-2 + 5i)$ **8.** $(-2 + 9i) - (7 + 8i)$ **9.** $3i(7 - 9i)$

10. $(8 - 3i)(-6 - 10i)$ **11.** $\frac{4i}{-6 - 11i}$ **12.** $\frac{3 - 2i}{-8 + 5i}$

Write the quadratic function in vertex form. Then identify the vertex. *(p. 284)*

13. $y = x^2 - 4x + 9$ **14.** $y = x^2 + 14x + 45$ **15.** $f(x) = x^2 - 10x + 17$

16. $g(x) = x^2 - 2x - 7$ **17.** $y = x^2 + x + 1$ **18.** $y = x^2 + 9x + 19$

19. **FALLING OBJECT** A student drops a ball from a school roof 45 feet above ground. How long is the ball in the air? *(p. 266)*

4.8 Use the Quadratic Formula and the Discriminant

Before You solved quadratic equations by completing the square.

Now You will solve quadratic equations using the quadratic formula.

Why? So you can model the heights of thrown objects, as in Example 5.

Key Vocabulary
- **quadratic formula**
- **discriminant**

In Lesson 4.7, you solved quadratic equations by completing the square for *each equation separately*. By completing the square *once* for the general equation $ax^2 + bx + c = 0$, you can develop a formula that gives the solutions of *any* quadratic equation. (See Exercise 67.) The formula for the solutions is called the **quadratic formula**.

KEY CONCEPT *For Your Notebook*

The Quadratic Formula

Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

EXAMPLE 1 Solve an equation with two real solutions

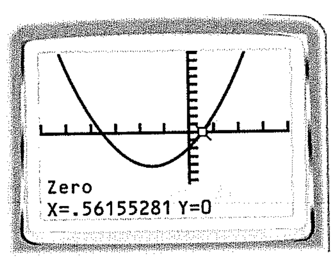
AVOID ERRORS
Remember to write the quadratic equation in standard form before applying the quadratic formula.

Solve $x^2 + 3x = 2$.

$x^2 + 3x = 2$	**Write original equation.**
$x^2 + 3x - 2 = 0$	**Write in standard form.**
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Quadratic formula**
$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$	$a = 1, b = 3, c = -2$
$x = \frac{-3 \pm \sqrt{17}}{2}$	**Simplify.**

▶ The solutions are $x = \frac{-3 + \sqrt{17}}{2} \approx 0.56$ and $x = \frac{-3 - \sqrt{17}}{2} \approx -3.56$.

CHECK Graph $y = x^2 + 3x - 2$ and note that the x-intercepts are about 0.56 and about −3.56. ✓



EXAMPLE 2 Solve an equation with one real solution

Solve $25x^2 - 18x = 12x - 9$.

ANOTHER WAY
You can also use factoring to solve this equation because the left side factors as $(5x - 3)^2$.

$$25x^2 - 18x = 12x - 9 \quad \textbf{Write original equation.}$$

$$25x^2 - 30x + 9 = 0 \quad \textbf{Write in standard form.}$$

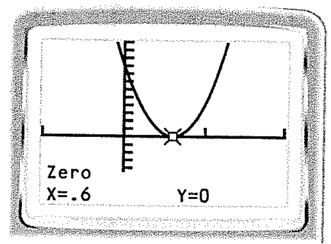
$$x = \frac{30 \pm \sqrt{(-30)^2 - 4(25)(9)}}{2(25)} \quad a = 25, b = -30, c = 9$$

$$x = \frac{30 \pm \sqrt{0}}{50} \quad \textbf{Simplify.}$$

$$x = \frac{3}{5} \quad \textbf{Simplify.}$$

▶ The solution is $\frac{3}{5}$.

CHECK Graph $y = 25x^2 - 30x + 9$ and note that the only x-intercept is $0.6 = \frac{3}{5}$. ✓



EXAMPLE 3 Solve an equation with imaginary solutions

Solve $-x^2 + 4x = 5$.

$$-x^2 + 4x = 5 \quad \textbf{Write original equation.}$$

$$-x^2 + 4x - 5 = 0 \quad \textbf{Write in standard form.}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)} \quad a = -1, b = 4, c = -5$$

$$x = \frac{-4 \pm \sqrt{-4}}{-2} \quad \textbf{Simplify.}$$

$$x = \frac{-4 \pm 2i}{-2} \quad \textbf{Rewrite using the imaginary unit } i.$$

$$x = 2 \pm i \quad \textbf{Simplify.}$$

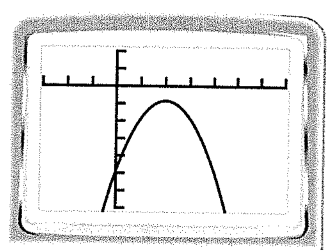
▶ The solutions are $2 + i$ and $2 - i$.

CHECK Graph $y = -x^2 + 4x - 5$. There are no x-intercepts. So, the original equation has no real solutions. The algebraic check for the imaginary solution $2 + i$ is shown.

$$-(2 + i)^2 + 4(2 + i) \stackrel{?}{=} 5$$

$$-3 - 4i + 8 + 4i \stackrel{?}{=} 5$$

$$5 = 5 \checkmark$$



GUIDED PRACTICE for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

1. $x^2 = 6x - 4$ **2.** $4x^2 - 10x = 2x - 9$ **3.** $7x - 5x^2 - 4 = 2x + 3$

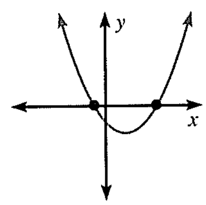
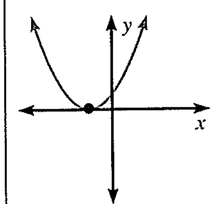
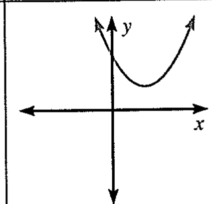
DISCRIMINANT In the quadratic formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **discriminant**

You can use the discriminant of a quadratic equation to determine the equation's number and type of solutions.

KEY CONCEPT *For Your Notebook*

Using the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

EXAMPLE 4 Use the discriminant

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $x^2 - 8x + 17 = 0$ **b.** $x^2 - 8x + 16 = 0$ **c.** $x^2 - 8x + 15 = 0$

Solution

Equation	**Discriminant**	**Solution(s)**
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a. $x^2 - 8x + 17 = 0$	$(-8)^2 - 4(1)(17) = -4$	Two imaginary: $4 \pm i$
b. $x^2 - 8x + 16 = 0$	$(-8)^2 - 4(1)(16) = 0$	One real: 4
c. $x^2 - 8x + 15 = 0$	$(-8)^2 - 4(1)(15) = 4$	Two real: 3, 5

✓ **GUIDED PRACTICE** for Example 4

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

4. $2x^2 + 4x - 4 = 0$ **5.** $3x^2 + 12x + 12 = 0$ **6.** $8x^2 = 9x - 11$

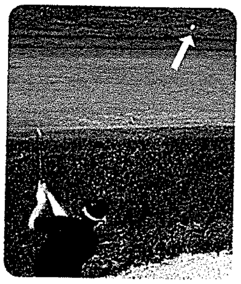
7. $7x^2 - 2x = 5$ **8.** $4x^2 + 3x + 12 = 3 - 3x$ **9.** $3x - 5x^2 + 1 = 6 - 7x$

MODELING LAUNCHED OBJECTS In Lesson 4.5, the function $h = -16t^2 + h_0$ was used to model the height of a *dropped* object. For an object that is *launched or thrown*, an extra term v_0t must be added to the model to account for the object's initial vertical velocity v_0 (in feet per second). Recall that h is the height (in feet), t is the time in motion (in seconds), and h_0 is the initial height (in feet).

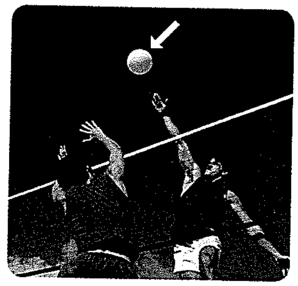
$h = -16t^2 + h_0$ **Object is dropped.**

$h = -16t^2 + v_0t + h_0$ **Object is launched or thrown.**

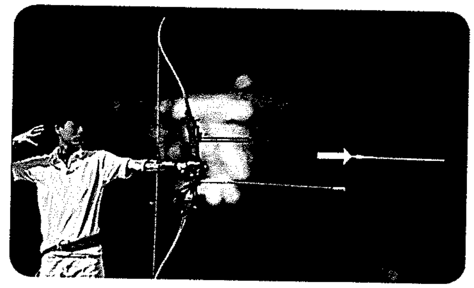
As shown below, the value of v_0 can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.



$v_0 > 0$



$v_0 < 0$



$v_0 = 0$

EXAMPLE 5 Solve a vertical motion problem

JUGGLING A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 40 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

Solution

Because the ball is thrown, use the model $h = -16t^2 + v_0t + h_0$. To find how long the ball is in the air, solve for t when $h = 3$.

$h = -16t^2 + v_0t + h_0$ **Write height model.**

$3 = -16t^2 + 40t + 4$ **Substitute 3 for h, 40 for v_0, and 4 for h_0.**

$0 = -16t^2 + 40t + 1$ **Write in standard form.**

$t = \dfrac{-40 \pm \sqrt{40^2 - 4(-16)(1)}}{2(-16)}$ **Quadratic formula**

$t = \dfrac{-40 \pm \sqrt{1664}}{-32}$ **Simplify.**

$t \approx -0.025$ or $t \approx 2.5$ **Use a calculator.**

▶ Reject the solution -0.025 because the ball's time in the air cannot be negative. So, the ball is in the air for about 2.5 seconds.

✓ **GUIDED PRACTICE** for Example 5

10. WHAT IF? In Example 5, suppose the ball leaves the juggler's hand with an initial vertical velocity of 50 feet per second. How long is the ball in the air?

4.8 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 19, 39, and 71
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 51, 55, 62, 69, 72, and 73

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: You can use the _?_ of a quadratic equation to determine the equation's number and type of solutions.

2. ★ **WRITING** *Describe* a real-life situation in which you can use the model $h = -16t^2 + v_0t + h_0$ but not the model $h = -16t^2 + h_0$.

EXAMPLES 1, 2, and 3 on pp. 292–293 for Exs. 3–30

EQUATIONS IN STANDARD FORM Use the quadratic formula to solve the equation.

3. $x^2 - 4x - 5 = 0$
4. $x^2 - 6x + 7 = 0$
5. $t^2 + 8t + 19 = 0$
6. $x^2 - 16x + 7 = 0$
7. $8w^2 - 8w + 2 = 0$
8. $5p^2 - 10p + 24 = 0$
9. $4x^2 - 8x + 1 = 0$
10. $6u^2 + 4u + 11 = 0$
11. $3r^2 - 8r - 9 = 0$

12. ★ **MULTIPLE CHOICE** What are the complex solutions of the equation $2x^2 - 16x + 50 = 0$?

Ⓐ $4 + 3i, 4 - 3i$
Ⓑ $4 + 12i, 4 - 12i$
Ⓒ $16 + 3i, 16 - 3i$
Ⓓ $16 + 12i, 16 - 12i$

EQUATIONS NOT IN STANDARD FORM Use the quadratic formula to solve the equation.

13. $3w^2 - 12w = -12$
14. $x^2 + 6x = -15$
15. $s^2 = -14 - 3s$
16. $-3y^2 = 6y - 10$
17. $3 - 8v - 5v^2 = 2v$
18. $7x - 5 + 12x^2 = -3x$
19. $4x^2 + 3 = x^2 - 7x$
20. $6 - 2t^2 = 9t + 15$
21. $4 + 9n - 3n^2 = 2 - n$

SOLVING USING TWO METHODS Solve the equation using the quadratic formula. Then solve the equation by factoring to check your solution(s).

22. $z^2 + 15z + 24 = -32$
23. $x^2 - 5x + 10 = 4$
24. $m^2 + 5m - 99 = 3m$
25. $s^2 - s - 3 = s$
26. $r^2 - 4r + 8 = 5r$
27. $3x^2 + 7x - 24 = 13x$
28. $45x^2 + 57x + 1 = 5$
29. $5p^2 + 40p + 100 = 25$
30. $9n^2 - 42n - 162 = 21n$

EXAMPLE 4 on p. 294 for Exs. 31–39

USING THE DISCRIMINANT Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

31. $x^2 - 8x + 16 = 0$
32. $s^2 + 7s + 11 = 0$
33. $8p^2 + 8p + 3 = 0$
34. $-4w^2 + w - 14 = 0$
35. $5x^2 + 20x + 21 = 0$
36. $8z - 10 = z^2 - 7z + 3$
37. $8n^2 - 4n + 2 = 5n - 11$
38. $5x^2 + 16x = 11x - 3x^2$
39. $7r^2 - 5 = 2r + 9r^2$

SOLVING QUADRATIC EQUATIONS Solve the equation using any method.

40. $16t^2 - 7t = 17t - 9$
41. $7x - 3x^2 = 85 + 2x^2 + 2x$
42. $4(x - 1)^2 = 6x + 2$
43. $25 - 16v^2 = 12v(v + 5)$
44. $\frac{3}{2}y^2 - 6y = \frac{3}{4}y - 9$
45. $3x^2 + \frac{9}{2}x - 4 = 5x + \frac{3}{4}$
46. $1.1(3.4x - 2.3)^2 = 15.5$
47. $19.25 = -8.5(2r - 1.75)^2$
48. $4.5 = 1.5(3.25 - s)^2$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

49.

$$3x^2 + 6x + 15 = 0$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(15)}}{2(3)}$$
$$= \frac{-6 \pm \sqrt{-144}}{6}$$
$$= \frac{-6 \pm 12}{6}$$
$$= 1 \text{ or } -3$$

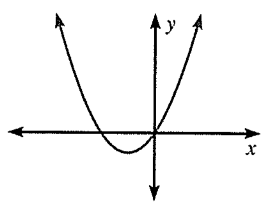
50.

$$x^2 + 6x + 8 = 2$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{4}}{2}$$
$$= \frac{-6 \pm 2}{2}$$
$$= -2 \text{ or } -4$$

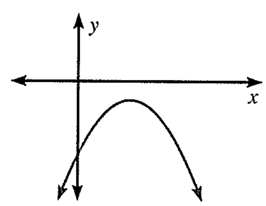
51. ★ **SHORT RESPONSE** For a quadratic equation $ax^2 + bx + c = 0$ with two real solutions, show that the mean of the solutions is $-\frac{b}{2a}$. How is this fact related to the symmetry of the graph of $y = ax^2 + bx + c$?

VISUAL THINKING In Exercises 52–54, the graph of a quadratic function $y = ax^2 + bx + c$ is shown. Tell whether the discriminant of $ax^2 + bx + c = 0$ is *positive, negative,* or *zero.*

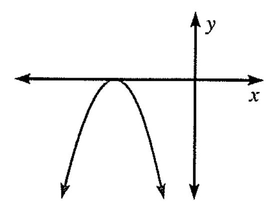
52.



53.



54.



55. ★ **MULTIPLE CHOICE** What is the value of c if the discriminant of $2x^2 + 5x + c = 0$ is -23?

(A) -23 (B) -6 (C) 6 (D) 14

THE CONSTANT TERM Use the discriminant to find all values of c for which the equation has (a) two real solutions, (b) one real solution, and (c) two imaginary solutions.

56. $x^2 - 4x + c = 0$ **57.** $x^2 + 8x + c = 0$ **58.** $-x^2 + 16x + c = 0$

59. $3x^2 + 24x + c = 0$ **60.** $-4x^2 - 10x + c = 0$ **61.** $x^2 - x + c = 0$

62. ★ **OPEN-ENDED MATH** Write a quadratic equation in standard form that has a discriminant of -10.

WRITING EQUATIONS Write a quadratic equation in the form $ax^2 + bx + c = 0$ such that $c = 4$ and the equation has the given solutions.

63. -4 and 3 **64.** $-\frac{4}{3}$ and -1 **65.** $-1 + i$ and $-1 - i$

66. **REASONING** Show that there is no quadratic equation $ax^2 + bx + c = 0$ such that a, b, and c are real numbers and $3i$ and $-2i$ are solutions.

67. **CHALLENGE** Derive the quadratic formula by completing the square to solve the general quadratic equation $ax^2 + bx + c = 0$.

PROBLEM SOLVING

EXAMPLE 5 on p. 295 for Exs. 68–69

68. FOOTBALL In a football game, a defensive player jumps up to block a pass by the opposing team's quarterback. The player bats the ball downward with his hand at an initial vertical velocity of -50 feet per second when the ball is 7 feet above the ground. How long do the defensive player's teammates have to intercept the ball before it hits the ground?

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69. ★ MULTIPLE CHOICE For the period 1990–2002, the number S (in thousands) of cellular telephone subscribers in the United States can be modeled by $S = 858t^2 + 1412t + 4982$ where t is the number of years since 1990. In what year did the number of subscribers reach 50 million?

Ⓐ 1991 Ⓑ 1992 Ⓒ 1996 Ⓓ 2000

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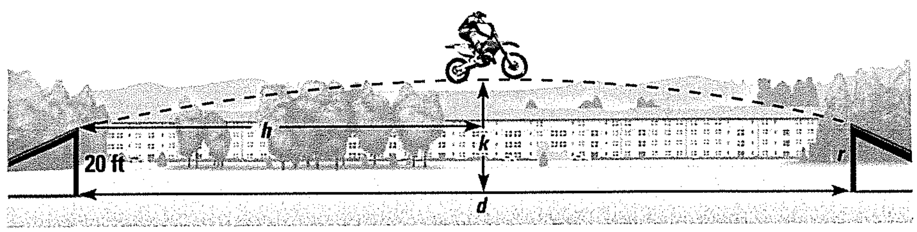
70. MULTI-STEP PROBLEM A stunt motorcyclist makes a jump from one ramp 20 feet off the ground to another ramp 20 feet off the ground. The jump between the ramps can be modeled by $y = -\frac{1}{640}x^2 + \frac{1}{4}x + 20$ where x is the horizontal distance (in feet) and y is the height above the ground (in feet).



a. What is the motorcycle's height r when it lands on the ramp?

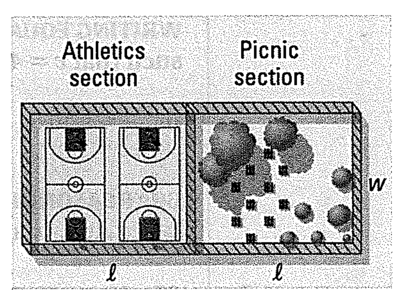
b. What is the distance d between the ramps?

c. What is the horizontal distance h the motorcycle has traveled when it reaches its maximum height?

d. What is the motorcycle's maximum height k above the ground?

71. BIOLOGY The number S of ant species in Kyle Canyon, Nevada, can be modeled by the function $S = -0.000013E^2 + 0.042E - 21$ where E is the elevation (in meters). Predict the elevation(s) at which you would expect to find 10 species of ants.

72. ★ SHORT RESPONSE A city planner wants to create adjacent sections for athletics and picnics in the yard of a youth center. The sections will be rectangular and will be surrounded by fencing as shown. There is 900 feet of fencing available. Each section should have an area of 12,000 square feet.



a. Show that $w = 300 - \frac{4}{3}\ell$.

b. Find the possible dimensions of each section.

73. ★ **EXTENDED RESPONSE** You can model the position (x, y) of a moving object using a pair of *parametric equations*. Such equations give x and y in terms of a third variable t that represents time. For example, suppose that when a basketball player attempts a free throw, the path of the basketball can be modeled by the parametric equations

$$x = 20t$$

$$y = -16t^2 + 21t + 6$$

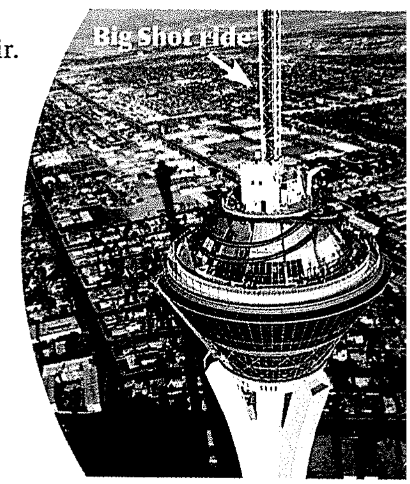
where x and y are measured in feet, t is measured in seconds, and the player's feet are at $(0, 0)$.

a. **Evaluate** Make a table of values giving the position (x, y) of the basketball after 0, 0.25, 0.5, 0.75, and 1 second.

b. **Graph** Use your table from part (a) to graph the parametric equations.

c. **Solve** The position of the basketball rim is $(15, 10)$. The top of the backboard is $(15, 12)$. Does the player make the free throw? *Explain.*

74. **CHALLENGE** The Stratosphere Tower in Las Vegas is 921 feet tall and has a "needle" at its top that extends even higher into the air. A thrill ride called the Big Shot catapults riders 160 feet up the needle and then lets them fall back to the launching pad.



a. The height h (in feet) of a rider on the Big Shot can be modeled by $h = -16t^2 + v_0t + 921$ where t is the elapsed time (in seconds) after launch and v_0 is the initial vertical velocity (in feet per second). Find v_0 using the fact that the maximum value of h is $921 + 160 = 1081$ feet.

b. A brochure for the Big Shot states that the ride up the needle takes two seconds. *Compare* this time with the time given by the model $h = -16t^2 + v_0t + 921$ where v_0 is the value you found in part (a). Discuss the model's accuracy.

PA

PENNSYLVANIA MIXED REVIEW

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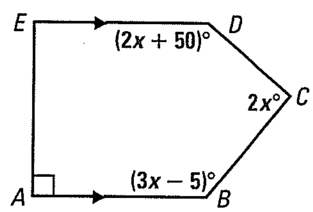
75. In the figure shown, $\overline{AB}$ is parallel to $\overline{ED}$. Which equation can be used to find the value of x?



Ⓐ $5x + 225 = 360$　　Ⓑ $5x + 235 = 540$

Ⓒ $7x + 235 = 360$　　Ⓓ $7x + 225 = 540$

76. Music recital tickets are \$4 for students and \$6 for adults. A total of 725 tickets are sold and \$3650 is collected. Which pair of equations can be used to determine the number of students, s, and the number of adults, a, who attended the music recital?

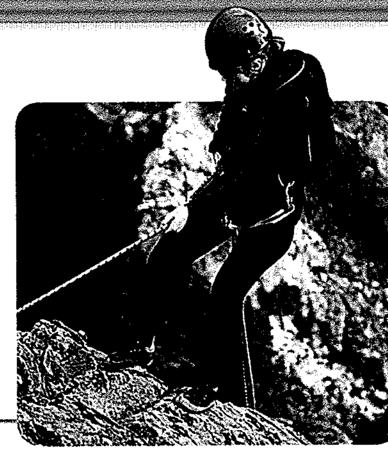
Ⓐ $s + a = 725$
$4s + 6a = 3650$

Ⓑ $s + a = 725$
$6s + 4a = 3650$

Ⓒ $s - a = 725$
$4s - 6a = 3650$

Ⓓ $4s + 6a = 725$
$s + a = 3650$

4.9 Graph and Solve Quadratic Inequalities



Before You graphed and solved linear inequalities.

Now You will graph and solve quadratic inequalities.

Why? So you can model the strength of a rope, as in Example 2.

Key Vocabulary
- **quadratic inequality in two variables**
- **quadratic inequality in one variable**

A **quadratic inequality in two variables** can be written in one of the following forms:

$$y < ax^2 + bx + c \qquad y \le ax^2 + bx + c \qquad y > ax^2 + bx + c \qquad y \ge ax^2 + bx + c$$

The graph of any such inequality consists of all solutions (x, y) of the inequality.

KEY CONCEPT *For Your Notebook*

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the forms above, follow these steps:

STEP 1 **Graph** the parabola with equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with $\le$ or $\ge$.

STEP 2 **Test** a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

STEP 3 **Shade** the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

EXAMPLE 1 Graph a quadratic inequality

Graph $y > x^2 + 3x - 4$.

Solution

AVOID ERRORS
Be sure to use a dashed parabola if the symbol is $>$ or $<$ and a solid parabola if the symbol is $\ge$ or $\le$.

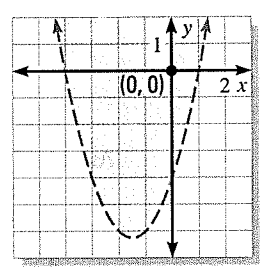
STEP 1 **Graph** $y = x^2 + 3x - 4$. Because the inequality symbol is $>$, make the parabola dashed.

STEP 2 **Test** a point inside the parabola, such as $(0, 0)$.

$$y > x^2 + 3x - 4$$
$$0 \overset{?}{>} 0^2 + 3(0) - 4$$
$$0 > -4 \checkmark$$

So, $(0, 0)$ is a solution of the inequality.

STEP 3 **Shade** the region inside the parabola.



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EXAMPLE 2 Use a quadratic inequality in real life

RAPPELLING A manila rope used for rappelling down a cliff can safely support a weight W (in pounds) provided

$$W \leq 1480d^2$$

where d is the rope's diameter (in inches). Graph the inequality.

Solution

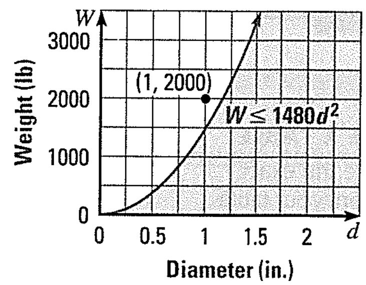
Graph $W = 1480d^2$ for nonnegative values of d. Because the inequality symbol is $\leq$, make the parabola solid. Test a point inside the parabola, such as (1, 2000).

$$W \leq 1480d^2$$

$$2000 \overset{?}{\leq} 1480(1)^2$$

$$2000 \leq 1480 \text{ ✗}$$

Because (1, 2000) is not a solution, shade the region below the parabola.



SYSTEMS OF QUADRATIC INEQUALITIES Graphing a *system* of quadratic inequalities is similar to graphing a system of linear inequalities. First graph each inequality in the system. Then identify the region in the coordinate plane common to all of the graphs. This region is called the *graph of the system.*

EXAMPLE 3 Graph a system of quadratic inequalities

Graph the system of quadratic inequalities.

$y \leq -x^2 + 4$ **Inequality 1**

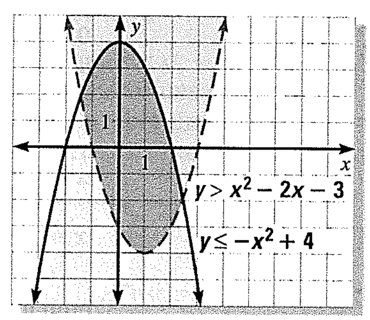
$y > x^2 - 2x - 3$ **Inequality 2**

Solution

STEP 1 **Graph** $y \leq -x^2 + 4$. The graph is the red region inside and including the parabola $y = -x^2 + 4$.

STEP 2 **Graph** $y > x^2 - 2x - 3$. The graph is the blue region inside (but not including) the parabola $y = x^2 - 2x - 3$.

STEP 3 **Identify** the **purple region** where the two graphs overlap. This region is the graph of the system.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the inequality.

1. $y > x^2 + 2x - 8$
2. $y \leq 2x^2 - 3x + 1$
3. $y < -x^2 + 4x + 2$
4. Graph the system of inequalities consisting of $y \geq x^2$ and $y < -x^2 + 5$.

4.9 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 17, 39, and 73

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 44, 45, 68, and 73

◆ = **MULTIPLE REPRESENTATIONS** Ex. 74

SKILL PRACTICE

1. **VOCABULARY** Give an example of a quadratic inequality in one variable and an example of a quadratic inequality in two variables.

2. ★ **WRITING** *Explain* how to solve $x^2 + 6x - 8 < 0$ using a table, by graphing, and algebraically.

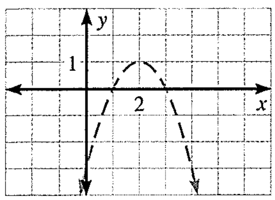
EXAMPLE 1 on p. 300 for Exs. 3–19

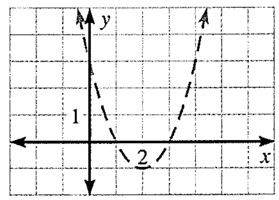
MATCHING INEQUALITIES WITH GRAPHS **Match the inequality with its graph.**

3. $y \le x^2 + 4x + 3$

4. $y > -x^2 + 4x - 3$

5. $y < x^2 - 4x + 3$

A.


B.


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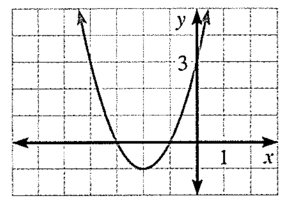


GRAPHING QUADRATIC INEQUALITIES **Graph the inequality.**

6. $y < -x^2$

7. $y \ge 4x^2$

8. $y > x^2 - 9$

9. $y \le x^2 + 5x$

10. $y < x^2 + 4x - 5$

11. $y > x^2 + 7x + 12$

12. $y \le -x^2 + 3x + 10$

13. $y \ge 2x^2 + 5x - 7$

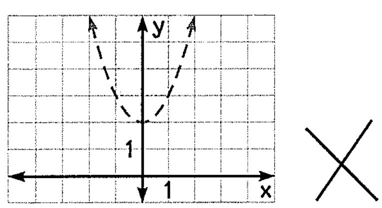
14. $y \ge -2x^2 + 9x - 4$

15. $y < 4x^2 - 3x - 5$

16. $y > 0.1x^2 - x + 1.2$

17. $y \le -\frac{2}{3}x^2 + 3x + 1$

ERROR ANALYSIS **Describe and correct the error in graphing** $y \ge x^2 + 2$.

18.


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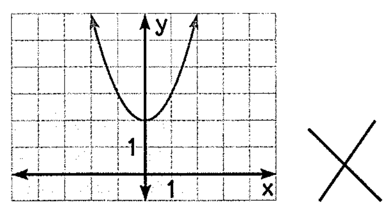


EXAMPLE 3 on p. 301 for Exs. 20–25

GRAPHING SYSTEMS **Graph the system of inequalities.**

20. $y \ge 2x^2$
$y < -x^2 + 1$

21. $y > -5x^2$
$y > 3x^2 - 2$

22. $y \ge x^2 - 4$
$y \le -2x^2 + 7x + 4$

23. $y \le -x^2 + 4x - 4$
$y < 2x^2 + x - 8$

24. $y > 3x^2 + 3x - 5$
$y < -x^2 + 5x + 10$

25. $y \ge x^2 - 3x - 6$
$y \ge 2x^2 + 7x + 6$

EXAMPLE 4 on p. 302 for Exs. 26–34

SOLVING USING A TABLE **Solve the inequality using a table.**

26. $x^2 - 5x < 0$

27. $x^2 + 2x - 3 > 0$

28. $x^2 + 3x \le 10$

29. $x^2 - 2x \ge 8$

30. $-x^2 + 15x - 50 > 0$

31. $x^2 - 10x < -16$

32. $x^2 - 4x > 12$

33. $3x^2 - 6x - 2 \le 7$

34. $2x^2 - 6x - 9 \ge 11$

EXAMPLE 5
on p. 302
for Exs. 35–43

SOLVING BY GRAPHING Solve the inequality by graphing.

35. $x^2 - 6x < 0$
36. $x^2 + 8x \leq -7$
37. $x^2 - 4x + 2 > 0$
38. $x^2 + 6x + 3 > 0$
39. $3x^2 + 2x - 8 \leq 0$
40. $3x^2 + 5x - 3 < 1$
41. $-6x^2 + 19x \geq 10$
42. $-\frac{1}{2}x^2 + 4x \geq 1$
43. $4x^2 - 10x - 7 < 10$

44. ★ **MULTIPLE CHOICE** What is the solution of $3x^2 - x - 4 > 0$?

Ⓐ $x < -1$ or $x > \frac{4}{3}$
Ⓑ $-1 < x < \frac{4}{3}$
Ⓒ $x < -\frac{4}{3}$ or $x > 1$
Ⓓ $1 < x < \frac{4}{3}$

45. ★ **MULTIPLE CHOICE** What is the solution of $2x^2 + 9x \leq 56$?

Ⓐ $x \leq -8$ or $x \geq 3.5$
Ⓑ $-8 \leq x \leq 3.5$
Ⓒ $x \leq 0$ or $x \geq 4.5$
Ⓓ $0 \leq x \leq 4.5$

EXAMPLE 7
on p. 303
for Exs. 46–57

SOLVING ALGEBRAICALLY Solve the inequality algebraically.

46. $4x^2 < 25$
47. $x^2 + 10x + 9 < 0$
48. $x^2 - 11x \geq -28$
49. $3x^2 - 13x > 10$
50. $2x^2 - 5x - 3 \leq 0$
51. $4x^2 + 8x - 21 \geq 0$
52. $-4x^2 - x + 3 \leq 0$
53. $5x^2 - 6x - 2 \leq 0$
54. $-3x^2 + 10x > -2$
55. $-2x^2 - 7x \geq 4$
56. $3x^2 + 1 < 15x$
57. $6x^2 - 5 > 8x$

58. **GRAPHING CALCULATOR** In this exercise, you will use a different graphical method to solve Example 6 on page 303.

a. Enter the equations $y = 7.51x^2 - 16.4x + 35.0$ and $y = 100$ into a graphing calculator.

b. Graph the equations from part (a) for $0 \leq x \leq 9$ and $0 \leq y \leq 300$.

c. Use the *intersect* feature to find the point where the graphs intersect.

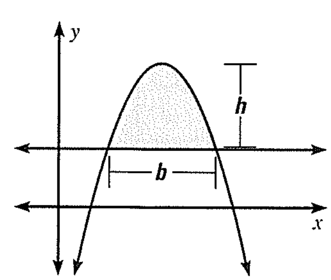
d. During what years was the number of participating teams greater than 100? *Explain* your reasoning.

CHOOSING A METHOD Solve the inequality using any method.

59. $8x^2 - 3x + 1 < 10$
60. $4x^2 + 11x + 3 \geq -3$
61. $-x^2 - 2x - 1 > 2$
62. $-3x^2 + 4x - 5 \leq 2$
63. $x^2 - 7x + 4 > 5x - 2$
64. $2x^2 + 9x - 1 \geq -3x + 1$
65. $3x^2 - 2x + 1 \leq -x^2 + 1$
66. $5x^2 + x - 7 < 3x^2 - 4x$
67. $6x^2 - 5x + 2 < -3x^2 + x$

68. ★ **OPEN-ENDED MATH** Write a quadratic inequality in one variable that has a solution of $x < -2$ or $x > 5$.

69. **CHALLENGE** The area A of the region bounded by a parabola and a horizontal line is given by $A = \frac{2}{3}bh$ where b and h are as defined in the diagram. Find the area of the region determined by each pair of inequalities.



a. $y \leq -x^2 + 4x$
$y \geq 0$

b. $y \geq x^2 - 4x - 5$
$y \leq 3$

PROBLEM SOLVING

EXAMPLE 2 on p. 301 for Exs. 70–71

70. ENGINEERING A wire rope can safely support a weight W (in pounds) provided $W \le 8000d^2$ where d is the rope's diameter (in inches). Graph the inequality.

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71. WOODWORKING A hardwood shelf in a wooden bookcase can safely support a weight W (in pounds) provided $W \le 115x^2$ where x is the shelf's thickness (in inches). Graph the inequality.

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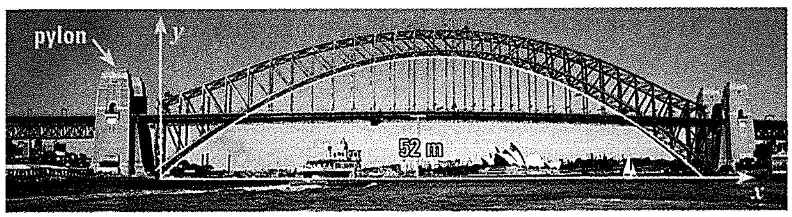
EXAMPLE 6 on p. 303 for Exs. 72–74

72. ARCHITECTURE The arch of the Sydney Harbor Bridge in Sydney, Australia, can be modeled by $y = -0.00211x^2 + 1.06x$ where x is the distance (in meters) from the left pylons and y is the height (in meters) of the arch above the water. For what distances x is the arch above the road?



73. ★ **SHORT RESPONSE** The length L (in millimeters) of the larvae of the black porgy fish can be modeled by

$$L(x) = 0.00170x^2 + 0.145x + 2.35,\ 0 \le x \le 40$$

where x is the age (in days) of the larvae. Write and solve an inequality to find at what ages a larvae's length tends to be greater than 10 millimeters. *Explain* how the given domain affects the solution.

74. ◆ **MULTIPLE REPRESENTATIONS** A study found that a driver's reaction time $A(x)$ to audio stimuli and his or her reaction time $V(x)$ to visual stimuli (both in milliseconds) can be modeled by

$$A(x) = 0.0051x^2 - 0.319x + 15,\ 16 \le x \le 70$$

$$V(x) = 0.005x^2 - 0.23x + 22,\ 16 \le x \le 70$$

where x is the driver's age (in years).

a. Writing an Inequality Write an inequality that you can use to find the x-values for which $A(x)$ is less than $V(x)$.

b. Making a Table Use a table to find the solution of the inequality from part (a). Your table should contain x-values from 16 to 70 in increments of 6.

c. Drawing a Graph Check the solution you found in part (b) by using a graphing calculator to solve the inequality $A(x) < V(x)$ graphically. *Describe* how you used the domain $16 \le x \le 70$ to determine a reasonable solution.

d. Interpret Based on your results from parts (b) and (c), do you think a driver would react more quickly to a traffic light changing from green to yellow or to the siren of an approaching ambulance? *Explain.*

75. SOCCER The path of a soccer ball kicked from the ground can be modeled by

$$y = -0.0540x^2 + 1.43x$$

where x is the horizontal distance (in feet) from where the ball was kicked and y is the corresponding height (in feet).

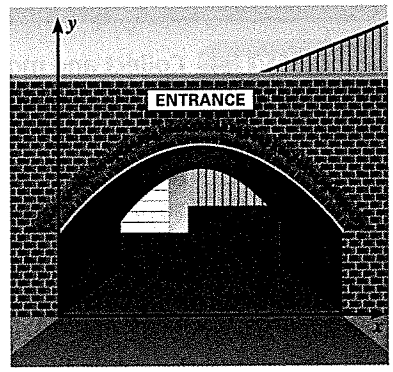
a. A soccer goal is 8 feet high. Write and solve an inequality to find at what values of x the ball is low enough to go into the goal.

b. A soccer player kicks the ball toward the goal from a distance of 15 feet away. No one is blocking the goal. Will the player score a goal? *Explain* your reasoning.

76. MULTI-STEP PROBLEM A truck that is 11 feet tall and 7 feet wide is traveling under an arch. The arch can be modeled by

$$y = -0.0625x^2 + 1.25x + 5.75$$

where x and y are measured in feet.



a. Will the truck fit under the arch? *Explain* your reasoning.

b. What is the maximum width that a truck 11 feet tall can have and still make it under the arch?

c. What is the maximum height that a truck 7 feet wide can have and still make it under the arch?

77. CHALLENGE For clear blue ice on lakes and ponds, the maximum weight w (in tons) that the ice can support is given by

$$w(x) = 0.1x^2 - 0.5x - 5$$

where x is the thickness of the ice (in inches).

a. **Calculate** What thicknesses of ice can support a weight of 20 tons?

b. **Interpret** *Explain* how you can use the graph of $w(x)$ to determine the minimum x-value in the domain for which the function gives meaningful results.

PA PENNSYLVANIA MIXED REVIEW

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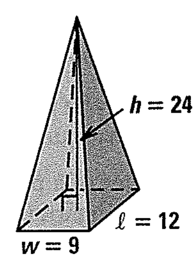
78. Rachel is a cross-country runner. Her coach recorded the data shown at the right during a timed practice run. If Rachel continues to run at the same rate, what is the approximate distance she will run in 25 minutes?

Time (minutes)	Distance (kilometers)
6	1.2
12	2.4
15	3

(A) 4.2 km (B) 5 km

(C) 6 km (D) 10 km

79. Which set of dimensions corresponds to a pyramid similar to the one shown?



(A) $w = 1$ unit, $\ell = 2$ units, $h = 4$ units

(B) $w = 2$ units, $\ell = 3$ units, $h = 6$ units

(C) $w = 3$ units, $\ell = 4$ units, $h = 8$ units

(D) $w = 4$ units, $\ell = 6$ units, $h = 12$ units

4 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

- quadratic function, *p. 236*
- standard form of a quadratic function, *p. 236*
- parabola, *p. 236*
- vertex, *p. 236*
- axis of symmetry, *p. 236*
- minimum, maximum value, *p. 238*
- vertex form, *p. 245*
- intercept form, *p. 246*
- monomial, binomial, trinomial, *p. 252*
- quadratic equation, *p. 253*
- standard form of a quadratic equation, *p. 253*
- root of an equation, *p. 253*
- zero of a function, *p. 254*
- square root, *p. 266*
- radical, radicand, *p. 266*
- rationalizing the denominator, *p. 267*
- conjugates, *p. 267*
- imaginary unit *i*, *p. 275*
- complex number, *p. 276*
- standard form of a complex number, *p. 276*
- imaginary number, *p. 276*
- pure imaginary number, *p. 276*
- complex conjugates, *p. 278*
- complex plane, *p. 278*
- absolute value of a complex number, *p. 279*
- completing the square, *p. 284*
- quadratic formula, *p. 292*
- discriminant, *p. 294*
- quadratic inequality in two variables, *p. 300*
- quadratic inequality in one variable, *p. 302*
- best-fitting quadratic model, *p. 311*

VOCABULARY EXERCISES

1. **WRITING** Given a quadratic function in standard form, explain how to determine whether the function has a maximum value or a minimum value.
2. Copy and complete: A(n) __?__ is a complex number $a + bi$ where $a = 0$ and $b \neq 0$.
3. Copy and complete: A function of the form $y = a(x - h)^2 + k$ is written in __?__.
4. Give an example of a quadratic equation that has a negative discriminant.

REVIEW EXAMPLES AND EXERCISES

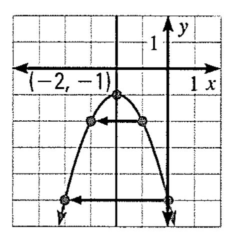
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 4.

4.1 Graph Quadratic Functions in Standard Form *pp. 236–243*

EXAMPLE

Graph $y = -x^2 - 4x - 5$.

Because $a < 0$, the parabola opens down. Find and plot the vertex $(-2, -1)$. Draw the axis of symmetry $x = -2$. Plot the y-intercept at $(0, -5)$, and plot its reflection $(-4, -5)$ in the axis of symmetry. Plot two other points: $(-1, -2)$ and its reflection $(-3, -2)$ in the axis of symmetry. Draw a parabola through the plotted points.



EXERCISES

EXAMPLE 3 on p. 238 for Exs. 5–7

Graph the function. Label the vertex and axis of symmetry.

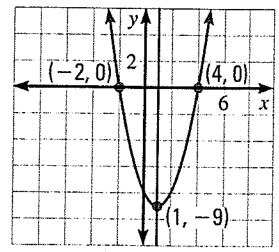
5. $y = x^2 + 2x - 3$
6. $y = -3x^2 + 12x - 7$
7. $f(x) = -x^2 - 2x - 6$

4.2 Graph Quadratic Functions in Vertex or Intercept Form *pp. 245–251*

EXAMPLE

Graph $y = (x - 4)(x + 2)$.

Identify the x-intercepts. The quadratic function is in intercept form $y = a(x - p)(x - q)$ where $a = 1$, $p = 4$, and $q = -2$. Plot the x-intercepts at $(4, 0)$ and $(-2, 0)$.



Find the coordinates of the vertex.

$$x = \frac{p+q}{2} = \frac{4 + (-2)}{2} = 1$$

$$y = (1 - 4)(1 + 2) = -9$$

Plot the vertex at $(1, -9)$. Draw a parabola through the plotted points as shown.

EXERCISES

EXAMPLES 1, 3, and 4 on pp. 245–247 for Exs. 8–14

Graph the function. Label the vertex and axis of symmetry.

8. $y = (x - 1)(x + 5)$
9. $g(x) = (x + 3)(x - 2)$
10. $y = -3(x + 1)(x - 6)$
11. $y = (x - 2)^2 + 3$
12. $f(x) = (x + 6)^2 + 8$
13. $y = -2(x + 8)^2 - 3$

14. **BIOLOGY** A flea's jump can be modeled by the function $y = -0.073x(x - 33)$ where x is the horizontal distance (in centimeters) and y is the corresponding height (in centimeters). How far did the flea jump? What was the flea's maximum height?

4.3 Solve $x^2 + bx + c = 0$ by Factoring *pp. 252–258*

EXAMPLE

Solve $x^2 - 13x - 48 = 0$.

Use factoring to solve for x.

$x^2 - 13x - 48 = 0$	**Write original equation.**
$(x - 16)(x + 3) = 0$	**Factor.**
$x - 16 = 0$ or $x + 3 = 0$	**Zero product property**
$x = 16$ or $x = -3$	**Solve for x.**

EXERCISES

EXAMPLE 3 on p. 254 for Exs. 15–21

Solve the equation.

15. $x^2 + 5x = 0$
16. $z^2 = 63z$
17. $s^2 - 6s - 27 = 0$
18. $k^2 + 12k - 45 = 0$
19. $x^2 + 18x = -81$
20. $n^2 + 5n = 24$

21. **URBAN PLANNING** A city wants to double the area of a rectangular playground that is 72 feet by 48 feet by adding the same distance x to the length and the width. Write and solve an equation to find the value of x.

4 CHAPTER REVIEW

4.4 Solve $ax^2 + bx + c = 0$ by Factoring — pp. 259–264

EXAMPLE

Solve $-30x^2 + 9x + 12 = 0$.

$-30x^2 + 9x + 12 = 0$	**Write original equation.**
$10x^2 - 3x - 4 = 0$	**Divide each side by -3.**
$(5x - 4)(2x + 1) = 0$	**Factor.**
$5x - 4 = 0$ or $2x + 1 = 0$	**Zero product property**
$x = \frac{4}{5}$ or $x = -\frac{1}{2}$	**Solve for x.**

EXAMPLE 5 on p. 261 for Exs. 22–24

EXERCISES

Solve the equation.

22. $16 = 38r - 12r^2$

23. $3x^2 - 24x - 48 = 0$

24. $20a^2 - 13a - 21 = 0$

4.5 Solve Quadratic Equations by Finding Square Roots — pp. 266–271

EXAMPLE

Solve $4(x - 7)^2 = 80$.

$4(x - 7)^2 = 80$	**Write original equation.**
$(x - 7)^2 = 20$	**Divide each side by 4.**
$x - 7 = \pm\sqrt{20}$	**Take square roots of each side.**
$x = 7 \pm 2\sqrt{5}$	**Add 7 to each side and simplify.**

EXAMPLES 3 and 4 on pp. 267–268 for Exs. 25–28

EXERCISES

Solve the equation.

25. $3x^2 = 108$

26. $5y^2 + 4 = 14$

27. $3(p + 1)^2 = 81$

28. GEOGRAPHY The total surface area of Earth is 510,000,000 square kilometers. Use the formula $S = 4\pi r^2$, which gives the surface area of a sphere with radius r, to find the radius of Earth.

4.6 Perform Operations with Complex Numbers — pp. 275–282

EXAMPLE

Write $(6 - 4i)(1 - 3i)$ as a complex number in standard form.

$(6 - 4i)(1 - 3i) = 6 - 18i - 4i + 12i^2$	**Multiply using FOIL.**
$= 6 - 22i + 12(-1)$	**Simplify and use $i^2 = -1$.**
$= -6 - 22i$	**Write in standard form.**

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Chapter Review Practice

EXAMPLES 2, 4, and 5 on pp. 276–278 for Exs. 29–34

EXERCISES

Write the expression as a complex number in standard form.

29. $-9i(2 - i)$ **30.** $(5 + i)(4 - 2i)$ **31.** $(2 - 5i)(2 + 5i)$

32. $(8 - 6i) + (7 + 4i)$ **33.** $(2 - 3i) - (6 - 5i)$ **34.** $\dfrac{4i}{-3 + 6i}$

4.7 Complete the Square

pp. 284–291

EXAMPLE

Solve $x^2 - 8x + 13 = 0$ by completing the square.

$x^2 - 8x + 13 = 0$	**Write original equation.**
$x^2 - 8x = -13$	**Write left side in the form $x^2 + bx$.**
$x^2 - 8x + 16 = -13 + 16$	**Add $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$ to each side.**
$(x - 4)^2 = 3$	**Write left side as a binomial squared.**
$x - 4 = \pm\sqrt{3}$	**Take square roots of each side.**
$x = 4 \pm \sqrt{3}$	**Solve for x.**

EXAMPLES 3 and 4 on pp. 285–286 for Exs. 35–37

EXERCISES

Solve the equation by completing the square.

35. $x^2 - 6x - 15 = 0$ **36.** $3x^2 - 12x + 1 = 0$ **37.** $x^2 + 3x - 1 = 0$

4.8 Use the Quadratic Formula and the Discriminant

pp. 292–299

EXAMPLE

Solve $3x^2 + 6x = -2$.

$3x^2 + 6x = -2$	**Write original equation.**
$3x^2 + 6x + 2 = 0$	**Write in standard form.**
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(3)(2)}}{2(3)}$	**Use $a = 3$, $b = 6$, and $c = 2$ in quadratic formula.**
$x = \dfrac{-3 \pm \sqrt{3}}{3}$	**Simplify.**

EXAMPLES 1, 2, 3, and 5 on pp. 292–295 for Exs. 38–41

EXERCISES

Use the quadratic formula to solve the equation.

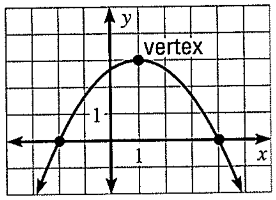
38. $x^2 + 4x - 3 = 0$ **39.** $9x^2 = -6x - 1$ **40.** $6x^2 - 8x = -3$

41. VOLLEYBALL A person spikes a volleyball over a net when the ball is 9 feet above the ground. The volleyball has an initial vertical velocity of −40 feet per second. The volleyball is allowed to fall to the ground. How long is the ball in the air after it is spiked?

PA Pennsylvania *Test Practice*

MULTIPLE CHOICE

In Exercises 1 and 2, use the parabola below.



1. Which statement is *not* true about the parabola?

   A The x-intercepts are -2 and 4.

   B The y-intercept is -2.

   C The maximum value is 3.

   D The axis of symmetry is $x = 1$.

2. What is an equation of the parabola?

   A $y = (x - 2)(x + 4)$

   B $y = -\frac{1}{3}(x + 2)(x - 4)$

   C $y = -(x + 2)(x - 4)$

   D $y = -3(x + 2)(x - 4)$

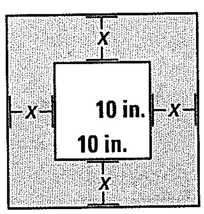
3. You are using glass tiles to make a picture frame for a square photograph with sides 10 inches long. You want the frame to form a uniform border around the photograph. You have enough tiles to cover 300 square inches. What is the largest possible frame width x?

   A 3.6 inches

   B 5 inches

   C 7.3 inches

   D 15 inches

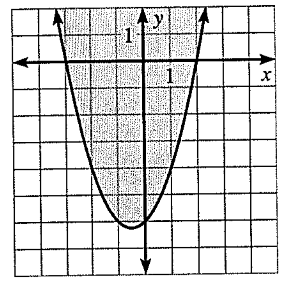


4. At a flea market held each weekend, an artist sells handmade earrings. The table below shows the average number of pairs of earrings sold for several prices. Given the pattern in the table, how much should the artist charge to maximize revenue?

Price	$15	$14	$13	$12
Pairs sold	50	60	70	80

   A $5 B $7.50 C $10 D $15

5. The graph of which inequality is shown?

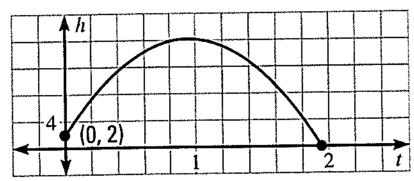


   A $y \geq -x^2 - x + 6$ B $y \geq x^2 + x - 6$

   C $y > 2x^2 + 2x - 12$ D $y \geq -2x^2 - 2x + 12$

In Exercises 6 and 7, use the information below.

The graph shows the height h (in feet) after t seconds of a horseshoe tossed during a game of horseshoes. The initial vertical velocity of the horseshoe is 30 feet per second.



6. To the nearest tenth of a second, how long is the horseshoe in the air?

   A 0.1 second B 1.9 seconds

   C 2.1 seconds D 3.9 seconds

7. To the nearest tenth of a foot, what is the maximum height of the horseshoe?

   A 1.9 feet B 8.1 feet

   C 16.1 feet D 32.2 feet

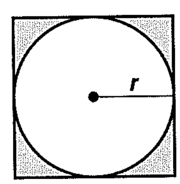
8. The diagram shows a circle inscribed in a square. The area of the shaded region is 21.5 square inches. To the nearest tenth of an inch, how long is a side of the square?

   A 4.6 inches

   B 8.7 inches

   C 9.7 inches

   D 10.0 inches



MULTIPLE CHOICE

9. What is the value of k in the equation $6x^2 - 11x - 10 = (3x + 2)(2x - k)$?

A -8 B -5

C 5 D 8

10. What is the real part of the standard form of the expression $(5 + i)(10 - i)$?

A 49 B 50

C 51 D 54

11. For what value of c is $x^2 - 7x + c$ a perfect square trinomial?

A $\frac{7}{2}$

B $\frac{49}{4}$

C $\frac{49}{2}$

D 49

12. What is the maximum value of the function $y = -3(x - 2)^2 + 6$?

A -6 B 2

C 6 D There is no maximum value.

13. What is the greatest zero of the function $y = x^2 - 25x + 66$?

A -22

B -3

C 3

D 22

14. What is the absolute value of $-5 + 12i$?

A 5

B 7

C 13

D 17

OPEN-ENDED

15. A parabola passes through the following points:
$(0, -22)$, $(2, -6)$, $(5, -12)$

A. What is the x-coordinate of the vertex of the parabola?

B. Use your calculator to write a quadratic equation that models the given data.

16. Given the function $f(x) = 4x^2 + 24x + 39$ —

A. Find the y-intercept of the function.

B. Find the minimum value of the function.

17. A volleyball is hit upward by a player in a game. The height h (in feet) of the volleyball after t seconds is given by the function $h = -16t^2 + 30t + 6$.

A. What is the maximum height of the volleyball? *Explain* your reasoning.

B. After how many seconds does the volleyball reach its maximum height?

C. After how many seconds does the volleyball hit the ground?

TEST PREPARATION

5 Polynomials and Polynomial Functions

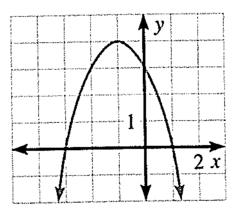
PA

M11.A.2.2.2	5.1 Use Properties of Exponents
	5.2 Evaluate and Graph Polynomial Functions
M11.D.2.2.1	5.3 Add, Subtract, and Multiply Polynomials
M11.D.2.2.2	5.4 Factor and Solve Polynomial Equations
	5.5 Apply the Remainder and Factor Theorems
	5.6 Find Rational Zeros
M11.A.3.2.1	5.7 Apply the Fundamental Theorem of Algebra
	5.8 Analyze Graphs of Polynomial Functions
	5.9 Write Polynomial Functions and Models

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 5: graphing functions, factoring, and solving equations.

Prerequisite Skills



VOCABULARY CHECK

Copy and complete the statement.

1. The **zeros** of the function graphed are _?_.
2. The **maximum value** of the function graphed is _?_.
3. The **standard form** of a quadratic equation in one variable is _?_ where $a \neq 0$.

SKILLS CHECK

Graph the function. Label the vertex and the axis of symmetry.
(Review pp. 236, 245 for 5.2.)

4. $y = -2(x - 1)^2 + 4$ 5. $y = 3(x - 2)(x + 3)$ 6. $y = -x^2 - 4x + 4$

Factor the expression. ***(Review pp. 252, 259 for 5.4.)***

7. $x^2 + 9x + 20$ 8. $2x^2 + 5x - 3$ 9. $9x^2 - 64$

Solve the equation. ***(Review pp. 252, 259 for 5.4–5.7.)***

10. $2x^2 + x + 6 = 0$ 11. $10x^2 + 13x = 3$ 12. $x^2 + 6x + 2 = 20$

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