6 Rational Exponents and Radical Functions

PA

Before

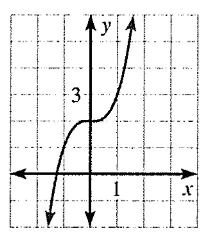
In previous chapters, you learned the following skills, which you'll use in Chapter 6: simplifying expressions involving exponents, rewriting equations, and graphing polynomial functions.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **square roots** of 81 are _?_ and _?_.
2. In the expression 2^5, the **exponent** is _?_.
3. For the polynomial function whose graph is shown, the sign of the **leading coefficient** is _?_.



SKILLS CHECK

Simplify the expression. ***(Review p. 330 for 6.2.)***

4. $\dfrac{5x^2y}{15x^3y^{-1}}$
5. $\dfrac{32x^{-3}y^4}{24x^{-3}y^{-2}} \cdot \dfrac{3x}{9y}$
6. $(2x^5y^{-3})^{-3}$

Solve the equation for *y*. ***(Review p. 26 for 6.4.)***

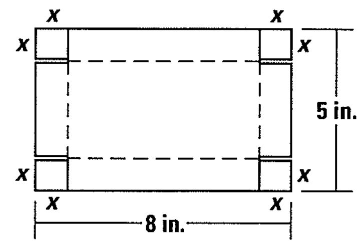
7. $-2x - 5y = 10$
8. $x - \frac{1}{3}y = -1$
9. $8x - 4xy = 3$

Graph the polynomial function. ***(Review p. 337 for 6.5.)***

10. $f(x) = x^3 - 4x + 6$
11. $f(x) = -x^5 + 7x^2 + 2$
12. $f(x) = x^4 - 4x^2 + x$

OPEN-ENDED

7. You are making an open box to hold paper clips out of a piece of cardboard that is 5 inches by 8 inches. The box will be formed by making the cuts shown in the diagram and folding up the sides. You want the box to have the greatest volume possible.



A. Use a graphing calculator to find how long you should make the cuts. *Explain* your reasoning.

B. What is the maximum volume of the box?

C. What will the dimensions of the finished box be?

8. From 1980 to 2002, the number of hospitals H in the United States and the average number of hospital beds B in each hospital can be modeled by

$H = -58.7t + 7070$ and $B = 0.0066t^3 - 0.192t^2 - 0.174t + 196$

where t is the number of years since 1980.

A. Write a model for the total number of hospital beds in U.S. hospitals.

B. According to the model, how many beds were in U.S. hospitals in 1995?

C. How does the model change if you want to find the number of hospital beds *in thousands*? *Explain* your reasoning.

MULTIPLE CHOICE

9. Which expression is equivalent to $\frac{x^2y}{z^4}$?

A $\frac{z^{-4}y^0}{x^{-2}}$

B $xyz \cdot \frac{x}{z^{-3}}$

C $(x^{-1}y^2z^2)^2(x^{-1}y^1z^2)^{-4}$

D $\frac{(x^2yz)^3}{x^4y^2z^7}$

10. What are all the real solutions of the equation $x^4 = 125x$?

A 0 B 0, 5, −5

C 0, 5 D 0, $5i$, $-5i$

11. Which polynomial function has −1, 3, and $-4i$ as zeros?

A $f(x) = x^4 - 2x^3 + 13x^2 - 32x - 48$

B $f(x) = x^4 + 2x^3 + 13x^2 + 32x - 48$

C $f(x) = x^4 - 2x^3 - 19x^2 + 32x + 48$

D $f(x) = x^4 + 2x^3 + 19x^2 - 32x + 48$

12. How many *real* zeros does the function $f(x) = 2x^4 + 3x^2 - 1$ have?

A 0 real zeros B 1 real zero

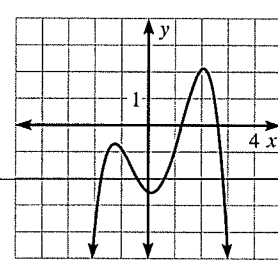
C 2 real zeros D 4 real zeros

13. Evaluate the expression $\left(\frac{3}{2}\right)^{-2}$.

A $\frac{4}{9}$ B $\frac{9}{4}$

C $\sqrt{\frac{2}{3}}$ D $\sqrt{\frac{3}{2}}$

14. The graph of a quartic function is shown. How many imaginary zeros does the function have?



A 0 imaginary zeros B 1 imaginary zero

C 2 imaginary zeros D 4 imaginary zeros

TEST PREPARATION

Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 465. You will also use the key vocabulary listed below.

Big Ideas

1. **Using rational exponents**
2. **Performing function operations and finding inverse functions**
3. **Graphing radical functions and solving radical equations**

KEY VOCABULARY

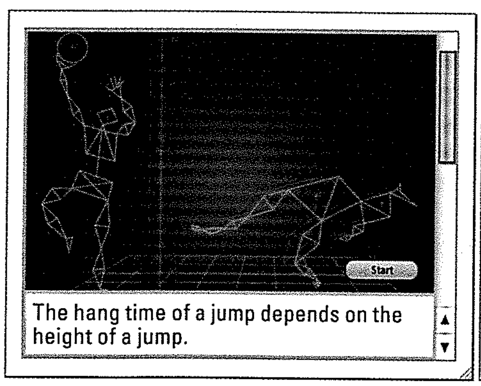
- nth root of a, *p. 414*
- index of a radical, *p. 414*
- simplest form of a radical, *p. 422*
- like radicals, *p. 422*
- power function, *p. 428*
- composition, *p. 430*
- inverse relation, *p. 438*
- inverse function, *p. 438*
- radical function, *p. 446*
- radical equation, *p. 452*

Why?

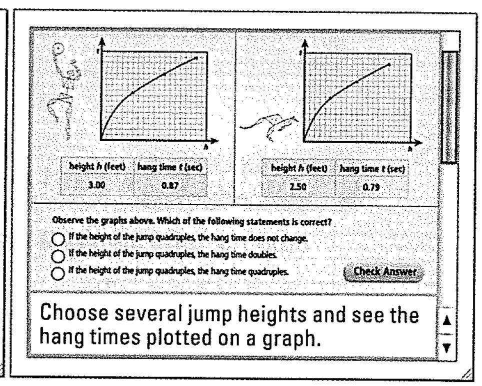
You can use a radical function to model the time you are suspended in the air during a jump. For example, the hang time of a basketball player can be modeled by a radical function.

Animated Algebra

The animation illustrated below for Exercise 60 on page 458 helps you answer this question: What is the relationship between the height of a jump and the time the jumper is suspended in air?



The hang time of a jump depends on the height of a jump.

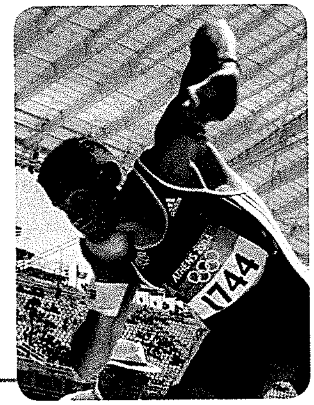


Choose several jump heights and see the hang times plotted on a graph.

Animated Algebra at classzone.com

Other animations for Chapter 6: pages 431, 444, 448, and 465

6.1 Evaluate nth Roots and Use Rational Exponents



PA **M11.A.2.2.1** Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value...

Before You evaluated square roots and used properties of exponents.

Now You will evaluate nth roots and study rational exponents.

Why? So you can find the radius of a spherical object, as in Ex. 60.

Key Vocabulary
- ***n*th root of *a***
- **index of a radical**

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$. In general, for an integer n greater than 1, if $b^n = a$, then b is an **nth root of a**. An nth root of a is written as $\sqrt[n]{a}$ where n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the power of a power property applies to rational exponents, then the following is true:

$$(a^{1/2})^2 = a^{(1/2) \cdot 2} = a^1 = a$$
$$(a^{1/3})^3 = a^{(1/3) \cdot 3} = a^1 = a$$
$$(a^{1/4})^4 = a^{(1/4) \cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

KEY CONCEPT *For Your Notebook*

Real nth Roots of a

Let n be an integer ($n > 1$) and let a be a real number.

n is an even integer.		n is an odd integer.	
$a < 0$	No real nth roots.	$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

EXAMPLE 1 Find nth roots

Find the indicated real nth root(s) of a.

a. $n = 3, a = -216$

b. $n = 4, a = 81$

Solution

a. Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root. Because $(-6)^3 = -216$, you can write $\sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.

b. Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\pm\sqrt[4]{81} = \pm 3$ or $\pm 81^{1/4} = \pm 3$.

RATIONAL EXPONENTS A rational exponent does not have to be of the form $\frac{1}{n}$. Other rational numbers such as $\frac{3}{2}$ and $-\frac{1}{2}$ can also be used as exponents. Two properties of rational exponents are shown below.

KEY CONCEPT *For Your Notebook*

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m},\ a \neq 0$$

EXAMPLE 2 Evaluate expressions with rational exponents

Evaluate (a) $16^{3/2}$ and (b) $32^{-3/5}$.

Solution

	Rational Exponent Form	Radical Form
a.	$16^{3/2} = (16^{1/2})^3 = 4^3 = 64$	$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$
b.	$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} = \frac{1}{2^3} = \frac{1}{8}$	$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$

AVOID ERRORS
Be sure to use parentheses to enclose a rational exponent: 9^(1/5) ≈ 1.552. Without them, the calculator evaluates a power and then divides: 9^1/5 = 1.8.

EXAMPLE 3 Approximate roots with a calculator

	Expression	Keystrokes	Display
a.	$9^{1/5}$	9 [^] [(] 1 [÷] 5 [)] [ENTER]	1.551845574
b.	$12^{3/8}$	12 [^] [(] 3 [÷] 8 [)] [ENTER]	2.539176951
c.	$(\sqrt[4]{7})^3 = 7^{3/4}$	7 [^] [(] 3 [÷] 4 [)] [ENTER]	4.303517071

✓ **GUIDED PRACTICE** for Examples 1, 2, and 3

Find the indicated real nth root(s) of a.

1. $n = 4, a = 625$

2. $n = 6, a = 64$

3. $n = 3, a = -64$

4. $n = 5, a = 243$

Evaluate the expression without using a calculator.

5. $4^{5/2}$

6. $9^{-1/2}$

7. $81^{3/4}$

8. $1^{7/8}$

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

9. $4^{2/5}$

10. $64^{-2/3}$

11. $(\sqrt[4]{16})^5$

12. $(\sqrt[3]{-30})^2$

EXAMPLE 4 Solve equations using *n*th roots

Solve the equation.

a. $4x^5 = 128$

$x^5 = 32$ **Divide each side by 4.**

$x = \sqrt[5]{32}$ **Take fifth root of each side.**

$x = 2$ **Simplify.**

b. $(x - 3)^4 = 21$

$x - 3 = \pm\sqrt[4]{21}$ **Take fourth roots of each side.**

$x = \pm\sqrt[4]{21} + 3$ **Add 3 to each side.**

$x = \sqrt[4]{21} + 3$ or $x = -\sqrt[4]{21} + 3$ **Write solutions separately.**

$x \approx 5.14$ or $x \approx 0.86$ **Use a calculator.**

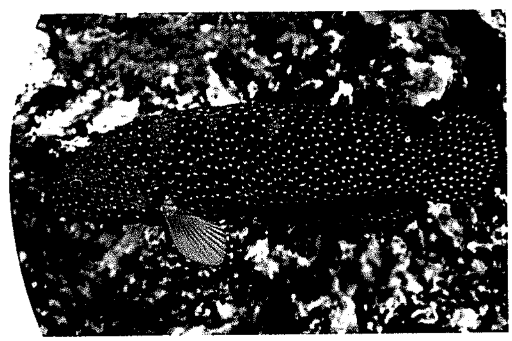
AVOID ERRORS
When n is even and $a > 0$, be sure to consider both the positive and negative nth roots of a.

EXAMPLE 5 Use *n*th roots in problem solving

BIOLOGY A study determined that the weight w (in grams) of coral cod near Palawan Island, Philippines, can be approximated using the model

$$w = 0.0167\ell^3$$

where ℓ is the coral cod's length (in centimeters). Estimate the length of a coral cod that weighs 200 grams.



Solution

$w = 0.0167\ell^3$ **Write model for weight.**

$\mathbf{200} = 0.0167\ell^3$ **Substitute 200 for *w*.**

$11{,}976 \approx \ell^3$ **Divide each side by 0.0167.**

$\sqrt[3]{11{,}976} \approx \ell$ **Take cube root of each side.**

$22.9 \approx \ell$ **Use a calculator.**

▶ A coral cod that weighs 200 grams is about 23 centimeters long.

✓ GUIDED PRACTICE for Examples 4 and 5

Solve the equation. Round the result to two decimal places when appropriate.

13. $x^3 = 64$ **14.** $\frac{1}{2}x^5 = 512$ **15.** $3x^2 = 108$

16. $\frac{1}{4}x^3 = 2$ **17.** $(x - 2)^3 = -14$ **18.** $(x + 5)^4 = 16$

19. WHAT IF? Use the information from Example 5 to estimate the length of a coral cod that has the given weight.

a. 275 grams **b.** 340 grams **c.** 450 grams

6.1 EXERCISES

HOMEWORK KEY ◯ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 9, 25, and 63
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 33, 46, 47, and 65

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In the expression $\sqrt[4]{10{,}000}$, the number 4 is called the _?_.

2. ★ **WRITING** *Explain* how the sign of a determines the number of real fourth roots of a and the number of real fifth roots of a.

EXAMPLE 1 on p. 414 for Exs. 3–20

MATCHING EXPRESSIONS **Match the expression in rational exponent notation with the equivalent expression in radical notation.**

3. $2^{1/3}$ 4. $2^{3/2}$ 5. $2^{2/3}$ 6. $2^{1/2}$

A. $(\sqrt{2})^3$ B. $\sqrt{2}$ C. $\sqrt[3]{2}$ D. $(\sqrt[3]{2})^2$

USING RATIONAL EXPONENT NOTATION **Rewrite the expression using rational exponent notation.**

7. $\sqrt[3]{12}$ 8. $\sqrt[5]{8}$ 9. $(\sqrt[3]{10})^7$ 10. $(\sqrt[8]{15})^3$

USING RADICAL NOTATION **Rewrite the expression using radical notation.**

11. $5^{1/4}$ 12. $7^{1/3}$ 13. $14^{2/5}$ 14. $21^{9/4}$

FINDING *N*TH ROOTS **Find the indicated real nth root(s) of a.**

15. $n = 2, a = 64$ 16. $n = 3, a = -27$ 17. $n = 4, a = 0$

18. $n = 3, a = 343$ 19. $n = 4, a = -16$ 20. $n = 5, a = -32$

EXAMPLE 2 on p. 415 for Exs. 21–33

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator.**

21. $\sqrt[6]{64}$ 22. $8^{1/3}$ 23. $16^{3/2}$ 24. $\sqrt[3]{-125}$

25. $27^{2/3}$ 26. $(-243)^{1/5}$ 27. $(\sqrt[3]{8})^{-2}$ 28. $(\sqrt[3]{-64})^4$

29. $(\sqrt[4]{16})^{-7}$ 30. $25^{3/2}$ 31. $64^{-2/3}$ 32. $\frac{1}{81^{-3/4}}$

33. ★ **MULTIPLE CHOICE** What is the value of $128^{5/7}$?

(A) 8 (B) 16 (C) 32 (D) 64

EXAMPLE 3 on p. 415 for Exs. 34–46

APPROXIMATING ROOTS **Evaluate the expression using a calculator. Round the result to two decimals places when appropriate.**

34. $\sqrt[5]{32{,}768}$ 35. $\sqrt[7]{1695}$ 36. $\sqrt[9]{-230}$ 37. $85^{1/6}$

38. $25^{-1/3}$ 39. $20{,}736^{1/4}$ 40. $(\sqrt[4]{187})^3$ 41. $(\sqrt{6})^{-5}$

42. $(\sqrt[5]{-8})^8$ 43. $86^{-5/6}$ 44. $1974^{2/7}$ 45. $\frac{1}{(-17)^{3/5}}$

46. ★ **MULTIPLE CHOICE** Which expression has the greatest value?

(A) $27^{3/5}$ (B) $5^{3/2}$ (C) $\sqrt[3]{81}$ (D) $(\sqrt[3]{2})^8$

47. ★ **OPEN-ENDED MATH** Write two different expressions of the form $a^{1/n}$ that equal 3, where a is a real number and n is an integer greater than 1.

EXAMPLE 4
on p. 416
for Exs. 48–58

ERROR ANALYSIS *Describe* and correct the error in solving the equation.

48.
$x^3 = 27$
$x = \sqrt[3]{27}$
$x = 9$

49.
$x^4 = 81$
$x = \sqrt[4]{81}$
$x = 3$

SOLVING EQUATIONS **Solve the equation. Round the result to two decimal places when appropriate.**

50. $x^3 = 125$ **51.** $5x^3 = 1080$ **52.** $x^6 + 36 = 100$

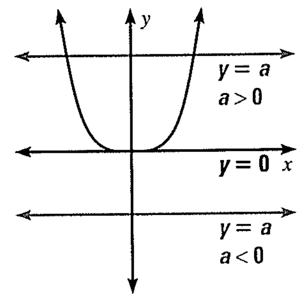
53. $(x - 5)^4 = 256$ **54.** $x^5 = -48$ **55.** $7x^4 = 56$

56. $x^3 + 40 = 25$ **57.** $(x + 10)^5 = 70$ **58.** $x^6 - 34 = 181$

59. CHALLENGE The general shape of the graph of $y = x^n$, where n is a positive *even* integer, is shown in red.

a. *Explain* how the graph justifies the results in the Key Concept box on page 414 when n is a positive *even* integer.

b. Draw a similar graph that justifies the results in the Key Concept box when n is a positive *odd* integer.



PROBLEM SOLVING

EXAMPLE 5
on p. 416
for Exs. 60–65

60. SHOT PUT The shot used in men's shot put has a volume of about 905 cubic centimeters. Find the radius of the shot. (*Hint:* Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere.)

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61. BOWLING A bowling ball has a surface area of about 232 square inches. Find the radius of the bowling ball. (*Hint:* Use the formula $S = 4\pi r^2$ for the surface area of a sphere.)

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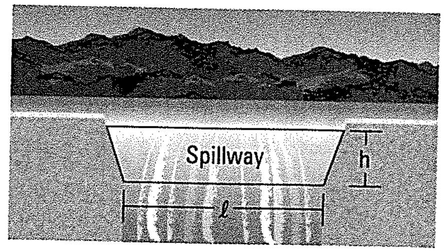
62. INFLATION If the average price of an item increases from p_1 to p_2 over a period of n years, the annual rate of inflation r (expressed as a decimal) is given by $r = \left(\frac{p_2}{p_1}\right)^{1/n} - 1$. Find the rate of inflation for each item in the table. Write each answer as a percent rounded to the nearest tenth.

Item	Price in 1950	Price in 1990
Butter (lb)	$.7420	$2.195
Chicken (lb)	$.4430	$1.087
Eggs (dozen)	$.6710	$1.356
Sugar (lb)	$.0936	$.4560

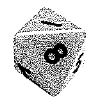
63. MULTI-STEP PROBLEM The power p (in horsepower) used by a fan with rotational speed s (in revolutions per minute) can be modeled by the formula $p = ks^3$ for some constant k. A certain fan uses 1.2 horsepower when its speed is 1700 revolutions per minute. First find the value of k for this fan. Then find the speed of the fan if it uses 1.5 horsepower.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

64. **WATER RATE** A *weir* is a dam that is built across a river to regulate the flow of water. The flow rate Q (in cubic feet per second) can be calculated using the formula $Q = 3.367\ell h^{3/2}$ where ℓ is the length (in feet) of the bottom of the spillway and h is the depth (in feet) of the water on the spillway. Determine the flow rate of a weir with a spillway that is 20 feet long and has a water depth of 5 feet.



65. ★ **EXTENDED RESPONSE** Some games use dice in the shape of regular polyhedra. You are designing dice and want them all to have the same volume as a cube with an edge length of 16 millimeters.

Name	Tetrahedron 	Octahedron 	Dodecahedron 	Icosahedron 
Number of faces	4	8	12	20
Volume formula	$V = 0.118x^3$	$V = 0.471x^3$	$V = 7.663x^3$	$V = 2.182x^3$

a. Find the volume of a cube with an edge length of 16 millimeters.

b. Find the edge length x for each of the polyhedra shown in the table.

c. Does the polyhedron with the greatest number of faces have the smallest edge length? *Explain.*

66. **CHALLENGE** The mass of the particles that a river can transport is proportional to the sixth power of the speed of the river. A certain river normally flows at a speed of 1 meter per second. What must its speed be in order to transport particles that are twice as massive as usual? 10 times as massive? 100 times as massive?

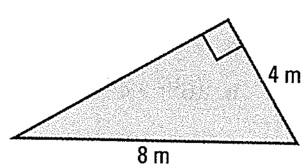
PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

67. Which expression is equivalent to $2x(4x + 1) - (7x + 3)(x - 4)$?

Ⓐ $x^2 - 23x - 12$
Ⓑ $15x^2 - 23x - 12$
Ⓒ $-x^2 + 27x + 12$
Ⓓ $x^2 + 27x + 12$

68. Frank digs a trench around the triangular garden shown. What is the approximate length of the trench that he digs?



Ⓐ 18.9 m
Ⓑ 19.3 m
Ⓒ 25.9 m
Ⓓ 37.9 m

6.2 Apply Properties of Rational Exponents

PA M11.A.2.2.1 Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value...

Before You simplified expressions involving integer exponents.

Now You will simplify expressions involving rational exponents.

Why? So you can find velocities, as in Ex. 84.

Key Vocabulary
- **simplest form of a radical**
- **like radicals**

The properties of integer exponents you learned in Lesson 5.1 can also be applied to rational exponents.

KEY CONCEPT *For Your Notebook*

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 330, but now apply to rational exponents as illustrated.

Property	Example
1. $a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2 + 3/2)} = 5^2 = 25$
2. $(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3. $(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2 - 1/2)} = 4^2 = 16$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

EXAMPLE 1 Use properties of exponents

Use the properties of rational exponents to simplify the expression.

a. $7^{1/4} \cdot 7^{1/2} = 7^{(1/4 + 1/2)} = 7^{3/4}$

b. $(6^{1/2} \cdot 4^{1/3})^2 = (6^{1/2})^2 \cdot (4^{1/3})^2 = 6^{(1/2 \cdot 2)} \cdot 4^{(1/3 \cdot 2)} = 6^1 \cdot 4^{2/3} = 6 \cdot 4^{2/3}$

c. $(4^5 \cdot 3^5)^{-1/5} = [(4 \cdot 3)^5]^{-1/5} = (12^5)^{-1/5} = 12^{[5 \cdot (-1/5)]} = 12^{-1} = \frac{1}{12}$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} = 5^{(1 - 1/3)} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\frac{42}{6}\right)^{1/3}\right]^2 = (7^{1/3})^2 = 7^{(1/3 \cdot 2)} = 7^{2/3}$

EXAMPLE 2 Apply properties of exponents

BIOLOGY A mammal's surface area S (in square centimeters) can be approximated by the model $S = km^{2/3}$ where m is the mass (in grams) of the mammal and k is a constant. The values of k for some mammals are shown below. Approximate the surface area of a rabbit that has a mass of 3.4 kilograms (3.4×10^3 grams).

Mammal	Sheep	Rabbit	Horse	Human	Monkey	Bat
k	8.4	9.75	10.0	11.0	11.8	57.5

Solution

$S = km^{2/3}$ — **Write model.**

$= 9.75(3.4 \times 10^3)^{2/3}$ — **Substitute 9.75 for k and 3.4×10^3 for m.**

$= 9.75(3.4)^{2/3}(10^3)^{2/3}$ — **Power of a product property**

$\approx 9.75(2.26)(10^2)$ — **Power of a power property**

≈ 2200 — **Simplify.**

▸ The rabbit's surface area is about 2200 square centimeters.

✓ GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $(5^{1/3} \cdot 7^{1/4})^3$
2. $2^{3/4} \cdot 2^{1/2}$
3. $\dfrac{3}{3^{1/4}}$
4. $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3$
5. **BIOLOGY** Use the information in Example 2 to approximate the surface area of a sheep that has a mass of 95 kilograms (9.5×10^4 grams).

PROPERTIES OF RADICALS The third and sixth properties on page 420 can be expressed using radical notation when $m = \frac{1}{n}$ for some integer n greater than 1.

KEY CONCEPT *For Your Notebook*

Properties of Radicals

Product property of radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient property of radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \ b \neq 0$$

EXAMPLE 3 Use properties of radicals

Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ — **Product property**

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\dfrac{80}{5}} = \sqrt[4]{16} = 2$ — **Quotient property**

SIMPLEST FORM A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

EXAMPLE 4 Write radicals in simplest form

Write the expression in simplest form.

a. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$ **Factor out perfect cube.**

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$ **Product property**

$= 3\sqrt[3]{5}$ **Simplify.**

REVIEW RADICALS For help with rationalizing denominators of radical expressions, see p. 266.

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}}$ **Make denominator a perfect fifth power.**

$= \frac{\sqrt[5]{28}}{\sqrt[5]{32}}$ **Product property**

$= \frac{\sqrt[5]{28}}{2}$ **Simplify.**

LIKE RADICALS Radical expressions with the same index and radicand are **like radicals.** To add or subtract like radicals, use the distributive property.

EXAMPLE 5 Add and subtract like radicals and roots

Simplify the expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5}) = (2 + 10)(8^{1/5}) = 12(8^{1/5})$

c. $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

✓ GUIDED PRACTICE for Examples 3, 4, and 5

Simplify the expression.

6. $\sqrt[4]{27} \cdot \sqrt[4]{3}$ **7.** $\frac{\sqrt[3]{250}}{\sqrt[3]{2}}$ **8.** $\sqrt[5]{\frac{3}{4}}$ **9.** $\sqrt[3]{5} + \sqrt[3]{40}$

VARIABLE EXPRESSIONS The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When n is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When n is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 6 Simplify expressions involving variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $(27p^3q^{12})^{1/3} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3} = 3p^{(3 \cdot 1/3)}q^{(12 \cdot 1/3)} = 3pq^4$

c. $\sqrt[4]{\frac{m^4}{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{(n^2)^4}} = \frac{m}{n^2}$

d. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

EXAMPLE 7 Write variable expressions in simplest form

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ **Factor out perfect fifth powers.**

$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ **Product property**

$= ab^2c\sqrt[5]{4a^3b^4}$ **Simplify.**

AVOID ERRORS
You must multiply both the numerator *and* denominator of the fraction by y so that the value of the fraction does not change.

b. $\sqrt[3]{\frac{x}{y^8}} = \sqrt[3]{\frac{x \cdot y}{y^8 \cdot y}}$ **Make denominator a perfect cube.**

$= \sqrt[3]{\frac{xy}{y^9}}$ **Simplify.**

$= \frac{\sqrt[3]{xy}}{\sqrt[3]{y^9}}$ **Quotient property**

$= \frac{\sqrt[3]{xy}}{y^3}$ **Simplify.**

EXAMPLE 8 Add and subtract expressions involving variables

Perform the indicated operation. Assume all variables are positive.

a. $\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w} = \left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w} = \frac{4}{5}\sqrt{w}$

b. $3xy^{1/4} - 8xy^{1/4} = (3 - 8)xy^{1/4} = -5xy^{1/4}$

c. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

✓ **GUIDED PRACTICE** for Examples 6, 7, and 8

Simplify the expression. Assume all variables are positive.

10. $\sqrt[3]{27q^9}$ **11.** $\sqrt[5]{\frac{x^{10}}{y^5}}$ **12.** $\frac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$ **13.** $\sqrt{9w^5} - w\sqrt{w^3}$

6.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 5, 27, and 85
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 51, 69, 86, and 89

SKILL PRACTICE

1. **VOCABULARY** Are $2\sqrt{5}$ and $2\sqrt[3]{5}$ like radicals? *Explain* why or why not.

2. ★ **WRITING** Under what conditions is a radical expression in simplest form?

EXAMPLE 1 on p. 420 for Exs. 3–14

PROPERTIES OF RATIONAL EXPONENTS Simplify the expression.

3. $5^{3/2} \cdot 5^{1/2}$
4. $(6^{2/3})^{1/2}$
5. $3^{1/4} \cdot 27^{1/4}$
6. $\dfrac{9}{9^{-4/5}}$
7. $\dfrac{80^{1/4}}{5^{-1/4}}$
8. $\left(\dfrac{7^3}{4^3}\right)^{-1/3}$
9. $\dfrac{11^{2/5}}{11^{4/5}}$
10. $(12^{3/5} \cdot 8^{3/5})^5$
11. $\dfrac{120^{-2/5} \cdot 120^{2/5}}{7^{-3/4}}$
12. $\dfrac{64^{5/9} \cdot 64^{2/9}}{4^{3/4}}$
13. $(16^{5/9} \cdot 5^{7/9})^{-3}$
14. $\dfrac{13^{3/7}}{13^{5/7}}$

EXAMPLE 3 on p. 421 for Exs. 15–22

PROPERTIES OF RADICALS Simplify the expression.

15. $\sqrt{20} \cdot \sqrt{5}$
16. $\sqrt[3]{16} \cdot \sqrt[3]{4}$
17. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
18. $(\sqrt[3]{3} \cdot \sqrt[4]{3})^{12}$
19. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$
20. $\dfrac{\sqrt{3}}{\sqrt{75}}$
21. $\dfrac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$
22. $\dfrac{\sqrt[4]{8} \cdot \sqrt[4]{16}}{\sqrt[8]{2} \cdot \sqrt[8]{3}}$

EXAMPLE 4 on p. 422 for Exs. 23–31

23. ★ **MULTIPLE CHOICE** What is the simplest form of the expression $3\sqrt[4]{32} \cdot (-6\sqrt[4]{5})$?

(A) $\sqrt[4]{10}$ (B) $-18\sqrt[4]{10}$ (C) $-36\sqrt[4]{10}$ (D) $36\sqrt[8]{10}$

SIMPLEST FORM Write the expression in simplest form.

24. $\sqrt{72}$
25. $\sqrt[6]{256}$
26. $\sqrt[3]{108} \cdot \sqrt[3]{4}$
27. $5\sqrt[4]{64} \cdot 2\sqrt[4]{8}$
28. $\sqrt[3]{\dfrac{1}{6}}$
29. $\dfrac{3}{\sqrt[4]{144}}$
30. $\sqrt[6]{\dfrac{81}{4}}$
31. $\dfrac{\sqrt[3]{9}}{\sqrt[5]{27}}$

EXAMPLE 5 on p. 422 for Exs. 32–41

COMBINING RADICALS AND ROOTS Simplify the expression.

32. $2\sqrt[6]{3} + 7\sqrt[6]{3}$
33. $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5}$
34. $25\sqrt[5]{2} - 15\sqrt[5]{2}$
35. $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$
36. $6\sqrt[3]{5} + 4\sqrt[3]{625}$
37. $-6\sqrt[7]{2} + 2\sqrt[7]{256}$
38. $12\sqrt[4]{2} - 7\sqrt[4]{512}$
39. $2\sqrt[4]{1250} - 8\sqrt[4]{32}$
40. $5\sqrt[3]{48} - \sqrt[3]{750}$

ERROR ANALYSIS *Describe* and correct the error in simplifying the expression.

41.
$$2\sqrt[3]{10} + 6\sqrt[3]{5} = (2 + 6)\sqrt[3]{15} = 8\sqrt[3]{15}$$

42.
$$\sqrt[3]{\frac{x}{y^2}} = \sqrt[3]{\frac{x}{y^2 \cdot y}} = \sqrt[3]{\frac{x}{y^3}} = \frac{\sqrt[3]{x}}{y}$$

EXAMPLE 6 on p. 423 for Exs. 43–51

VARIABLE EXPRESSIONS **Simplify the expression. Assume all variables are positive.**

43. $x^{1/4} \cdot x^{1/3}$
44. $(y^4)^{1/6}$
45. $\sqrt[4]{81x^4}$
46. $\dfrac{2}{x^{-3/2}}$
47. $\dfrac{x^{2/5}y}{xy^{-1/3}}$
48. $\sqrt[3]{\dfrac{x^{15}}{y^6}}$
49. $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$
50. $\dfrac{\sqrt[3]{x} \cdot \sqrt{x^5}}{\sqrt{25x^{16}}}$

51. ★ **OPEN-ENDED MATH** Write two variable expressions with noninteger exponents whose quotient is $x^{3/4}$.

EXAMPLE 7 on p. 423 for Exs. 52–59

SIMPLEST FORM **Write the expression in simplest form. Assume all variables are positive.**

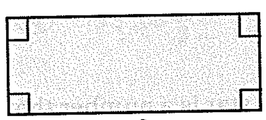
52. $\sqrt{49x^5}$
53. $\sqrt[4]{12x^2y^6z^{12}}$
54. $\sqrt[3]{4x^3y^5} \cdot \sqrt[3]{12y^2}$
55. $\sqrt{x^2yz^3} \cdot \sqrt{x^3z^5}$
56. $\dfrac{-3}{\sqrt[5]{x^6}}$
57. $\sqrt[3]{\dfrac{x^3}{y^4}}$
58. $\sqrt{\dfrac{20x^3y^2}{9xz^3}}$
59. $\dfrac{\sqrt[4]{x^6}}{\sqrt[7]{x^5}}$

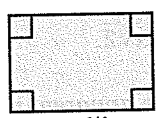
EXAMPLE 8 on p. 423 for Exs. 60–65

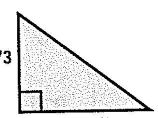
COMBINING VARIABLE EXPRESSIONS **Perform the indicated operation. Assume all variables are positive.**

60. $3\sqrt[5]{x} + 9\sqrt[5]{x}$
61. $\frac{3}{4}y^{3/2} - \frac{1}{4}y^{3/2}$
62. $-7\sqrt[3]{y} + 16\sqrt[3]{y}$
63. $(x^4y)^{1/2} + (xy^{1/4})^2$
64. $x\sqrt{9x^3} - 2\sqrt{x^5}$
65. $y\sqrt[4]{32x^6} + \sqrt[4]{162x^2y^4}$

GEOMETRY **Find simplified expressions for the perimeter and area of the given figure.**

66.


67.


68.


69. ★ **MULTIPLE CHOICE** What is the simplified form of $-\frac{1}{6}\sqrt{4x} - \frac{1}{6}\sqrt{9x}$?

(A) $-\frac{1}{3}\sqrt{x}$ (B) $-\frac{1}{3}\sqrt{36x}$ (C) $-\frac{5}{6}\sqrt{x}$ (D) $-\frac{5}{6}\sqrt{36x}$

DECIMAL EXPONENTS **Simplify the expression. Assume all variables are positive.**

70. $x^{0.5} \cdot x^2$
71. $y^{-0.6} \cdot y^{-6}$
72. $(x^6y^2)^{-0.75}$
73. $\dfrac{x^{0.3}}{x^{1.5}}$
74. $(x^5y^{-3})^{-0.25}$
75. $\dfrac{y^{-0.5}}{y^{0.8}}$
76. $10x^{0.6} + (4x^{0.3})^2$
77. $15z^{0.3} - (2z^{0.1})^3$

IRRATIONAL EXPONENTS **The properties in this lesson can also be applied to irrational exponents. Simplify the expression. Assume all variables are positive.**

78. $\dfrac{x^{5\sqrt{3}}}{x^{2\sqrt{3}}}$
79. $(x^{\sqrt{2}})^{\sqrt{3}}$
80. $\left(\dfrac{x^{\pi}}{x^{\pi/3}}\right)^2$
81. $x^2y^{\sqrt{2}} + 3x^2y^{\sqrt{2}}$

82. **CHALLENGE** Solve the equation using the properties of rational exponents.

a. $\dfrac{3}{9^x} = 243$ b. $2^x \cdot 2^{x+1} = \dfrac{1}{16}$ c. $(4^x)^{x+2} = 64$

6.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 3, 13, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 38, 39, and 44

◆ = **MULTIPLE REPRESENTATIONS** Ex. 46

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The function $h(x) = g(f(x))$ is called the _?_ of the function g with the function f.

2. ★ **WRITING** Tell whether the sum of two power functions is *sometimes*, *always*, or *never* a power function. *Explain* your reasoning.

EXAMPLE 1 on p. 428 for Exs. 3–11

ADD AND SUBTRACT FUNCTIONS Let $f(x) = -3x^{1/3} + 4x^{1/2}$ and $g(x) = 5x^{1/3} + 4x^{1/2}$. Perform the indicated operation and state the domain.

3. $f(x) + g(x)$
4. $g(x) + f(x)$
5. $f(x) + f(x)$
6. $g(x) + g(x)$
7. $f(x) - g(x)$
8. $g(x) - f(x)$
9. $f(x) - f(x)$
10. $g(x) - g(x)$

11. ★ **MULTIPLE CHOICE** What is $f(x) + g(x)$ if $f(x) = -7x^{2/3} - 1$ and $g(x) = 2x^{2/3} + 6$?

Ⓐ $5x^{2/3} - 5$ Ⓑ $-5x^{2/3} + 5$ Ⓒ $9x^{2/3} + 7$ Ⓓ $-9x^{2/3} - 7$

EXAMPLE 2 on p. 429 for Exs. 12–19

MULTIPLY AND DIVIDE FUNCTIONS Let $f(x) = 4x^{2/3}$ and $g(x) = 5x^{1/2}$. Perform the indicated operation and state the domain.

12. $f(x) \cdot g(x)$
13. $g(x) \cdot f(x)$
14. $f(x) \cdot f(x)$
15. $g(x) \cdot g(x)$
16. $\frac{f(x)}{g(x)}$
17. $\frac{g(x)}{f(x)}$
18. $\frac{f(x)}{f(x)}$
19. $\frac{g(x)}{g(x)}$

EXAMPLE 4 on p. 430 for Exs. 20–27

EVALUATE COMPOSITIONS OF FUNCTIONS Let $f(x) = 3x + 2$, $g(x) = -x^2$, and $h(x) = \frac{x - 2}{5}$. Find the indicated value.

20. $f(g(-3))$
21. $g(f(2))$
22. $h(f(-9))$
23. $g(h(8))$
24. $h(g(5))$
25. $f(f(7))$
26. $h(h(-4))$
27. $g(g(-5))$

EXAMPLE 5 on p. 430 for Exs. 28–38

FIND COMPOSITIONS OF FUNCTIONS Let $f(x) = 3x^{-1}$, $g(x) = 2x - 7$, and $h(x) = \frac{x + 4}{3}$. Perform the indicated operation and state the domain.

28. $f(g(x))$
29. $g(f(x))$
30. $h(f(x))$
31. $g(h(x))$
32. $h(g(x))$
33. $f(f(x))$
34. $h(h(x))$
35. $g(g(x))$

ERROR ANALYSIS Let $f(x) = x^2 - 3$ and $g(x) = 4x$. *Describe* and correct the error in the composition.

36.
$$\begin{aligned} f(g(x)) &= f(4x) \\ &= (x^2 - 3)(4x) \\ &= 4x^3 - 12x \end{aligned}$$
✗

37.
$$\begin{aligned} g(f(x)) &= g(x^2 - 3) \\ &= 4x^2 - 3 \end{aligned}$$
✗

38. ★ **MULTIPLE CHOICE** What is $g(f(x))$ if $f(x) = 7x^2$ and $g(x) = 3x^{-2}$?

Ⓐ $\frac{3}{49x^4}$ Ⓑ 21 Ⓒ $21x^4$ Ⓓ $\frac{7}{9x^4}$

39. ★ **OPEN-ENDED MATH** Find two different functions f and g such that $f(g(x)) = g(f(x))$.

CHALLENGE Find functions f and g such that $f(g(x)) = h(x)$, $g(x) \neq x$, and $f(x) \neq x$.

40. $h(x) = \sqrt[3]{x + 2}$

41. $h(x) = \dfrac{4}{3x^2 + 7}$

42. $h(x) = |2x + 9|$

PROBLEM SOLVING

EXAMPLE 3 on p. 429 for Exs. 43, 46

43. **BIOLOGY** For a mammal that weighs w grams, the volume b (in milliliters) of air breathed in and the volume d (in milliliters) of "dead space" (the portion of the lungs not filled with air) can be modeled by:

$$b(w) = 0.007w \qquad d(w) = 0.002w$$

The breathing rate r (in breaths per minute) of a mammal that weighs w grams can be modeled by:

$$r(w) = \frac{1.1w^{0.734}}{b(w) - d(w)}$$

Simplify $r(w)$ and calculate the breathing rate for body weights of 6.5 grams, 300 grams, and 70,000 grams.

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EXAMPLE 6 on p. 431 for Exs. 44–45

44. ★ **SHORT RESPONSE** The cost (in dollars) of producing x sneakers in a factory is given by $C(x) = 60x + 750$. The number of sneakers produced in t hours is given by $x(t) = 50t$. Find $C(x(t))$. Evaluate $C(x(5))$ and explain what this number represents.

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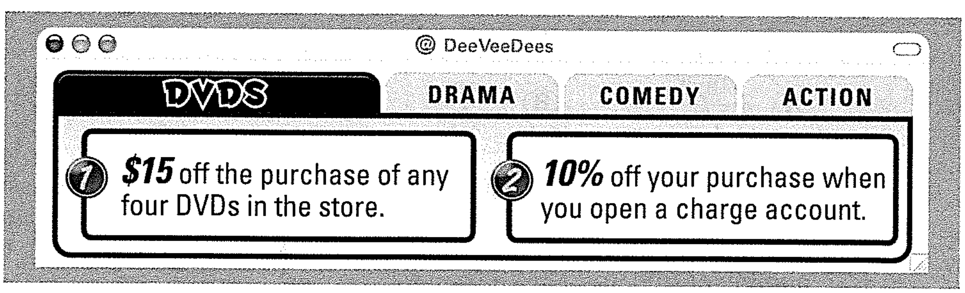
45. **MULTI-STEP PROBLEM** An online movie store is having a sale. You decide to open a charge account and buy four DVDs.

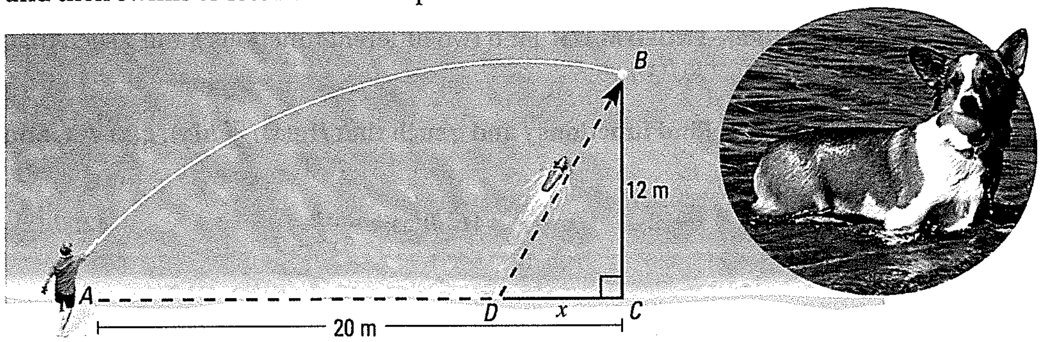


a. Use composition of functions to find the sale price of \$85 worth of DVDs when the \$15 discount is applied before the 10% discount.

b. Use composition of functions to find the sale price of \$85 worth of DVDs when the 10% discount is applied before the \$15 discount.

c. Which order of discounts gives you a better deal? *Explain.*

46. **MULTIPLE REPRESENTATIONS** A mathematician at a lake throws a tennis ball from point A along the water's edge to point B in the water, as shown. His dog, Elvis, first runs along the beach from point A to point D and then swims to fetch the ball at point B.



a. **Using a Diagram** Elvis's running speed is about 6.4 meters per second. Write a function $r(x)$ for the time he spends running from point A to point D. Elvis's swimming speed is about 0.9 meter per second. Write a function $s(x)$ for the time he spends swimming from point D to point B.

b. **Writing a Function** Write a function $t(x)$ that represents the total time Elvis spends traveling from point A to point D to point B.

c. **Using a Graph** Use a graphing calculator to graph $t(x)$. Find the value of x that minimizes $t(x)$. *Explain* the meaning of this value.

47. **CHALLENGE** To approximate the square root of a number n, the Babylonians used a method that involves starting with an initial guess x and calculating a sequence of values that approaches the exact answer. Their method was based on the function shown at the right.

$$f(x) = \frac{x + \frac{n}{x}}{2}$$

a. Let $n = 2$, and choose $x = 1$ as an initial guess for $\sqrt{n} = \sqrt{2}$. Calculate $f(x)$, $f(f(x))$, $f(f(f(x)))$, and $f(f(f(f(x))))$.

b. How many times do you need to compose the function in order for the result to approximate $\sqrt{2}$ to three decimal places? six decimal places?

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48. Which expression is equivalent to $(6x^3y^5z^{-1})(-3x^{-4}y^2)$?

Ⓐ $-\frac{18y^{10}}{x^{12}z}$ Ⓑ $-\frac{18z}{x^7y^3}$ Ⓒ $-\frac{18y^7}{xz}$ Ⓓ $-\frac{y^7}{18xz}$

49. In a high school marching band, 68% of the members are underclassmen. The rest of the members of the marching band are seniors. Which equation best represents the number of seniors, s, in the band in terms of the total number of students, t, in the band?

Ⓐ $s = \frac{8}{17}t$ Ⓑ $s = \frac{8}{25}t$

Ⓒ $s = \frac{17}{8}t$ Ⓓ $s = \frac{25}{8}t$

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Keystrokes

6.3 Use Operations with Functions

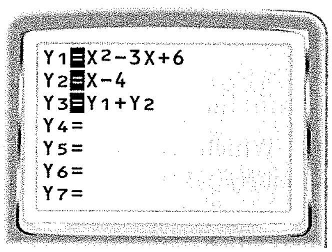
QUESTION **How can you use a graphing calculator to perform operations with functions?**

EXAMPLE **Perform function operations**

Let $f(x) = x^2 - 3x + 6$ and $g(x) = x - 4$. Find $f(4) + g(4)$ and $f(g(-2))$.

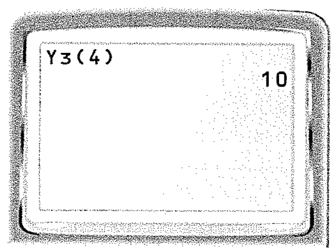
STEP 1 ***Form sum***

Enter $y_1 = x^2 - 3x + 6$ and $y_2 = x - 4$. The sum can be entered as $y_3 = y_1 + y_2$. To do so, press VARS, choose the Y-Vars menu, and select Function.



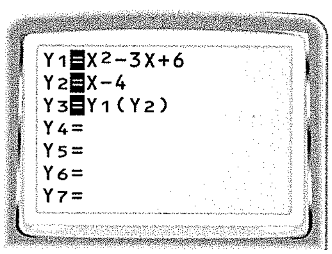
STEP 2 ***Evaluate sum***

On the home screen, enter $y_3(4)$ and press ENTER. The screen shows that $y_3(4) = 10$, so $f(4) + g(4) = 10$.



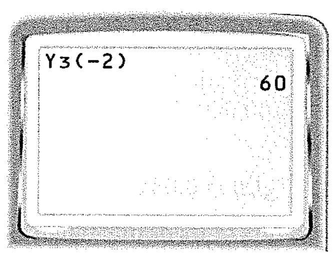
STEP 3 ***Form composition***

The composition $f(g(x))$ can be entered as $y_3 = y_1(y_2)$.



STEP 4 ***Evaluate composition***

On the home screen, enter $y_3(-2)$ and press ENTER. The screen shows that $y_3(-2) = 60$, so $f(g(-2)) = 60$.

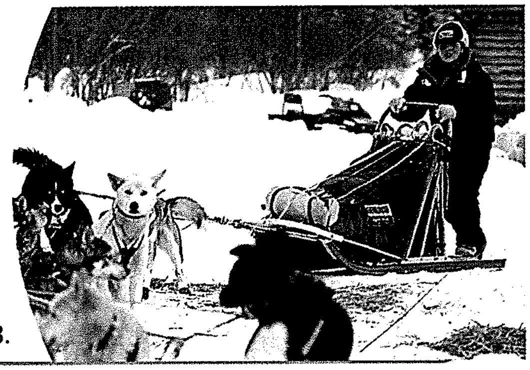


PRACTICE

Use a graphing calculator and the functions f and g to find the indicated value.

1. $f(x) = x^3 + 5x - 3$, $g(x) = -3x^2 - x$: $g(7) + f(7)$
2. $f(x) = x^{1/3}$, $g(x) = 9x$: $\dfrac{f(-8)}{g(-8)}$
3. $f(x) = 5x^3 - 3x^2$, $g(x) = -2x^2 - 5$: $g(2) - f(2)$
4. $f(x) = 2x^2 + 7x - 2$, $g(x) = x - 6$: $f(g(5))$

6.4 Use Inverse Functions



Before You performed operations with functions.

Now You will find inverse functions.

Why? So you can convert temperatures, as in Ex. 48.

Key Vocabulary
- **inverse relation**
- **inverse function**

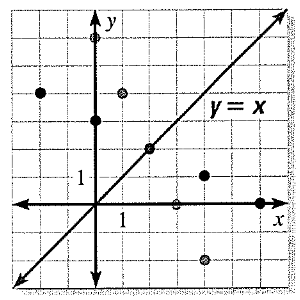
In Lesson 2.1, you learned that a relation is a pairing of input values with output values. An **inverse relation** interchanges the input and output values of the original relation. This means that the domain and range are also interchanged.

Original relation

x	0	1	2	3	4
y	6	4	2	0	−2

Inverse relation

x	6	4	2	0	−2
y	0	1	2	3	4



The graph of an inverse relation is a *reflection* of the graph of the original relation. The line of reflection is $y = x$. To find the inverse of a relation given by an equation in x and y, switch the roles of x and y and solve for y.

EXAMPLE 1 Find an inverse relation

Find an equation for the inverse of the relation $y = 3x - 5$.

$y = 3x - 5$ **Write original relation.**

$x = 3y - 5$ **Switch x and y.**

$x + 5 = 3y$ **Add 5 to each side.**

$\frac{1}{3}x + \frac{5}{3} = y$ **Solve for y. This is the inverse relation.**

In Example 1, both the original relation and the inverse relation happen to be functions. In such cases, the two functions are called **inverse functions**.

READING
The symbol −1 in f^{-1} is not to be interpreted as an exponent. In other words, $f^{-1}(x) \neq \frac{1}{f(x)}$.

KEY CONCEPT *For Your Notebook*

Inverse Functions

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

EXAMPLE 2 Verify that functions are inverses

Verify that $f(x) = 3x - 5$ and $f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$ are inverse functions.

Solution

STEP 1 **Show** that $f(f^{-1}(x)) = x$.

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x + \frac{5}{3}\right)$$
$$= 3\left(\frac{1}{3}x + \frac{5}{3}\right) - 5$$
$$= x + 5 - 5$$
$$= x \checkmark$$

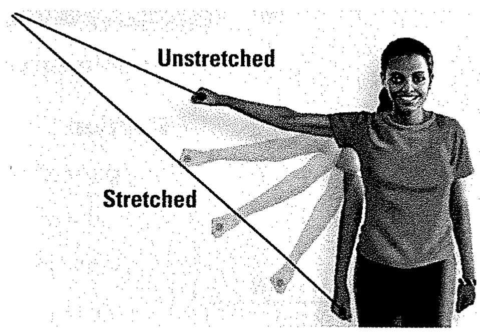
STEP 2 **Show** that $f^{-1}(f(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}(3x - 5)$$
$$= \frac{1}{3}(3x - 5) + \frac{5}{3}$$
$$= x - \frac{5}{3} + \frac{5}{3}$$
$$= x \checkmark$$

EXAMPLE 3 Solve a multi-step problem

FITNESS Elastic bands can be used in exercising to provide a range of resistance. A band's resistance R (in pounds) can be modeled by $R = \frac{3}{8}L - 5$ where L is the total length of the stretched band (in inches).

- Find the inverse of the model.
- Use the inverse function to find the length at which the band provides 19 pounds of resistance.



FIND INVERSES
Notice that you do not switch the variables when you are finding inverses of models. This would be confusing because the letters are chosen to remind you of the real-life quantities they represent.

Solution

STEP 1 **Find** the inverse function.

$$R = \frac{3}{8}L - 5 \qquad \textbf{Write original model.}$$
$$R + 5 = \frac{3}{8}L \qquad \textbf{Add 5 to each side.}$$
$$\frac{8}{3}R + \frac{40}{3} = L \qquad \textbf{Multiply each side by } \frac{8}{3}.$$

STEP 2 **Evaluate** the inverse function when $R = 19$.

$$L = \frac{8}{3}R + \frac{40}{3} = \frac{8}{3}(19) + \frac{40}{3} = \frac{152}{3} + \frac{40}{3} = \frac{192}{3} = 64$$

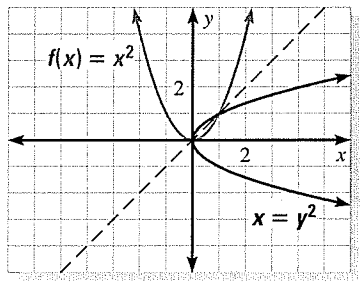
▶ The band provides 19 pounds of resistance when it is stretched to 64 inches.

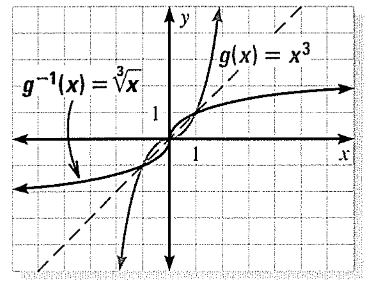
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Find the inverse of the given function. Then verify that your result and the original function are inverses.

1. $f(x) = x + 4$
2. $f(x) = 2x - 1$
3. $f(x) = -3x + 1$
4. **FITNESS** Use the inverse function in Example 3 to find the length at which the band provides 13 pounds of resistance.

INVERSES OF NONLINEAR FUNCTIONS The graphs of the power functions $f(x) = x^2$ and $g(x) = x^3$ are shown below along with their reflections in the line $y = x$. Notice that the inverse of $g(x) = x^3$ is a function, but that the inverse of $f(x) = x^2$ is *not* a function.





If the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, then the inverse of f *is* a function.

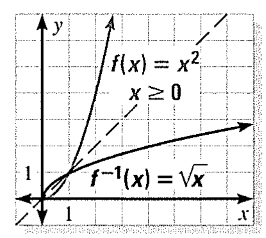
EXAMPLE 4 Find the inverse of a power function

Find the inverse of $f(x) = x^2, x \geq 0$. Then graph f and f^{-1}.

Solution

$f(x) = x^2$	**Write original function.**
$y = x^2$	**Replace $f(x)$ with y.**
$x = y^2$	**Switch x and y.**
$\pm\sqrt{x} = y$	**Take square roots of each side.**

The domain of f is restricted to nonnegative values of x. So, the range of f^{-1} must also be restricted to nonnegative values, and therefore the inverse is $f^{-1}(x) = \sqrt{x}$. (If the domain was restricted to $x \leq 0$, you would choose $f^{-1}(x) = -\sqrt{x}$.)



CHECK SOLUTION

You can check the solution of Example 4 by noting that the graph of

$f^{-1}(x) = \sqrt{x}$

is the reflection of the graph of $f(x) = x^2, x \geq 0$, in the line $y = x$.

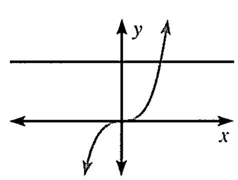
HORIZONTAL LINE TEST You can use the graph of a function f to determine whether the inverse of f is a function by applying the *horizontal line test.*

KEY CONCEPT *For Your Notebook*

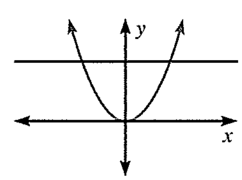
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function



Inverse is not a function



38. ★ **MULTIPLE CHOICE** What is $g(f(x))$ if $f(x) = 7x^2$ and $g(x) = 3x^{-2}$?

Ⓐ $\frac{3}{49x^4}$ Ⓑ 21 Ⓒ $21x^4$ Ⓓ $\frac{7}{9x^4}$

39. ★ **OPEN-ENDED MATH** Find two different functions f and g such that $f(g(x)) = g(f(x))$.

CHALLENGE Find functions f and g such that $f(g(x)) = h(x)$, $g(x) \neq x$, and $f(x) \neq x$.

40. $h(x) = \sqrt[3]{x+2}$ 41. $h(x) = \frac{4}{3x^2+7}$ 42. $h(x) = |2x+9|$

PROBLEM SOLVING

EXAMPLE 3 on p. 429 for Exs. 43, 46

43. **BIOLOGY** For a mammal that weighs w grams, the volume b (in milliliters) of air breathed in and the volume d (in milliliters) of "dead space" (the portion of the lungs not filled with air) can be modeled by:

$$b(w) = 0.007w \qquad d(w) = 0.002w$$

The breathing rate r (in breaths per minute) of a mammal that weighs w grams can be modeled by:

$$r(w) = \frac{1.1w^{0.734}}{b(w) - d(w)}$$

Simplify $r(w)$ and calculate the breathing rate for body weights of 6.5 grams, 300 grams, and 70,000 grams.

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EXAMPLE 6 on p. 431 for Exs. 44–45

44. ★ **SHORT RESPONSE** The cost (in dollars) of producing x sneakers in a factory is given by $C(x) = 60x + 750$. The number of sneakers produced in t hours is given by $x(t) = 50t$. Find $C(x(t))$. Evaluate $C(x(5))$ and explain what this number represents.

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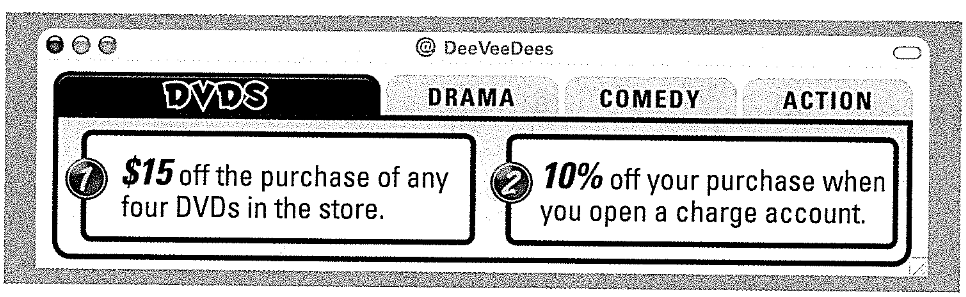
45. **MULTI-STEP PROBLEM** An online movie store is having a sale. You decide to open a charge account and buy four DVDs.

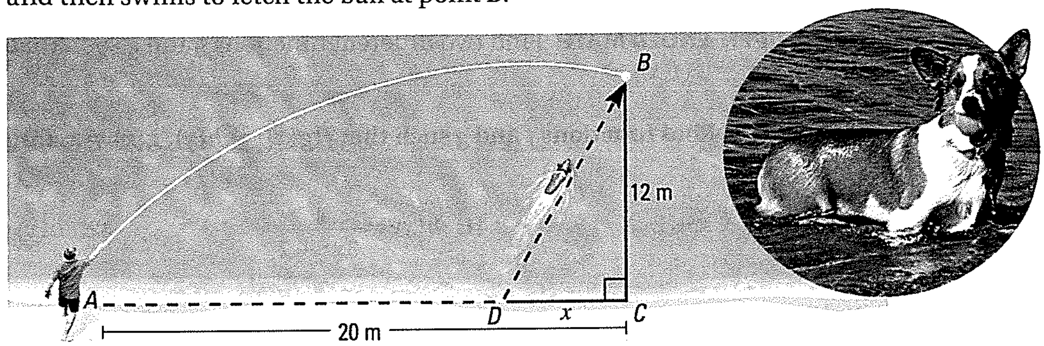


a. Use composition of functions to find the sale price of \$85 worth of DVDs when the \$15 discount is applied before the 10% discount.

b. Use composition of functions to find the sale price of \$85 worth of DVDs when the 10% discount is applied before the \$15 discount.

c. Which order of discounts gives you a better deal? *Explain.*

46. **MULTIPLE REPRESENTATIONS** A mathematician at a lake throws a tennis ball from point A along the water's edge to point B in the water, as shown. His dog, Elvis, first runs along the beach from point A to point D and then swims to fetch the ball at point B.



a. **Using a Diagram** Elvis's running speed is about 6.4 meters per second. Write a function $r(x)$ for the time he spends running from point A to point D. Elvis's swimming speed is about 0.9 meter per second. Write a function $s(x)$ for the time he spends swimming from point D to point B.

b. **Writing a Function** Write a function $t(x)$ that represents the total time Elvis spends traveling from point A to point D to point B.

c. **Using a Graph** Use a graphing calculator to graph $t(x)$. Find the value of x that minimizes $t(x)$. *Explain* the meaning of this value.

47. **CHALLENGE** To approximate the square root of a number n, the Babylonians used a method that involves starting with an initial guess x and calculating a sequence of values that approaches the exact answer. Their method was based on the function shown at the right.

$$f(x) = \frac{x + \frac{n}{x}}{2}$$

a. Let $n = 2$, and choose $x = 1$ as an initial guess for $\sqrt{n} = \sqrt{2}$. Calculate $f(x)$, $f(f(x))$, $f(f(f(x)))$, and $f(f(f(f(x))))$.

b. How many times do you need to compose the function in order for the result to approximate $\sqrt{2}$ to three decimal places? six decimal places?

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

48. Which expression is equivalent to $(6x^3y^5z^{-1})(-3x^{-4}y^2)$?

(A) $-\frac{18y^{10}}{x^{12}z}$ (B) $-\frac{18z}{x^7y^3}$ (C) $-\frac{18y^7}{xz}$ (D) $-\frac{y^7}{18xz}$

49. In a high school marching band, 68% of the members are underclassmen. The rest of the members of the marching band are seniors. Which equation best represents the number of seniors, s, in the band in terms of the total number of students, t, in the band?

(A) $s = \frac{8}{17}t$ (B) $s = \frac{8}{25}t$

(C) $s = \frac{17}{8}t$ (D) $s = \frac{25}{8}t$

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Keystrokes

6.3 Use Operations with Functions

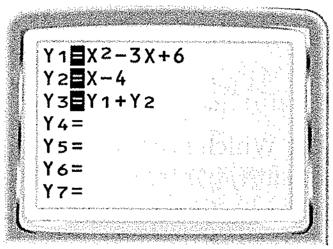
QUESTION **How can you use a graphing calculator to perform operations with functions?**

EXAMPLE **Perform function operations**

Let $f(x) = x^2 - 3x + 6$ and $g(x) = x - 4$. Find $f(4) + g(4)$ and $f(g(-2))$.

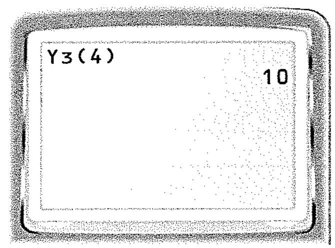
STEP 1 ***Form sum***

Enter $y_1 = x^2 - 3x + 6$ and $y_2 = x - 4$. The sum can be entered as $y_3 = y_1 + y_2$. To do so, press VARS, choose the Y-Vars menu, and select Function.



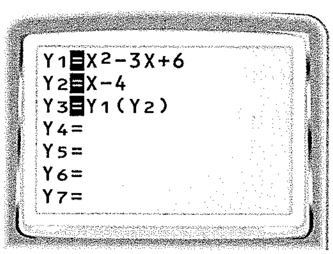
STEP 2 ***Evaluate sum***

On the home screen, enter $y_3(4)$ and press ENTER. The screen shows that $y_3(4) = 10$, so $f(4) + g(4) = 10$.



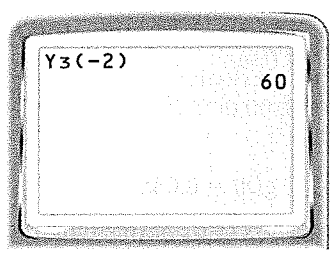
STEP 3 ***Form composition***

The composition $f(g(x))$ can be entered as $y_3 = y_1(y_2)$.



STEP 4 ***Evaluate composition***

On the home screen, enter $y_3(-2)$ and press ENTER. The screen shows that $y_3(-2) = 60$, so $f(g(-2)) = 60$.



PRACTICE

Use a graphing calculator and the functions f and g to find the indicated value.

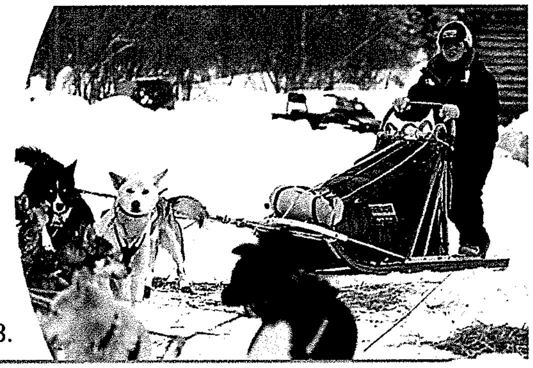
1. $f(x) = x^3 + 5x - 3,\ g(x) = -3x^2 - x$: $g(7) + f(7)$

2. $f(x) = x^{1/3},\ g(x) = 9x$: $\dfrac{f(-8)}{g(-8)}$

3. $f(x) = 5x^3 - 3x^2,\ g(x) = -2x^2 - 5$: $g(2) - f(2)$

4. $f(x) = 2x^2 + 7x - 2,\ g(x) = x - 6$: $f(g(5))$

6.4 Use Inverse Functions



Before You performed operations with functions.

Now You will find inverse functions.

Why? So you can convert temperatures, as in Ex. 48.

Key Vocabulary
- **inverse relation**
- **inverse function**

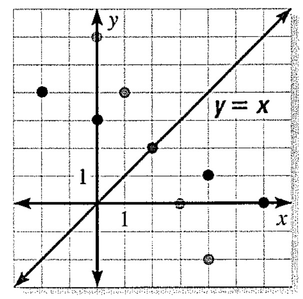
In Lesson 2.1, you learned that a relation is a pairing of input values with output values. An **inverse relation** interchanges the input and output values of the original relation. This means that the domain and range are also interchanged.

Original relation

x	0	1	2	3	4
y	6	4	2	0	−2

Inverse relation

x	6	4	2	0	−2
y	0	1	2	3	4



The graph of an inverse relation is a *reflection* of the graph of the original relation. The line of reflection is $y = x$. To find the inverse of a relation given by an equation in x and y, switch the roles of x and y and solve for y.

EXAMPLE 1 Find an inverse relation

Find an equation for the inverse of the relation $y = 3x - 5$.

$y = 3x - 5$	**Write original relation.**
$x = 3y - 5$	**Switch x and y.**
$x + 5 = 3y$	**Add 5 to each side.**
$\frac{1}{3}x + \frac{5}{3} = y$	**Solve for y. This is the inverse relation.**

In Example 1, both the original relation and the inverse relation happen to be functions. In such cases, the two functions are called **inverse functions**.

READING
The symbol −1 in f^{-1} is not to be interpreted as an exponent. In other words, $f^{-1}(x) \neq \frac{1}{f(x)}$.

KEY CONCEPT *For Your Notebook*

Inverse Functions

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \qquad \text{and} \qquad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

EXAMPLE 2 Verify that functions are inverses

Verify that $f(x) = 3x - 5$ and $f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$ are inverse functions.

Solution

STEP 1 **Show** that $f(f^{-1}(x)) = x$.

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x + \frac{5}{3}\right)$$
$$= 3\left(\frac{1}{3}x + \frac{5}{3}\right) - 5$$
$$= x + 5 - 5$$
$$= x \checkmark$$

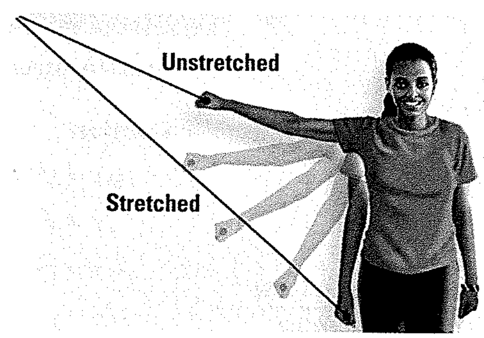
STEP 2 **Show** that $f^{-1}(f(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}(3x - 5)$$
$$= \frac{1}{3}(3x - 5) + \frac{5}{3}$$
$$= x - \frac{5}{3} + \frac{5}{3}$$
$$= x \checkmark$$

EXAMPLE 3 Solve a multi-step problem

FITNESS Elastic bands can be used in exercising to provide a range of resistance. A band's resistance R (in pounds) can be modeled by $R = \frac{3}{8}L - 5$ where L is the total length of the stretched band (in inches).

- Find the inverse of the model.
- Use the inverse function to find the length at which the band provides 19 pounds of resistance.



Solution

STEP 1 **Find** the inverse function.

$R = \frac{3}{8}L - 5$	**Write original model.**
$R + 5 = \frac{3}{8}L$	**Add 5 to each side.**
$\frac{8}{3}R + \frac{40}{3} = L$	**Multiply each side by $\frac{8}{3}$.**

STEP 2 **Evaluate** the inverse function when $R = 19$.

$$L = \frac{8}{3}R + \frac{40}{3} = \frac{8}{3}(19) + \frac{40}{3} = \frac{152}{3} + \frac{40}{3} = \frac{192}{3} = 64$$

▶ The band provides 19 pounds of resistance when it is stretched to 64 inches.

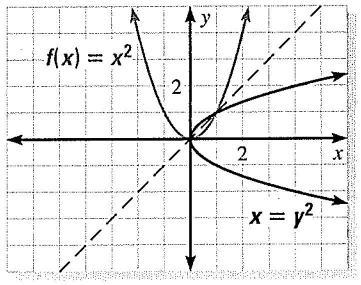
FIND INVERSES
Notice that you do not switch the variables when you are finding inverses of models. This would be confusing because the letters are chosen to remind you of the real-life quantities they represent.

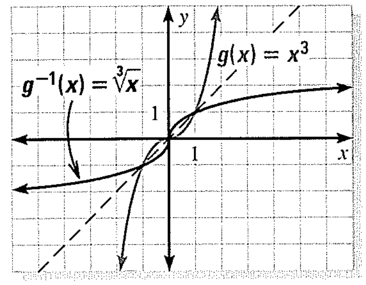
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Find the inverse of the given function. Then verify that your result and the original function are inverses.

1. $f(x) = x + 4$
2. $f(x) = 2x - 1$
3. $f(x) = -3x + 1$
4. **FITNESS** Use the inverse function in Example 3 to find the length at which the band provides 13 pounds of resistance.

INVERSES OF NONLINEAR FUNCTIONS The graphs of the power functions $f(x) = x^2$ and $g(x) = x^3$ are shown below along with their reflections in the line $y = x$. Notice that the inverse of $g(x) = x^3$ is a function, but that the inverse of $f(x) = x^2$ is *not* a function.





If the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, then the inverse of f *is* a function.

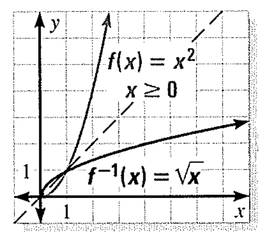
EXAMPLE 4 Find the inverse of a power function

Find the inverse of $f(x) = x^2, x \ge 0$. Then graph f and f^{-1}.

Solution

$f(x) = x^2$	**Write original function.**
$y = x^2$	**Replace $f(x)$ with y.**
$x = y^2$	**Switch x and y.**
$\pm\sqrt{x} = y$	**Take square roots of each side.**

The domain of f is restricted to nonnegative values of x. So, the range of f^{-1} must also be restricted to nonnegative values, and therefore the inverse is $f^{-1}(x) = \sqrt{x}$. (If the domain was restricted to $x \le 0$, you would choose $f^{-1}(x) = -\sqrt{x}$.)



CHECK SOLUTION
You can check the solution of Example 4 by noting that the graph of
$f^{-1}(x) = \sqrt{x}$
is the reflection of the graph of $f(x) = x^2, x \ge 0$, in the line $y = x$.

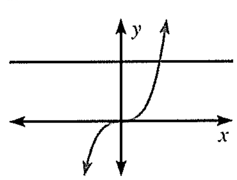
HORIZONTAL LINE TEST You can use the graph of a function f to determine whether the inverse of f is a function by applying the *horizontal line test.*

KEY CONCEPT *For Your Notebook*

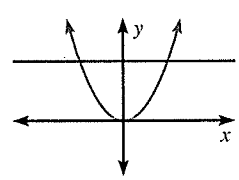
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function



Inverse is not a function



Extension

Use after Lesson 6.6

Solve Radical Inequalities

GOAL Solve radical inequalities by using tables and graphs.

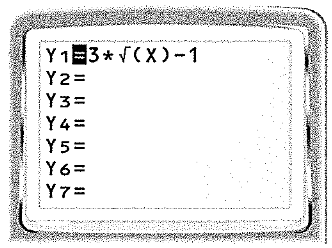
In Chapter 4, you learned how to use tables and graphs to solve quadratic inequalities. You can also use tables and graphs to solve radical inequalities.

EXAMPLE 1 Solve a radical inequality using a table

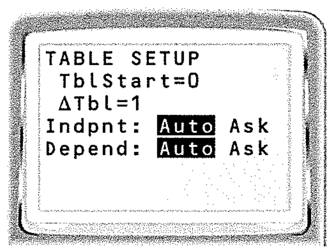
Use a table to solve $3\sqrt{x} - 1 \leq 11$.

Solution

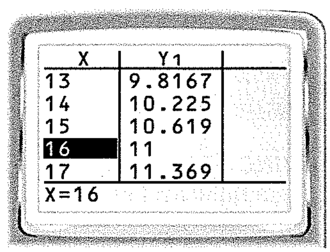
STEP 1 **Enter** the function $y = 3\sqrt{x} - 1$ into a graphing calculator.



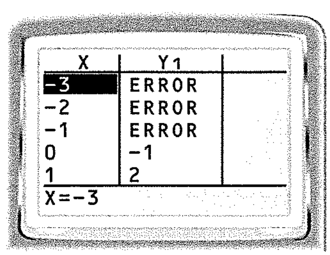
STEP 2 **Set up** the table to display x-values starting at 0 and increasing in increments of 1.



STEP 3 **Make** the table of values for $y = 3\sqrt{x} - 1$. Scroll through the table to find the x-value for which $y = 11$. This x-value is 16. It appears that $3\sqrt{x} - 1 \leq 11$ when $x \leq 16$.



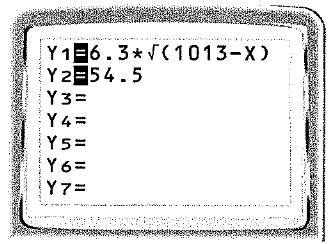
STEP 4 **Check** the domain of $y = 3\sqrt{x} - 1$. The domain is $x \geq 0$, so the solutions of $3\sqrt{x} - 1 \leq 11$ cannot be negative. (This is indicated by the word ERROR next to the negative x-values.)



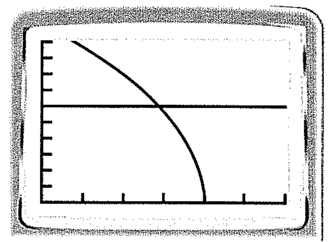
▸ The solution of the inequality is $x \leq 16$ *and* $x \geq 0$, which you can write as $0 \leq x \leq 16$.

METHOD 2 **Using a Graph** You can also use a graph to solve the equation $6.3\sqrt{1013 - p} = 54.5$.

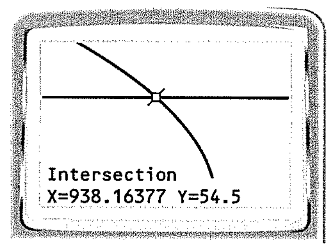
STEP 1 **Enter** the functions $y = 6.3\sqrt{1013 - x}$ and $y = 54.5$ into a graphing calculator.



STEP 2 **Graph** the functions from Step 1. Adjust the viewing window so that it shows the interval $800 \le x \le 1100$ with a scale of 50 and the interval $25 \le y \le 75$ with a scale of 5.



STEP 3 **Find** the intersection point of the two graphs using the *intersect* feature. The graphs intersect at about (938, 54.5).



▸ The mean sustained wind velocity is 54.5 meters per second when the air pressure is about 938 millibars.

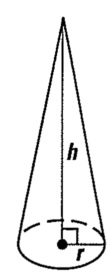
PRACTICE

SOLVING EQUATIONS Solve the radical equation using a table and using a graph.

1. $\sqrt{25 - x} = 8$
2. $2.3\sqrt{x - 1} = 11.5$
3. $4.3\sqrt{x - 7} = 30$
4. $6\sqrt{2 - 7x} - 1.2 = 22.8$
5. **ROCKETS** A model rocket is launched 25 feet from you. When the rocket is at height h, the distance d between you and the rocket is given by $d = \sqrt{625 + h^2}$ where h and d are measured in feet. What is the rocket's height when the distance between you and the rocket is 100 feet?
6. **WHAT IF?** In the problem on page 460, what is the air pressure at the center of a hurricane when the mean sustained wind velocity is 25 meters per second?
7. **GEOMETRY** The lateral surface area L of a right circular cone is given by

$$L = \pi r\sqrt{r^2 + h^2}$$

where r is the radius and h is the height. Find the height of a right circular cone with a radius of 7.5 centimeters and a lateral surface area of 900 square centimeters.



PROBLEM SOLVING WORKSHOP
LESSON 6.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 2, page 453

MULTIPLE REPRESENTATIONS In Example 2 on page 453, you solved a radical equation algebraically. You can also solve a radical equation using a table or a graph.

PROBLEM

WIND VELOCITY In a hurricane, the mean sustained wind velocity v (in meters per second) is given by

$$v(p) = 6.3\sqrt{1013 - p}$$

where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 54.5 meters per second.

METHOD 1

Using a Table The problem requires solving the radical equation $6.3\sqrt{1013 - p} = 54.5$. One way to solve this equation is to make a table of values. You can use a graphing calculator to make the table.

STEP 1 **Enter** the function $y = 6.3\sqrt{1013 - x}$ into a graphing calculator. Note that x represents air pressure and y represents wind velocity. Set up a table to display x-values starting at 900 and increasing in increments of 10.

```
Y1=6.3*√(1013-X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
TABLE SETUP
  TblStart=900
  ΔTbl=10
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 2 **Make** a table of values for the function. The first table below shows that $y = 54.5$ between $x = 930$ and $x = 940$. To approximate x more precisely, set up the table to display x-values starting at 930 and increasing in increments of 1. The second table below shows that $y = 54.5$ between $x = 938$ and $x = 939$.

X	Y1	
900	66.97	
910	63.938	
920	60.755	
930	57.396	
940	53.827	

X=930

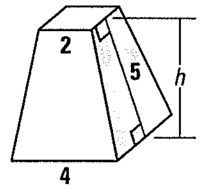
X	Y1	
935	55.64	
936	55.282	
937	54.922	
938	54.56	
939	54.195	

X=938

▶ The mean sustained wind velocity is 54.5 meters per second when the air pressure is between 938 and 939 millibars.

62. **CHALLENGE** You are trying to determine a truncated pyramid's height, which cannot be measured directly. The height h and slant height ℓ of the truncated pyramid are related by the formula shown below.

$$\ell = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2}$$



In the given formula, b_1 and b_2 are the side lengths of the upper and lower bases of the pyramid, respectively. If $\ell = 5$, $b_1 = 2$, and $b_2 = 4$, what is the height of the pyramid?

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

63. What are the zeros of the function $y = 12x^2 + 11x - 15$?

   (A) $-\frac{5}{3}, \frac{3}{4}$ (B) $\frac{5}{3}, -\frac{3}{4}$ (C) $-1, \frac{5}{4}$ (D) $2, \frac{5}{2}$

64. Which equation represents the line that contains the point $(-4, 2)$ and has slope $-\frac{5}{2}$?

   (A) $-5x - 2y = 1$ (B) $-2x + 5y = 18$

   (C) $2x - 5y = -16$ (D) $5x + 2y = -16$

QUIZ for Lessons 6.5–6.6

Graph the function. Then state the domain and range. *(p. 446)*

1. $y = 4\sqrt{x}$
2. $y = \sqrt{x} + 3$
3. $g(x) = \sqrt{x+2} - 5$
4. $y = -\frac{1}{2}\sqrt[3]{x}$
5. $f(x) = \sqrt[3]{x} - 4$
6. $y = \sqrt[3]{x-3} + 2$

Solve the equation. Check for extraneous solutions. *(p. 452)*

7. $\sqrt{6x+15} = 9$
8. $\frac{1}{4}(7x+8)^{3/2} = 54$
9. $\sqrt[3]{3x+5} + 2 = 5$
10. $x - 3 = \sqrt{10x - 54}$
11. $\sqrt{4x-4} = \sqrt{5x-1} - 1$
12. $\sqrt[3]{\frac{4}{5}x - 9} = \sqrt[3]{x-6}$

13. **ASTRONOMY** According to Kepler's third law of planetary motion, the function $P = 0.199a^{3/2}$ relates a planet's orbital period P (in days) to the length a (in millions of kilometers) of the orbit's minor axis. The orbital period of Mars is about 1.88 years. What is the length of the orbit's minor axis? *(p. 452)*

57. **BURNING RATE** A burning candle has a radius of r inches and was initially h_0 inches tall. After t minutes, the height of the candle has been reduced to h inches. These quantities are related by the formula

$$r = \sqrt{\frac{kt}{\pi(h_0 - h)}}$$

where k is a constant. How long will it take for the entire candle to burn if its radius is 0.875 inch, its initial height is 6.5 inches, and $k = 0.04$?

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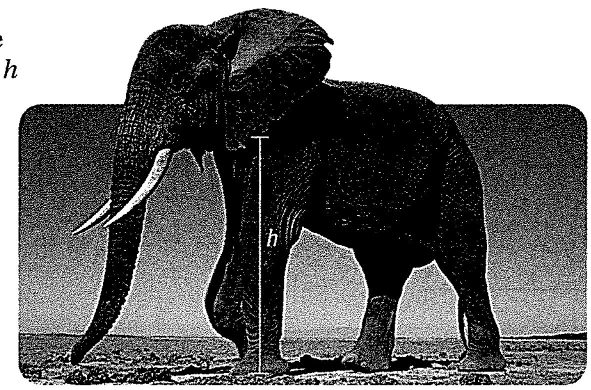
58. **CONSTRUCTION** The length ℓ (in inches) of a standard nail can be modeled by $\ell = 54d^{3/2}$ where d is the diameter (in inches) of the nail. What is the diameter of a standard nail that is 3 inches long?

59. ★ **SHORT RESPONSE** Biologists have discovered that the shoulder height h (in centimeters) of a male African elephant can be modeled by

$$h = 62.5\sqrt[3]{t} + 75.8$$

where t is the age (in years) of the elephant. *Compare* the ages of two elephants, one with a shoulder height of 150 centimeters and the other with a shoulder height of 250 centimeters.



60. ★ **EXTENDED RESPONSE** "Hang time" is the time you are suspended in the air during a jump. Your hang time t (in seconds) is given by the function $t = 0.5\sqrt{h}$ where h is the height of the jump (in feet). A basketball player jumps and has a hang time of 0.81 second. A kangaroo jumps and has a hang time of 1.12 seconds.

a. **Solve** Find the heights that the basketball player and the kangaroo jumped.

b. **Calculate** Double the hang times of the basketball player and the kangaroo and calculate the corresponding heights of each jump.

c. **Interpret** If the hang time doubles, does the height of the jump double? *Explain.*

Animated Algebra at classzone.com

61. **MULTI-STEP PROBLEM** The Beaufort wind scale was devised to measure wind speed. The Beaufort numbers B, which range from 0 to 12, can be modeled by

$$B = 1.69\sqrt{s + 4.25} - 3.55$$

where s is the speed (in miles per hour) of the wind.

a. Find the wind speed that corresponds to the Beaufort number $B = 0$.

b. Find the wind speed that corresponds to the Beaufort number $B = 12$.

c. Write an inequality that describes the range of wind speeds represented by the Beaufort model.

Beaufort Wind Scale

Beaufort number	Force of wind
0	Calm
3	Gentle breeze
6	Strong breeze
9	Strong gale
12	Hurricane

○ = **WORKED-OUT SOLUTIONS on p. WS1**   ★ = **STANDARDIZED TEST PRACTICE**

EXAMPLE 5
on p. 454
for Exs. 34–44

SOLVING RADICAL EQUATIONS **Solve the equation. Check for extraneous solutions.**

34. $x - 6 = \sqrt{3x}$

35. $x - 10 = \sqrt{9x}$

36. $x = \sqrt{16x + 225}$

37. $\sqrt{21x + 1} = x + 5$

38. $\sqrt{44 - 2x} = x - 10$

39. $\sqrt{x^2 + 4} = x + 5$

40. $x - 2 = \sqrt{\frac{3}{2}x - 2}$

41. $\sqrt[4]{3 - 8x^2} = 2x$

42. $\sqrt[3]{8x^3 - 1} = 2x - 1$

43. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of $\sqrt{32x - 64} = 2x$?

(A) 4 (B) −16 (C) 4, −16 (D) 1, 3

44. ★ **SHORT RESPONSE** *Explain* how you can tell that $\sqrt{x + 4} = -5$ has no solution without solving it.

EXAMPLE 6
on p. 455
for Exs. 45–52

EQUATIONS WITH TWO RADICALS **Solve the equation. Check for extraneous solutions.**

45. $\sqrt{4x + 1} = \sqrt{x + 10}$

46. $\sqrt[3]{12x - 5} - \sqrt[3]{8x + 15} = 0$

47. $\sqrt{3x - 8} + 1 = \sqrt{x + 5}$

48. $\sqrt{\frac{2}{3}x - 4} = \sqrt{\frac{2}{5}x - 7}$

49. $\sqrt{x + 2} = 2 - \sqrt{x}$

50. $\sqrt{2x + 3} + 2 = \sqrt{6x + 7}$

51. $\sqrt{2x + 5} = \sqrt{x + 2} + 1$

52. $\sqrt{5x + 6} + 3 = \sqrt{3x + 3} + 4$

SOLVING SYSTEMS **Solve the system of equations.**

53. $3\sqrt{x} + 5\sqrt{y} = 31$
$5\sqrt{x} - 5\sqrt{y} = -15$

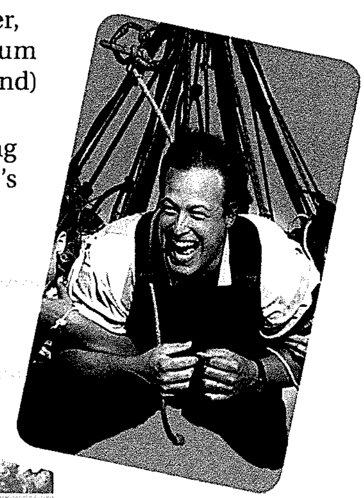
54. $5\sqrt{x} - 2\sqrt{y} = 4\sqrt{2}$
$2\sqrt{x} + 3\sqrt{y} = 13\sqrt{2}$

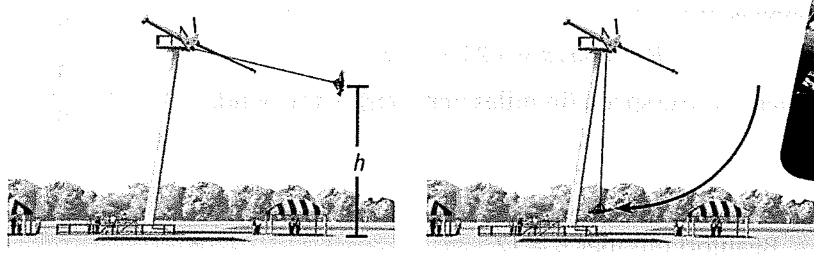
55. **CHALLENGE** Give an example of a radical equation that has two extraneous solutions.

PROBLEM SOLVING

EXAMPLE 2
on p. 453
for Exs. 56–57

56. **MAXIMUM SPEED** In an amusement park ride called the Sky Flyer, a rider suspended by a cable swings back and forth like a pendulum from a tall tower. A rider's maximum speed v (in meters per second) occurs at the bottom of each swing and can be approximated by $v = \sqrt{2gh}$ where h is the height (in meters) at the top of each swing and g is the acceleration due to gravity $(g \approx 9.8 \text{ m/sec}^2)$. If a rider's maximum speed was 15 meters per second, what was the rider's height at the top of the swing?





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6.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 5, 13, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 22, 43, 44, 59, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: When you solve an equation algebraically, an apparent solution that must be rejected because it does not satisfy the original equation is called a(n) __?__ solution.

2. ★ **WRITING** A student was asked to solve $\sqrt{3x-1} - \sqrt{9x-5} = 0$. His first step was to square each side. While trying to isolate x, he gave up in frustration. What could the student have done to avoid this situation?

EXAMPLE 1 on p. 452 for Exs. 3–21

EQUATIONS WITH SQUARE ROOTS Solve the equation. Check your solution.

3. $\sqrt{5x+1} = 6$
4. $\sqrt{3x+10} = 8$
5. $\sqrt{9x} + 11 = 14$
6. $\sqrt{2x} - \frac{2}{3} = 0$
7. $-2\sqrt{24x} + 13 = -11$
8. $8\sqrt{10x} - 7 = 9$
9. $\sqrt{x-25} + 3 = 5$
10. $-4\sqrt{x} - 6 = -20$
11. $\sqrt{-2x+3} - 2 = 10$

12. ★ **MULTIPLE CHOICE** What is the solution of $\sqrt{8x+3} = 3$?

Ⓐ $-\frac{3}{4}$ Ⓑ 0 Ⓒ $\frac{3}{4}$ Ⓓ $\frac{9}{8}$

EQUATIONS WITH CUBE ROOTS Solve the equation. Check your solution.

13. $\sqrt[3]{x} - 10 = -3$
14. $\sqrt[3]{x-16} = 2$
15. $\sqrt[3]{12x} - 13 = -7$
16. $3\sqrt[3]{16x} - 7 = 17$
17. $-5\sqrt[3]{8x} + 12 = -8$
18. $\sqrt[3]{4x+5} = \frac{1}{2}$
19. $\sqrt[3]{x-3} + 2 = 4$
20. $\sqrt[3]{4x+2} - 6 = -10$
21. $-4\sqrt[3]{x+10} + 3 = 15$

22. ★ **OPEN-ENDED MATH** Write a radical equation of the form $\sqrt[3]{ax+b} = c$ that has -3 as a solution. *Explain* the method you used to find your equation.

EXAMPLES 3 and 4 on pp. 453–454 for Exs. 23–33

EQUATIONS WITH RATIONAL EXPONENTS Solve the equation. Check your solution.

23. $2x^{3/2} = 16$
24. $\frac{1}{2}x^{5/2} = 16$
25. $9x^{3/5} = 72$
26. $(16x)^{3/4} + 44 = 556$
27. $\frac{1}{7}(x+9)^{3/2} = 49$
28. $(x-5)^{5/3} - 73 = 170$
29. $\left(\frac{1}{3}x - 11\right)^{1/2} = 5$
30. $(5x-19)^{5/6} = 32$
31. $(3x+5)^{7/3} + 22 = 150$

ERROR ANALYSIS *Describe* and correct the error in solving the equation.

32.

$$\sqrt[3]{x} + 2 = 4$$
$$(\sqrt[3]{x} + 2)^3 = 4^3$$
$$x + 8 = 64$$
$$x = 56$$

33.

$$(x+7)^{1/2} = 5$$
$$[(x+7)^{1/2}]^2 = 5$$
$$x + 7 = 5$$
$$x = -2$$

SQUARING TWICE When an equation contains two radicals, you may need to square each side twice in order to eliminate both radicals.

EXAMPLE 6 Solve an equation with two radicals

Solve $\sqrt{x+2} + 1 = \sqrt{3-x}$.

Solution

METHOD 1 Solve using algebra.

$\sqrt{x+2} + 1 = \sqrt{3-x}$	**Write original equation.**
$(\sqrt{x+2} + 1)^2 = (\sqrt{3-x})^2$	**Square each side.**
$x + 2 + 2\sqrt{x+2} + 1 = 3 - x$	**Expand left side and simplify right side.**
$2\sqrt{x+2} = -2x$	**Isolate radical expression.**
$\sqrt{x+2} = -x$	**Divide each side by 2.**
$(\sqrt{x+2})^2 = (-x)^2$	**Square each side again.**
$x + 2 = x^2$	**Simplify.**
$0 = x^2 - x - 2$	**Write in standard form.**
$0 = (x-2)(x+1)$	**Factor.**
$x - 2 = 0$ or $x + 1 = 0$	**Zero-product property**
$x = 2$ or $x = -1$	**Solve for *x*.**

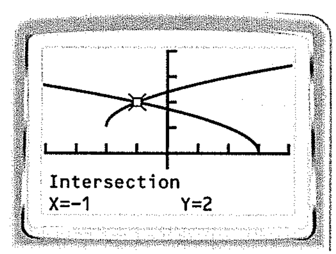
REVIEW FOIL METHOD
For help with multiplying algebraic expressions using the FOIL method, see p. 245.

Check $x = 2$ in the original equation.	Check $x = -1$ in the original equation.
$\sqrt{x+2} + 1 = \sqrt{3-x}$	$\sqrt{x+2} + 1 = \sqrt{3-x}$
$\sqrt{2+2} + 1 \stackrel{?}{=} \sqrt{3-2}$	$\sqrt{-1+2} + 1 \stackrel{?}{=} \sqrt{3-(-1)}$
$\sqrt{4} + 1 \stackrel{?}{=} \sqrt{1}$	$\sqrt{1} + 1 \stackrel{?}{=} \sqrt{4}$
$3 \neq 1$	$2 = 2$ ✓

▶ The only solution is −1. (The apparent solution 2 is extraneous.)

METHOD 2 Use a graph to solve the equation.

Use a graphing calculator to graph $y_1 = \sqrt{x+2} + 1$ and $y_2 = \sqrt{3-x}$. Then find the intersection points of the two graphs by using the *intersect* feature. You will find that the only point of intersection is (−1, 2). Therefore, −1 is the only solution of the equation $\sqrt{x+2} + 1 = \sqrt{3-x}$.



✓ GUIDED PRACTICE for Examples 5 and 6

Solve the equation. Check for extraneous solutions.

11. $x - \frac{1}{2} = \sqrt{\frac{1}{4}x}$

12. $\sqrt{10x+9} = x + 3$

13. $\sqrt{2x+5} = \sqrt{x+7}$

14. $\sqrt{x+6} - 2 = \sqrt{x-2}$

EXAMPLE 4 Solve an equation with a rational exponent

Solve $(x + 2)^{3/4} - 1 = 7$.

$(x + 2)^{3/4} - 1 = 7$	**Write original equation.**
$(x + 2)^{3/4} = 8$	**Add 1 to each side.**
$\left[(x + 2)^{3/4}\right]^{4/3} = 8^{4/3}$	**Raise each side to the power $\frac{4}{3}$.**
$x + 2 = (8^{1/3})^4$	**Apply properties of exponents.**
$x + 2 = 2^4$	**Simplify.**
$x + 2 = 16$	**Simplify.**
$x = 14$	**Subtract 2 from each side.**

▶ The solution is 14. Check this in the original equation.

✓ GUIDED PRACTICE for Examples 3 and 4

Solve the equation. Check your solution.

5. $3x^{3/2} = 375$ **6.** $-2x^{3/4} = -16$ **7.** $-\frac{2}{3}x^{1/5} = -2$

8. $(x + 3)^{5/2} = 32$ **9.** $(x - 5)^{5/3} = 243$ **10.** $(x + 2)^{1/3} + 3 = 7$

EXTRANEOUS SOLUTIONS Raising each side of an equation to the same power may introduce extraneous solutions. When you use this procedure, you should always check each apparent solution in the *original* equation.

EXAMPLE 5 Solve an equation with an extraneous solution

Solve $x + 1 = \sqrt{7x + 15}$.

$x + 1 = \sqrt{7x + 15}$	**Write original equation.**
$(x + 1)^2 = (\sqrt{7x + 15})^2$	**Square each side.**
$x^2 + 2x + 1 = 7x + 15$	**Expand left side and simplify right side.**
$x^2 - 5x - 14 = 0$	**Write in standard form.**
$(x - 7)(x + 2) = 0$	**Factor.**
$x - 7 = 0$ or $x + 2 = 0$	**Zero-product property**
$x = 7$ or $x = -2$	**Solve for x.**

REVIEW FACTORING
For help with factoring, see p. 252.

CHECK

Check $x = 7$ in the original equation.

$x + 1 = \sqrt{7x + 15}$

$7 + 1 \stackrel{?}{=} \sqrt{7(7) + 15}$

$8 \stackrel{?}{=} \sqrt{64}$

$8 = 8$ ✓

Check $x = -2$ in the original equation.

$x + 1 = \sqrt{7x + 15}$

$-2 + 1 \stackrel{?}{=} \sqrt{7(-2) + 15}$

$-1 \stackrel{?}{=} \sqrt{1}$

$-1 \neq 1$

▶ The only solution is 7. (The apparent solution -2 is extraneous.)

EXAMPLE 2 Solve a radical equation given a function

WIND VELOCITY In a hurricane, the mean sustained wind velocity v (in meters per second) is given by

$$v(p) = 6.3\sqrt{1013 - p}$$

where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 54.5 meters per second.

ANOTHER WAY

For alternative methods for solving the problem in Example 2, turn to page 460 for the **Problem Solving Workshop**.

Solution

$v(p) = 6.3\sqrt{1013 - p}$	**Write given function.**
$54.5 = 6.3\sqrt{1013 - p}$	**Substitute 54.5 for $v(p)$.**
$8.65 \approx \sqrt{1013 - p}$	**Divide each side by 6.3.**
$(8.65)^2 \approx (\sqrt{1013 - p})^2$	**Square each side.**
$74.8 \approx 1013 - p$	**Simplify.**
$-938.2 \approx -p$	**Subtract 1013 from each side.**
$938.2 \approx p$	**Divide each side by −1.**

▸ The air pressure at the center of the hurricane is about 938 millibars.

✓ GUIDED PRACTICE for Example 2

4. **WHAT IF?** Use the function in Example 2 to estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 48.3 meters per second.

RATIONAL EXPONENTS When an equation contains a power with a rational exponent, you can solve the equation using a procedure similar to the one for solving radical equations. In this case, you first isolate the power and then raise each side of the equation to the reciprocal of the rational exponent.

★ EXAMPLE 3 Standardized Test Practice

What is the solution of the equation $4x^{3/2} = 108$?

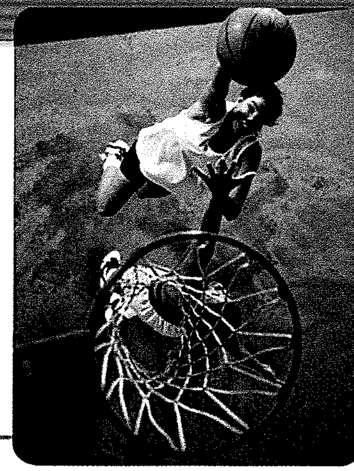
(A) 3 (B) 6 (C) 9 (D) 27

Solution

$4x^{3/2} = 108$	**Write original equation.**
$x^{3/2} = 27$	**Divide each side by 4.**
$(x^{3/2})^{2/3} = 27^{2/3}$	**Raise each side to the power $\frac{2}{3}$.**
$x = 9$	**Simplify.**

▸ The correct answer is C. (A) (B) (C) (D)

6.6 Solve Radical Equations



Before You solved polynomial equations.
Now You will solve radical equations.
Why? So you can calculate hang time, as in Ex. 60.

Key Vocabulary
- **radical equation**
- **extraneous solution,** *p. 52*

Equations with radicals that have variables in their radicands are called **radical equations**. An example of a radical equation is $\sqrt[3]{2x+7} = 3$.

KEY CONCEPT *For Your Notebook*

Solving Radical Equations

To solve a radical equation, follow these steps:

STEP 1 **Isolate** the radical on one side of the equation, if necessary.

STEP 2 **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

STEP 3 **Solve** the polynomial equation using techniques you learned in previous chapters. Check your solution.

EXAMPLE 1 Solve a radical equation

Solve $\sqrt[3]{2x+7} = 3$.

$\sqrt[3]{2x+7} = 3$	**Write original equation.**
$(\sqrt[3]{2x+7})^3 = 3^3$	**Cube each side to eliminate the radical.**
$2x + 7 = 27$	**Simplify.**
$2x = 20$	**Subtract 7 from each side.**
$x = 10$	**Divide each side by 2.**

CHECK Check $x = 10$ in the original equation.

$\sqrt[3]{2(10)+7} \stackrel{?}{=} 3$	**Substitute 10 for *x*.**
$\sqrt[3]{27} \stackrel{?}{=} 3$	**Simplify.**
$3 = 3$ ✓	**Solution checks.**

✓ GUIDED PRACTICE for Example 1

Solve the equation. Check your solution.

1. $\sqrt[3]{x} - 9 = -1$ **2.** $\sqrt{x+25} = 4$ **3.** $2\sqrt[3]{x-3} = 4$

38. **DRAG RACING** For a given total weight, the speed of a car at the end of a drag race is a function of the car's power. For a car with a total weight of 3500 pounds, the speed s (in miles per hour) can be modeled by $s = 14.8\sqrt[3]{p}$ where p is the power (in horsepower). Graph the model. Then determine the power of a 3500 pound car that reaches a speed of 200 miles per hour.

39. **MULTIPLE REPRESENTATIONS** Under certain conditions, a skydiver's terminal velocity v_t (in feet per second) is given by

$$v_t = 33.7\sqrt{\frac{W}{A}}$$

where W is the weight of the skydiver (in pounds) and A is the skydiver's cross-sectional surface area (in square feet). Note that skydivers can vary their cross-sectional surface area by changing positions as they fall.

a. **Writing an Equation** Write an equation that gives v_t as a function of A for a skydiver who weighs 165 pounds.

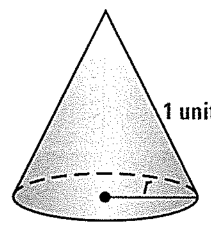
b. **Making a Table** Make a table of values for the equation from part (a).

c. **Drawing a Graph** Use your table to graph the equation.

40. **CHALLENGE** The surface area S of a right circular cone with a slant height of 1 unit is given by $S = \pi r + \pi r^2$ where r is the cone's radius.

a. Use completing the square to show the following:

$$r = \frac{1}{\sqrt{\pi}}\sqrt{S + \frac{\pi}{4}} - \frac{1}{2}$$



b. Graph the equation from part (a) using a graphing calculator.

c. Find the radius of a right circular cone with a slant height of 1 unit and a surface area of $\frac{3\pi}{4}$ square units.

PA PENNSYLVANIA MIXED REVIEW

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41. Which equation best represents the relationship between x and y shown in the table?

x	y
0	0
1	37
2	58
3	63

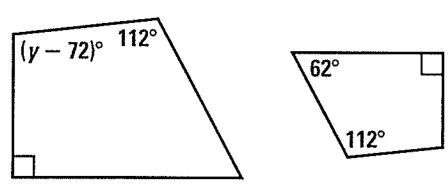
Ⓐ $y = 25x + 12$

Ⓑ $y = 45x - 8x^2$

Ⓒ $y = 8x^2 - 45x$

Ⓓ $y = 70 - 33x^3$

42. The two polygons are similar. What is the value of y?



Ⓐ 24 Ⓑ 134

Ⓒ 168 Ⓓ 204

26. **ERROR ANALYSIS** A student tried to explain how the graphs of $y = -2\sqrt[3]{x}$ and $y = -2\sqrt[3]{x+1} - 3$ are related. *Describe* and correct the error.

The graph of $y = -2\sqrt[3]{x+1} - 3$
is the graph of $y = -2\sqrt[3]{x}$ translated
right 1 unit and down 3 units.

27. ★ **MULTIPLE CHOICE** If the graph of $y = 3\sqrt[3]{x}$ is shifted left 2 units, what is the equation of the translated graph?

(A) $y = 3\sqrt[3]{x-2}$ (B) $y = 3\sqrt[3]{x} - 2$ (C) $y = 3\sqrt[3]{x+2}$ (D) $y = 3\sqrt[3]{x} + 2$

REASONING Find the domain and range of the function without graphing. *Explain* how you found your answers.

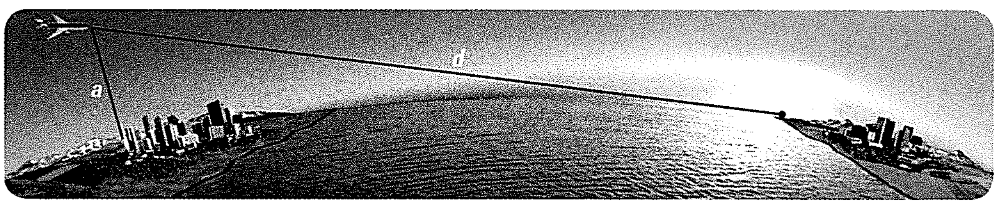
28. $y = \sqrt{x+5}$
29. $y = \sqrt{x-12}$
30. $y = \frac{1}{3}\sqrt{x} - 4$
31. $y = \frac{1}{2}\sqrt[3]{x+7}$
32. $g(x) = \sqrt[3]{x+7}$
33. $f(x) = \frac{1}{4}\sqrt{x-3} + 6$

34. **CHALLENGE** Graph $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}$, $y = \sqrt[6]{x}$, and $y = \sqrt[7]{x}$ on a graphing calculator. Make generalizations about the graph of $y = \sqrt[n]{x}$ when n is even and when n is odd.

PROBLEM SOLVING

EXAMPLE 3
on p. 447
for Exs. 35–36

35. **INDIRECT MEASUREMENT** The distance d (in miles) that a pilot can see to the horizon can be modeled by $d = 1.22\sqrt{a}$ where a is the plane's altitude (in feet above sea level). Graph the model on a graphing calculator. Then determine at what altitude the pilot can see 8 miles.



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36. **PENDULUMS** Use the model $T = 1.11\sqrt{\ell}$ for the period of a pendulum from Example 3 on page 447.

    a. Find the period of a pendulum with a length of 2 feet.

    b. Find the length of a pendulum with a period of 2 seconds.

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37. ★ **SHORT RESPONSE** The speed v (in meters per second) of sound waves in air depends on the temperature K (in kelvins) and can be modeled by:

$$v = 331.5\sqrt{\frac{K}{273.15}},\ K \geq 0$$

    a. Kelvin temperature K is related to Celsius temperature C by the formula $K = 273.15 + C$. Write an equation that gives the speed v of sound waves in air as a function of the temperature C in degrees Celsius.

    b. What are a reasonable domain and range for the function from part (a)?

✓ **GUIDED PRACTICE** for Examples 4 and 5

Graph the function. Then state the domain and range.

6. $y = -4\sqrt{x} + 2$ **7.** $y = 2\sqrt{x+1}$ **8.** $f(x) = \frac{1}{2}\sqrt{x-3} - 1$

9. $y = 2\sqrt[3]{x-4}$ **10.** $y = \sqrt[3]{x} - 5$ **11.** $g(x) = -\sqrt[3]{x+2} - 3$

6.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 11, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 25, 27, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

1. VOCABULARY Copy and complete: Square root functions and cube root functions are examples of _?_ functions.

2. ★ WRITING The graph of $y = \sqrt{x}$ is the graph of $y = a\sqrt{x-h} + k$ with $a = 1$, $h = 0$, and $k = 0$. Predict how the graph of $y = \sqrt{x}$ will change if:

a. $a = -3$ **b.** $h = 2$ **c.** $k = 4$

EXAMPLE 1 on p. 446 for Exs. 3–9

SQUARE ROOT FUNCTIONS Graph the function. Then state the domain and range.

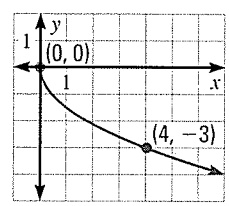
3. $y = -4\sqrt{x}$ **4.** $f(x) = \frac{1}{2}\sqrt{x}$ **5.** $y = -\frac{4}{5}\sqrt{x}$

6. $y = -6\sqrt{x}$ **7.** $y = 5\sqrt{x}$ **8.** $g(x) = 9\sqrt{x}$

9. ★ MULTIPLE CHOICE The graph of which function is shown?

Ⓐ $y = \frac{3}{4}\sqrt{x}$ Ⓑ $y = -\frac{3}{4}\sqrt{x}$

Ⓒ $y = \frac{3}{2}\sqrt{x}$ Ⓓ $y = -\frac{3}{2}\sqrt{x}$



EXAMPLE 2 on p. 447 for Exs. 10–15

CUBE ROOT FUNCTIONS Graph the function. Then state the domain and range.

10. $y = \frac{1}{4}\sqrt[3]{x}$ **11.** $y = 2\sqrt[3]{x}$ **12.** $f(x) = -5\sqrt[3]{x}$

13. $h(x) = -\frac{1}{7}\sqrt[3]{x}$ **14.** $g(x) = 6\sqrt[3]{x}$ **15.** $y = \frac{7}{9}\sqrt[3]{x}$

EXAMPLES 4 and 5 on p. 448 for Exs. 16–24

RADICAL FUNCTIONS Graph the function. Then state the domain and range.

16. $f(x) = 2\sqrt{x-1} + 3$ **17.** $y = (x+1)^{1/2} + 8$ **18.** $y = -4\sqrt{x-5} + 1$

19. $y = \frac{3}{4}x^{1/3} - 1$ **20.** $y = -2\sqrt[3]{x+5} + 5$ **21.** $h(x) = -3\sqrt[3]{x+7} - 6$

22. $y = -\sqrt{x-4} - 7$ **23.** $g(x) = -\frac{1}{3}\sqrt[3]{x} - 6$ **24.** $y = 4\sqrt[3]{x-4} + 5$

25. ★ SHORT RESPONSE *Explain* why there are limitations on the domain and range of the function $y = \sqrt{x-5} + 4$.

TRANSLATIONS OF RADICAL FUNCTIONS The procedure for graphing functions of the form $y = a\sqrt{x-h} + k$ and $y = a\sqrt[3]{x-h} + k$ is described below.

KEY CONCEPT *For Your Notebook*

Graphs of Radical Functions

To graph $y = a\sqrt{x-h} + k$ or $y = a\sqrt[3]{x-h} + k$, follow these steps:

STEP 1 **Sketch** the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

STEP 2 **Translate** the graph horizontally h units and vertically k units.

EXAMPLE 4 Graph a translated square root function

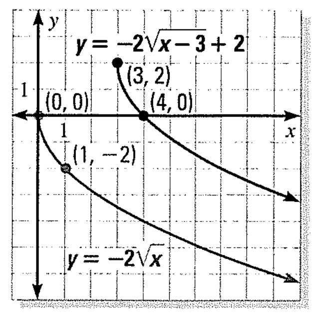
Graph $y = -2\sqrt{x-3} + 2$. Then state the domain and range.

Solution

REVIEW TRANSLATIONS
For help with translating graphs, see p. 123.

STEP 1 **Sketch** the graph of $y = -2\sqrt{x}$ (shown in blue). Notice that it begins at the origin and passes through the point $(1, -2)$.

STEP 2 **Translate** the graph. For $y = -2\sqrt{x-3} + 2$, $h = 3$ and $k = 2$. So, shift the graph of $y = -2\sqrt{x}$ right 3 units and up 2 units. The resulting graph starts at $(3, 2)$ and passes through $(4, 0)$.



From the graph, you can see that the domain of the function is $x \geq 3$ and the range of the function is $y \leq 2$.

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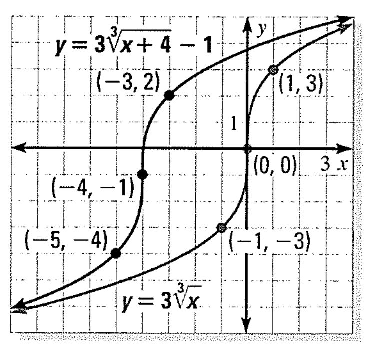
EXAMPLE 5 Graph a translated cube root function

Graph $y = 3\sqrt[3]{x+4} - 1$. Then state the domain and range.

Solution

STEP 1 **Sketch** the graph of $y = 3\sqrt[3]{x}$ (shown in blue). Notice that it passes through the origin and the points $(-1, -3)$ and $(1, 3)$.

STEP 2 **Translate** the graph. Note that for $y = 3\sqrt[3]{x+4} - 1$, $h = -4$ and $k = -1$. So, shift the graph of $y = 3\sqrt[3]{x}$ left 4 units and down 1 unit. The resulting graph passes through the points $(-5, -4)$, $(-4, -1)$, and $(-3, 2)$.



From the graph, you can see that the domain and range of the function are both all real numbers.

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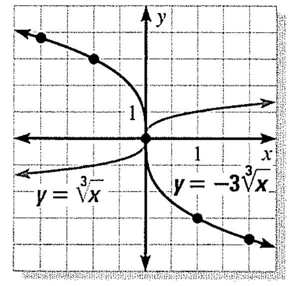
EXAMPLE 2 Graph a cube root function

Graph $y = -3\sqrt[3]{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt[3]{x}$.

Solution

Make a table of values and sketch the graph.

x	−2	−1	0	1	2
y	3.78	3	0	−3	−3.78

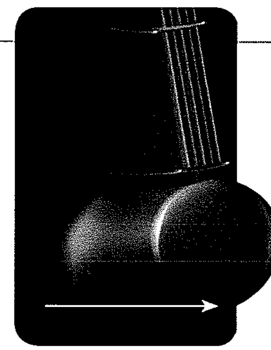


REVIEW STRETCHES AND SHRINKS
For help with vertical stretches and shrinks, see p. 123.

The domain and range are all real numbers.

The graph of $y = -3\sqrt[3]{x}$ is a vertical stretch of the graph of $y = \sqrt[3]{x}$ by a factor of 3 followed by a reflection in the x-axis.

EXAMPLE 3 Solve a multi-step problem

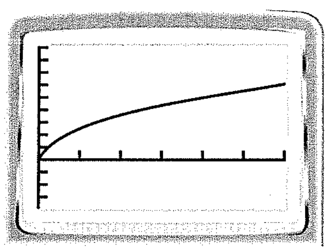


PENDULUMS The *period* of a pendulum is the time the pendulum takes to complete one back-and-forth swing. The period T (in seconds) can be modeled by $T = 1.11\sqrt{\ell}$ where ℓ is the pendulum's length (in feet).

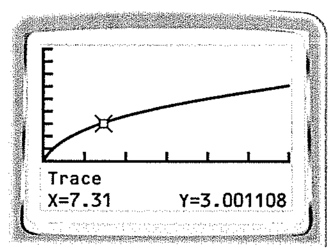
- Use a graphing calculator to graph the model.
- How long is a pendulum with a period of 3 seconds?

Solution

STEP 1 **Graph** the model. Enter the equation $y = 1.11\sqrt{x}$. The graph is shown below.



STEP 2 **Use** the *trace* feature to find the value of x when $y = 3$. The graph shows $x \approx 7.3$.



▸ A pendulum with a period of 3 seconds is about 7.3 feet long.

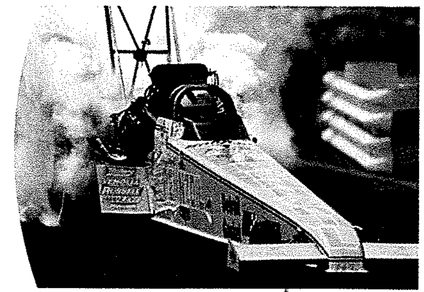
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function. Then state the domain and range.

1. $y = -3\sqrt{x}$ **2.** $f(x) = \frac{1}{4}\sqrt{x}$ **3.** $y = -\frac{1}{2}\sqrt[3]{x}$ **4.** $g(x) = 4\sqrt[3]{x}$

5. WHAT IF? Use the model in Example 3 to find the length of a pendulum with a period of 1 second.

6.5 Graph Square Root and Cube Root Functions



PA M11.D.1.1.3 Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

Before You graphed polynomial functions.

Now You will graph square root and cube root functions.

Why? So you can graph the speed of a racing car, as in Ex. 38.

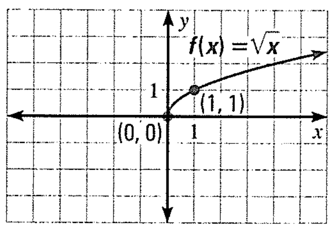
Key Vocabulary
- **radical function**
- **parent function,** *p. 89*

In Lesson 6.4, you saw the graphs of $y = \sqrt{x}$ and $y = \sqrt[3]{x}$. These are examples of **radical functions.** In this lesson, you will learn to graph functions of the form $y = a\sqrt{x - h} + k$ and $y = a\sqrt[3]{x - h} + k$.

KEY CONCEPT *For Your Notebook*

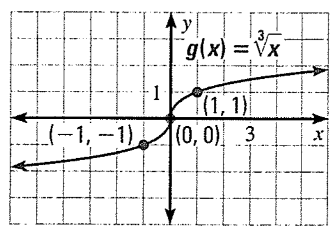
Parent Functions for Square Root and Cube Root Functions

The parent function for the family of square root functions is $f(x) = \sqrt{x}$.



Domain: $x \geq 0$, Range: $y \geq 0$

The parent function for the family of cube root functions is $g(x) = \sqrt[3]{x}$.



Domain and range: all real numbers

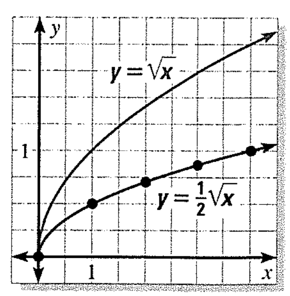
EXAMPLE 1 Graph a square root function

Graph $y = \frac{1}{2}\sqrt{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

Solution

Make a table of values and sketch the graph.

x	0	1	2	3	4
y	0	0.5	0.71	0.87	1



REVIEW DOMAIN AND RANGE
For help with the domain and range of a function, see p. 72.

The radicand of a square root must be nonnegative. So, the domain is $x \geq 0$. The range is $y \geq 0$.

The graph of $y = \frac{1}{2}\sqrt{x}$ is a vertical shrink of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{2}$.

51. **CHALLENGE** Consider the function $g(x) = -x$.

   a. Graph $g(x) = -x$ and explain why it is its own inverse. Also verify that $g(x) = g^{-1}(x)$ algebraically.

   b. Graph other linear functions that are their own inverses. Write equations of the lines you graphed.

   c. Use your results from part (b) to write a general equation describing the family of linear functions that are their own inverses.

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52. What is the value of $f(x) = -5x^4 + 3x^3 + 10x^2 - x - 8$ when $x = -1$?

   (A) -5 (B) -1 (C) 1 (D) 3

53. At a school's annual choir competition, there are a total of 750 adults and students in the audience. The number of students, s, is 30 more than three times the number of adults, a. Which system of linear equations could be used to determine the numbers of students and adults in the audience?

   (A) $s + a = 30$, $s = 750 - 3a$

   (B) $s + a = 750$, $s = 30 + 3a$

   (C) $s + a = 750$, $a = 30 + 3s$

   (D) $s + a = 30$, $a = 750 - 3s$

QUIZ for Lessons 6.3–6.4

Let $f(x) = 4x^2 - x$ and $g(x) = 2x^2$. Perform the indicated operation and state the domain. *(p. 428)*

1. $f(x) + g(x)$
2. $g(x) - f(x)$
3. $f(x) \cdot g(x)$
4. $\frac{f(x)}{g(x)}$
5. $f(g(x))$
6. $g(f(x))$
7. $f(f(x))$
8. $g(g(x))$

Verify that f and g are inverse functions. *(p. 438)*

9. $f(x) = x - 9$, $g(x) = x + 9$
10. $f(x) = 5x^3$, $g(x) = \sqrt[3]{\frac{x}{5}}$
11. $f(x) = -\frac{3}{2}x + \frac{1}{4}$, $g(x) = -\frac{2}{3}x + \frac{1}{6}$
12. $f(x) = 6x^2 + 1, x \geq 0$; $g(x) = \left(\frac{x-1}{6}\right)^{1/2}$

Find the inverse of the function. *(p. 438)*

13. $f(x) = -\frac{1}{3}x + 5$
14. $f(x) = x^2 - 16, x \geq 0$
15. $f(x) = -\frac{2}{9}x^5$
16. $f(x) = 5x + 12$
17. $f(x) = -3x^3 - 4$
18. $f(x) = 9x^4 - 49, x \leq 0$

19. **GASOLINE COSTS** The cost (in dollars) of g gallons of gasoline can be modeled by $C(g) = 2.15g$. The amount of gasoline used by a car can be modeled by $g(d) = 0.02d$ where d is the distance (in miles) that the car has been driven. Find $C(g(d))$ and $C(g(400))$. What does $C(g(400))$ represent? *(p. 428)*

PROBLEM SOLVING

EXAMPLE 3
on p. 439
for Exs. 46–48

46. **EXCHANGE RATES** The *euro* is the unit of currency for the European Union. On a certain day, the number E of euros that could be obtained for D dollars was given by this function:

$$E = 0.81419D$$

Find the inverse of the function. Then use the inverse to find the number of dollars that could be obtained for 250 euros on that day.

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47. **MULTI-STEP PROBLEM** When calibrating a spring scale, you need to know how far the spring stretches for various weights. Hooke's law states that the length a spring stretches is proportional to the weight attached to it. A model for one scale is $\ell = 0.5w + 3$ where ℓ is the total length (in inches) of the stretched spring and w is the weight (in pounds) of the object.

    a. Find the inverse of the given model.

    b. If you place a weight on the scale and the spring stretches to a total length of 6.5 inches, how heavy is the weight?

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48. ★ **EXTENDED RESPONSE** At the start of a dog sled race in Anchorage, Alaska, the temperature was 5°C. By the end of the race, the temperature was −10°C. The formula for converting temperatures from degrees Fahrenheit F to degrees Celsius C is $C = \frac{5}{9}(F - 32)$.

    a. Find the inverse of the given model. *Describe* what information you can obtain from the inverse.

    b. Find the Fahrenheit temperatures at the start and end of the race.

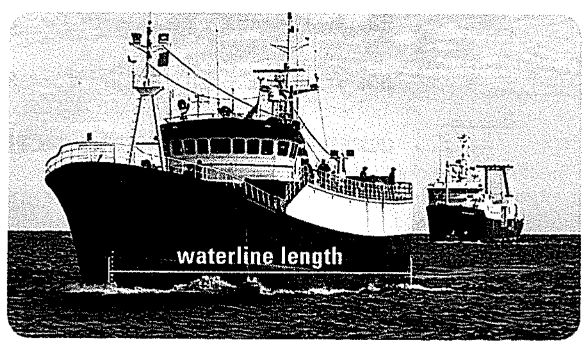
    c. Use a graphing calculator to graph the original function and its inverse. Find the temperature that is the same on both temperature scales.

EXAMPLES 6 and 7
on pp. 441–442
for Exs. 49–50

49. **BOAT SPEED** The maximum hull speed v (in knots) of a boat with a displacement hull can be approximated by

$$v = 1.34\sqrt{\ell}$$

where ℓ is the length (in feet) of the boat's waterline. Find the inverse of the model. Then find the waterline length needed to achieve a maximum speed of 7.5 knots.



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50. **BIOLOGY** The body surface area A (in square meters) of a person with a mass of 60 kilograms can be approximated by the model

$$A = 0.2195h^{0.3964}$$

where h is the person's height (in centimeters). Find the inverse of the model. Then estimate the height of a 60 kilogram person who has a body surface area of 1.6 square meters.

14. ★ **OPEN-ENDED MATH** Write a function f such that the graph of f^{-1} is a line with a slope of 3.

EXAMPLE 2
on p. 439
for Exs. 15–21

VERIFYING INVERSE FUNCTIONS **Verify that f and g are inverse functions.**

15. $f(x) = x + 4,\ g(x) = x - 4$

16. $f(x) = 2x + 3,\ g(x) = \frac{1}{2}x - \frac{3}{2}$

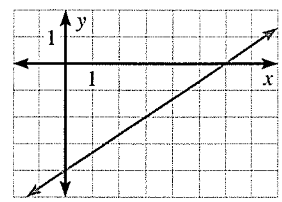
17. $f(x) = \frac{1}{4}x^3,\ g(x) = (4x)^{1/3}$

18. $f(x) = \frac{1}{5}x - 1,\ g(x) = 5x + 5$

19. $f(x) = 4x + 9,\ g(x) = \frac{1}{4}x - \frac{9}{4}$

20. $f(x) = 5x^2 - 2,\ x \ge 0;\ g(x) = \left(\frac{x+2}{5}\right)^{1/2}$

21. ★ **MULTIPLE CHOICE** What is the inverse of the function whose graph is shown?



Ⓐ $g(x) = \frac{3}{2}x - 6$ Ⓑ $g(x) = \frac{3}{2}x + 6$

Ⓒ $g(x) = \frac{2}{3}x - 6$ Ⓓ $g(x) = \frac{3}{2}x + 12$

EXAMPLE 4
on p. 440
for Exs. 22–28

INVERSES OF POWER FUNCTIONS **Find the inverse of the power function.**

22. $f(x) = x^7$

23. $f(x) = 4x^4,\ x \ge 0$

24. $f(x) = -10x^6,\ x \le 0$

25. $f(x) = 32x^5$

26. $f(x) = -\frac{2}{5}x^3$

27. $f(x) = \frac{16}{25}x^2,\ x \le 0$

28. ★ **MULTIPLE CHOICE** What is the inverse of $f(x) = -\frac{1}{64}x^3$?

Ⓐ $g(x) = -4x^3$ Ⓑ $g(x) = 4\sqrt[3]{x}$ Ⓒ $g(x) = -4\sqrt[3]{x}$ Ⓓ $g(x) = \sqrt[3]{-4x}$

EXAMPLE 5
on p. 441
for Exs. 29–43

HORIZONTAL LINE TEST **Graph the function f. Then use the graph to determine whether the inverse of f is a function.**

29. $f(x) = 3x + 1$

30. $f(x) = -x - 5$

31. $f(x) = \frac{1}{4}x^2 - 1$

32. $f(x) = -6x^2,\ x \ge 0$

33. $f(x) = \frac{1}{3}x^3$

34. $f(x) = x^3 - 2$

35. $f(x) = (x - 4)(x + 1)$

36. $f(x) = |x| + 4$

37. $f(x) = 4x^4 - 5x^2 - 6$

INVERSES OF NONLINEAR FUNCTIONS **Find the inverse of the function.**

38. $f(x) = \frac{3}{2}x^4,\ x \ge 0$

39. $f(x) = x^3 - 2$

40. $f(x) = \frac{3}{4}x^5 + 5$

41. $f(x) = -\frac{2}{5}x^6 + 8,\ x \le 0$

42. $f(x) = \frac{2x^3 - 6}{9}$

43. $f(x) = x^4 - 9,\ x \ge 0$

44. **REASONING** Determine whether the statement is *true* or *false*. *Explain* your reasoning.
    a. If $f(x) = x^n$ where n is a positive even integer, then the inverse of f is a function.
    b. If $f(x) = x^n$ where n is a positive odd integer, then the inverse of f is a function.

45. **CHALLENGE** Show that the inverse of any linear function $f(x) = mx + b$, where $m \neq 0$, is also a linear function. Give the slope and y-intercept of the graph of f^{-1} in terms of m and b.

EXAMPLE 7 Use an inverse power model to make a prediction

Use the inverse power model from Example 6 to predict the year when the average ticket price will reach $58.

Solution

$t = \left(\frac{P}{35}\right)^{5.2}$ **Write inverse power model.**

$= \left(\frac{58}{35}\right)^{5.2}$ **Substitute 58 for *P*.**

≈ 14 **Use a calculator.**

▸ You can predict that the average ticket price will reach $58 about 14 years after 1995, or in 2009.

✓ **GUIDED PRACTICE** for Examples 6 and 7

11. **TICKET PRICES** The average price P (in dollars) for a Major League Baseball ticket can be modeled by $P = 10.7t^{0.272}$ where t is the number of years since 1995. Write the inverse model. Then use the inverse to predict the year when the average ticket price will reach $25.

6.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 7, 15, and 49
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 21, 28, and 48

SKILL PRACTICE

1. **VOCABULARY** State the definition of an inverse relation.

2. ★ **WRITING** *Explain* how to determine whether a function g is an inverse of f.

EXAMPLE 1 on p. 438 for Exs. 3–13

INVERSE RELATIONS Find an equation for the inverse relation.

3. $y = 4x - 1$
4. $y = -2x + 5$
5. $y = 7x - 6$
6. $y = 10x - 28$
7. $y = 12x + 7$
8. $y = -18x - 5$
9. $y = 5x + \frac{1}{3}$
10. $y = -\frac{2}{3}x + 2$
11. $y = -\frac{3}{5}x + \frac{7}{5}$

ERROR ANALYSIS *Describe* and correct the error in finding the inverse of the relation.

12.
$$y = 6x - 11$$
$$x = 6y - 11$$
$$x + 11 = 6y$$
$$\frac{x}{6} + 11 = y$$

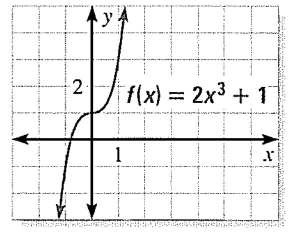
13.
$$y = -x + 3$$
$$-x = y + 3$$
$$-x - 3 = y$$

EXAMPLE 5 Find the inverse of a cubic function

Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

Solution

Graph the function f. Notice that no horizontal line intersects the graph more than once. So, the inverse of f is itself a function. To find an equation for f^{-1}, complete the following steps:



$f(x) = 2x^3 + 1$	**Write original function.**
$y = 2x^3 + 1$	**Replace $f(x)$ with y.**
$x = 2y^3 + 1$	**Switch x and y.**
$x - 1 = 2y^3$	**Subtract 1 from each side.**
$\frac{x-1}{2} = y^3$	**Divide each side by 2.**
$\sqrt[3]{\frac{x-1}{2}} = y$	**Take cube root of each side.**

▶ The inverse of f is $f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$.

✓ GUIDED PRACTICE for Examples 4 and 5

Find the inverse of the function. Then graph the function and its inverse.

5. $f(x) = x^6, x \geq 0$
6. $g(x) = \frac{1}{27}x^3$
7. $f(x) = -\frac{64}{125}x^3$
8. $f(x) = -x^3 + 4$
9. $f(x) = 2x^5 + 3$
10. $g(x) = -7x^5 + 7$

EXAMPLE 6 Find the inverse of a power model

TICKET PRICES The average price P (in dollars) for a National Football League ticket can be modeled by

$$P = 35t^{0.192}$$

where t is the number of years since 1995. Find the inverse model that gives time as a function of the average ticket price.

Solution

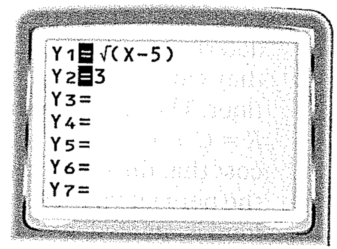
$P = 35t^{0.192}$	**Write original model.**
$\frac{P}{35} = t^{0.192}$	**Divide each side by 35.**
$\left(\frac{P}{35}\right)^{1/0.192} = \left(t^{0.192}\right)^{1/0.192}$	**Raise each side to the power $\frac{1}{0.192}$.**
$\left(\frac{P}{35}\right)^{5.2} \approx t$	**Simplify. This is the inverse model.**

EXAMPLE 2 Solve a radical inequality using a graph

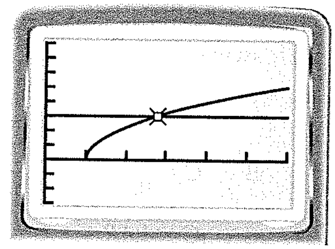
Use a graph to solve $\sqrt{x-5} > 3$.

Solution

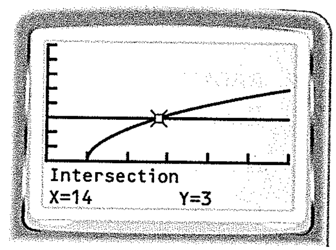
STEP 1 **Enter** the functions $y = \sqrt{x-5}$ and $y = 3$ into a graphing calculator.



STEP 2 **Graph** the functions from Step 1. Adjust the viewing window so that the x-axis shows $0 \le x \le 30$ with a scale of 5 and the y-axis shows $-3 \le y \le 8$ with a scale of 1.



STEP 3 **Identity** the x-values for which the graph of $y = \sqrt{x-5}$ lies above the graph of $y = 3$. You can use the *intersect* feature to show that the graphs intersect when $x = 14$. The graph of $y = \sqrt{x-5}$ lies above the graph of $y = 3$ when $x > 14$.



▶ The solution of the inequality is $x > 14$.

INTERPRET DOMAIN
In Example 2, note that the domain of $y = \sqrt{x-5}$ is $x \ge 5$. Therefore, the domain does not affect the solution.

PRACTICE

EXAMPLE 1
on p. 462
for Exs. 1–6

Use a table to solve the inequality.

1. $2\sqrt{x} - 5 \ge 3$
2. $\sqrt{x-4} \le 5$
3. $4\sqrt{x} + 1 \le 9$
4. $\sqrt{x+7} \ge 3$
5. $\sqrt{x} + \sqrt{x+3} \ge 3$
6. $\sqrt{x} + \sqrt{x-5} \le 5$

EXAMPLE 2
on p. 463
for Exs. 7–12

Use a graph to solve the inequality.

7. $2\sqrt{x} + 3 \le 8$
8. $\sqrt{x+3} \ge 2.6$
9. $7\sqrt{x} + 1 < 9$
10. $4\sqrt{3x-7} > 7.8$
11. $\sqrt{x} - \sqrt{x+5} < -1$
12. $\sqrt{x+2} + \sqrt{x-1} \le 9$

13. **SAILBOAT RACE** In order to compete in the America's Cup sailboat race, a boat must satisfy the rule

$$\ell + 1.25\sqrt{s} - 9.8\sqrt[3]{d} \le 16$$

where ℓ is the length (in meters) of the boat, s is the area (in square meters) of the sails, and d is the volume (in cubic meters) of water displaced by the boat. A boat has a length of 20 meters and displaces 27 cubic meters of water. What is the maximum allowable value for s?

6 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

• *n*th root of *a*, *p. 414*
• index of a radical, *p. 414*
• simplest form of a radical, *p. 422*
• like radicals, *p. 422*
• power function, *p. 428*
• composition, *p. 430*
• inverse relation, *p. 438*
• inverse function, *p. 438*
• radical function, *p. 446*
• radical equation, *p. 452*

VOCABULARY EXERCISES

1. Copy and complete: The index of the radical $\sqrt[4]{7}$ is _?_.
2. List two different pairs of like radicals.
3. Copy and complete: A(n) _?_ function has the form $y = ax^b$ where a is a real number and b is a rational number.
4. **WRITING** *Explain* how the graph of a function and the graph of its inverse are related.
5. **WRITING** *Explain* how to use the horizontal line test to determine whether the inverse of a function f is also a function.
6. **WRITING** *Describe* how the graph of $y = \sqrt[3]{x-4} + 5$ is related to the graph of the parent function $y = \sqrt[3]{x}$.
7. **REASONING** A student began solving the equation $x^{2/3} = 5$ by cubing each side. What will the student have to do next? What could the student have done to solve the equation in just one step?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Evaluate *n*th Roots and Use Rational Exponents *pp. 414–419*

EXAMPLE

Evaluate the expression.

a. $(\sqrt[4]{16})^5 = 2^5 = 32$

b. $27^{-4/3} = \frac{1}{27^{4/3}} = \frac{1}{(27^{1/3})^4} = \frac{1}{3^4} = \frac{1}{81}$

EXERCISES

EXAMPLE 2 on p. 415 for Exs. 8–15

Evaluate the expression without using a calculator.

8. $81^{1/4}$ **9.** $0^{1/3}$ **10.** $\sqrt[3]{-64}$ **11.** $\sqrt[3]{125}$

12. $256^{3/4}$ **13.** $27^{-2/3}$ **14.** $(\sqrt[3]{8})^7$ **15.** $\frac{1}{(\sqrt[5]{-32})^{-3}}$

6.2 Apply Properties of Rational Exponents

pp. 420–427

EXAMPLE

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = \sqrt[3]{8} \cdot \sqrt[3]{6} = 2\sqrt[3]{6}$

b. $\left(\frac{x^4}{y^8}\right)^{1/2} = \frac{(x^4)^{1/2}}{(y^8)^{1/2}} = \frac{x^{4 \cdot 1/2}}{y^{8 \cdot 1/2}} = \frac{x^2}{y^4}$

EXAMPLES 4, 6, and 7 on pp. 422–423 for Exs. 16–19

EXERCISES

Write the expression in simplest form. Assume all variables are positive.

16. $\sqrt[3]{80}$ **17.** $(3^4 \cdot 5^4)^{-1/4}$ **18.** $(25a^{10}b^{16})^{1/2}$ **19.** $\sqrt{\frac{18x^5y^4}{49xz^3}}$

6.3 Perform Function Operations and Composition

pp. 428–434

EXAMPLE

Let $f(x) = 3x^2 + 1$ and $g(x) = x + 4$. Perform the indicated operation.

a. $f(x) + g(x) = (3x^2 + 1) + (x + 4) = 3x^2 + x + 5$

b. $f(x) \cdot g(x) = (3x^2 + 1)(x + 4) = 3x^3 + 12x^2 + x + 4$

c. $f(g(x)) = f(x + 4) = 3(x + 4)^2 + 1 = 3(x^2 + 8x + 16) + 1 = 3x^2 + 24x + 49$

EXAMPLES 1, 2, and 5 on pp. 428–430 for Exs. 20–23

EXERCISES

Let $f(x) = 4x - 6$ and $g(x) = x + 8$. Perform the indicated operation.

20. $f(x) + g(x)$ **21.** $f(x) - g(x)$ **22.** $f(x) \cdot g(x)$ **23.** $f(g(x))$

6.4 Use Inverse Functions

pp. 438–445

EXAMPLE

Find the inverse of the function $y = 3x + 7$.

$y = 3x + 7$	**Write original function.**
$x = 3y + 7$	**Switch x and y.**
$x - 7 = 3y$	**Subtract 7 from each side.**
$\frac{1}{3}x - \frac{7}{3} = y$	**Divide each side by 3.**

EXAMPLES 1, 4, and 5 on pp. 438–441 for Exs. 24–26

EXERCISES

Find the inverse of the function.

24. $y = \frac{1}{3}x + 4$ **25.** $y = 4x^2 + 9, x \geq 0$ **26.** $f(x) = x^3 - 4$

MULTIPLE CHOICE QUESTIONS

If you have difficulty solving a multiple choice problem directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. The surface area of a sphere is given by $S = 4\pi r^2$. Which choice correctly expresses S as a function of V?

A $4\pi V^2$ B $\sqrt[3]{\frac{9V^2}{4\pi}}$ C $\sqrt[3]{36\pi V^2}$ D $\sqrt[3]{\frac{3V^2}{4\pi}}$

METHOD 1

SOLVE DIRECTLY Solve for r in terms of V and substitute this expression in the formula for S.

STEP 1 Solve for r in terms of V.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{3}{4\pi}V = r^3$$

$$r = \sqrt[3]{\frac{3}{4\pi}V}$$

STEP 2 Substitute the above expression for r in the formula for S.

$$S = 4\pi r^2$$

$$r = \sqrt[3]{\frac{3}{4\pi}V}$$

STEP 3 Simplify the expression into a form similar to the answer choices.

$$S = 4\pi\left(\sqrt[3]{\frac{3}{4\pi}V}\right)^2 = 4\pi\sqrt[3]{\frac{9V^2}{4^2\pi^2}}$$

$$= \sqrt[3]{\frac{\quad}{4^2\pi^2}} = \sqrt[3]{36\pi V^2}$$

▶ The correct answer is C.

METHOD 2

ELIMINATE CHOICES Choose a value of r that will produce associated values of V and S. Then check the answer choices.

STEP 1 Compute the volume and surface area for a single value of r, such as $r = 2$.

Volume: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 \approx 33.51$

Surface Area: $S = 4\pi r^2 = 4\pi(2)^2 \approx 50.27$

STEP 2 Substitute $V \approx 33.51$ into the answer choices. The correct choice will yield a surface area of 50.27.

Choice A: $4\pi V^2 = 4\pi(33.51)^2 \approx 14{,}111 \neq 50.27$

Choice B: $\sqrt[3]{\frac{9V^2}{4\pi}} = \sqrt[3]{\frac{9(33.51)^2}{4\pi}} \approx 9.30 \neq 50.27$

Choice C: $\sqrt[3]{36\pi V^2} = \sqrt[3]{36\pi(50.27)} \approx 50.27$

Choice D: $\sqrt[3]{\frac{3V^2}{4\pi}} = \sqrt[3]{\frac{3(33.51)^2}{4\pi}} \approx 6.45 \neq 50.27$

▶ The correct answer is C.

PROBLEM 2

Find all solutions of the equation $\sqrt{2x-1}+3=6$.

A 1 B 2 C 4 D 5

METHOD 1

SOLVE DIRECTLY Solve the equation for x.

STEP 1 Isolate x.

$$\sqrt{2x-1}+3=6$$
$$\sqrt{2x-1}=3$$
$$2x-1=9$$
$$2x=10$$
$$x=5$$

STEP 2 **Check,** in order to avoid including an extraneous root.

$$\begin{aligned}\sqrt{2x-1}+3 &= \sqrt{2(5)-1}+3\\ &= \sqrt{9}+3\\ &= 3+3\\ &= 6\end{aligned}$$

▸ The correct answer is D.

METHOD 2

ELIMINATE CHOICES Another method is to test which value of x satisfies the equation.

Choice A: $$\begin{aligned}\sqrt{2x-1}+3 &= \sqrt{2(1)-1}+3\\ &= \sqrt{1}+3\\ &= 1+3\\ &\neq 6\end{aligned}$$

Choice B: $$\begin{aligned}\sqrt{2x-1}+3 &= \sqrt{2(2)-1}+3\\ &= \sqrt{3}+3\\ &\neq 6\end{aligned}$$

Choice C: $$\begin{aligned}\sqrt{2x-1}+3 &= \sqrt{2(4)-1}+3\\ &= \sqrt{7}+3\\ &\neq 6\end{aligned}$$

Choice D: $$\begin{aligned}\sqrt{2x-1}+3 &= \sqrt{2(5)-1}+3\\ &= \sqrt{9}+3\\ &= 3+3\\ &= 6\end{aligned}$$

▸ The correct answer is D.

TEST PREPARATION

PRACTICE

***Explain* why you can eliminate the highlighted answer choice.**

1. What is the solution of the equation $\sqrt{2x-3}-3=6$?

A 0 B 6 C 39 D 42

2. Which expression is equivalent to $(9x^{3/2})^{-1/2}$?

A $\dfrac{9}{2x^{3/4}}$ B $\dfrac{1}{3x^{3/4}}$ C $9x^3$ D $3x^{3/4}$

TEST PREPARATION

MULTIPLE CHOICE

1. Simplify the expression $4^{1/2} \cdot 2^4$.

   A 16 B $8^{4^{1/2}}$

   C 32 D 64

2. If $f(x) = 2x^{-3/2}$, then $f(0.25)$ equals what?

   A -8 B -4

   C -0.25 D 16

3. What is the solution of the equation $(8x)^{3/5} = 8$?

   A $8^{-2/5}$ B 1

   C $8^{2/5}$ D 4

4. Most carnivorous dinosaurs, called theropods, walked on 2 legs. The height at the hip of a theropod can be modeled by the function $h(\ell) = 3.49\ell^{1.14}$, where ℓ is the length (in centimeters) of the dinosaur's instep.

   The length of the instep can be modeled by $\ell(p) = 1.2p$, where p is the footprint length (in centimeters). Which expression represents h in terms of p?

   A $4.188p$ B $4.188p^{1.14}$

   C $4.30p$ D $4.30p^{1.14}$

5. In the equation $\sqrt[4]{16x^3y^4z} = 2x^ayz^b$, what is the sum of a and b? Assume all variables are positive.

   A $\frac{1}{2}$ B 1

   C $\frac{7}{4}$ D 4

6. The expression $\frac{5y^2 - 15y}{3y^2 - y^3}$ is equivalent to:

   A $-\frac{5}{y}$ B $\frac{5}{y}$

   C $\frac{10}{3}$ D $\frac{5}{3} - \frac{15}{y^2}$

7. What is the value of $\frac{1}{(\sqrt[4]{625})^{-2}}$?

   A $\frac{1}{25}$ B $\frac{1}{5}$

   C 25 D $78{,}125$

8. Let $f(x) = \frac{2}{5}x^{-0.25}$ and $g(x) = 5x^{3.25}$. Approximated to three decimal places, what is the value of $f(x) \cdot g(x)$ when $x = 3$?

   A 0.054 B 0.110

   C 54.00 D 93.531

9. What is the y-intercept of the graph of the function $y = \sqrt[3]{x - 8}$?

   A -8 B -2

   C 2 D 8

10. Let $f(x) = \sqrt{x - 2} + 3$ and $g(x) = \sqrt{x + 13}$. For what value of x does $f(x) = g(x)$?

    A -4 B -3

    C 3 D 9

11. Consider the function $y = 3x + 9$. What is the slope of the graph of the inverse function?

    A -3 B $-\frac{1}{3}$

    C $\frac{1}{3}$ D 3

12. The expression $(k^{-1/2})\sqrt[3]{k^4}$ is equivalent to:

    A $k^{-2/3}$ B k^3

    C $k^{3/5}$ D $k^{5/6}$

13. Simplify $(-4x^3)(5x^4)$.

    A $-20x^{12}$ B $-2x^{12}$

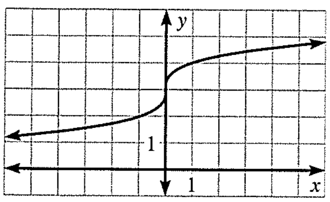
    C $-20x^7$ D $-2x^7$

MULTIPLE CHOICE

14. What is the solution of the equation $\sqrt{2x+3} = 5$?

A -14 B -4
C 1 D 11

15. The graph of which function is shown?



A $y = \sqrt[3]{x+3}$ B $y = \sqrt[3]{x-3}$
C $y = \sqrt[3]{x} + 3$ D $y = \sqrt[3]{x} - 3$

16. What is the inverse of $y = -2x^5 + 10$?

A $y = \sqrt[5]{5 - \frac{1}{2}x}$ B $y = 20 - \sqrt[5]{2x}$
C $y = \sqrt[5]{2x} - 20$ D $y = \sqrt[5]{\frac{1}{2}x - 5}$

17. Let $f(x) = 2x^{1/2}$ and $g(x) = 4x^2$. What is the value of $g(f(9))$?

A -144 B -36
C 36 D 144

18. What is the value of $\left(\frac{5^5}{2^5}\right)^{-1/5}$?

A $\frac{1}{160}$ B $\frac{1}{10}$
C $\frac{2}{5}$ D $\frac{5}{2}$

19. Which expression is equivalent to $\sqrt{9x^2y^3}$ for *all* real numbers x and y?

A $3xy^{3/2}$
B $3|x|y^{3/2}$
C $3|x|(|y|)^{3/2}$
D all of the above

OPEN-ENDED

20. Consider the following equation:
$$-\sqrt{x+4} = \sqrt{x-1} + 1$$
A. Solve the equation algebraically.
B. Solve the equation by graphing.
C. Are the results the same? *Explain* why or why not.

21. You have a \$10 gift card to spend at a local toy store. The store has a sale offering 15% off all board games.
A. Use composition of functions to find the final price of a board game that originally costs \$27 when the \$10 is subtracted before the 15% discount is applied.
B. Use composition of functions to find the final price of the board game when the 15% discount is applied before the \$10 is subtracted.
C. How much more money can you save if the store applies the 15% discount first?

CUMULATIVE REVIEW Chapters 1–6

Write an equation of the line that passes through the given point and has the given slope. *(p. 98)*

1. $(3, 1), m = 4$ **2.** $(4, 6), m = 7$ **3.** $(-3, 2), m = -8$

4. $(1, -5), m = 9$ **5.** $(-5, 8), m = \frac{4}{5}$ **6.** $(2, -10), m = -\frac{3}{4}$

Solve the equation. Check your solution(s).

7. $-2x + 7 = 15$ *(p. 18)* **8.** $|4x - 6| = 14$ *(p. 51)* **9.** $x^2 - 9x + 14 = 0$ *(p. 252)*

10. $4x^2 - 6x + 9 = 0$ *(p. 292)* **11.** $x^3 + 3x^2 - 10x = 0$ *(p. 353)* **12.** $\sqrt{8x + 1} = 7$ *(p. 452)*

Graph the equation or inequality in a coordinate plane.

13. $y = 3x - 5$ *(p. 89)* **14.** $y = -|x + 4| + 3$ *(p. 123)* **15.** $y < -2x + 5$ *(p. 132)*

16. $y = x^2 - 2x - 4$ *(p. 236)* **17.** $y = 2(x - 6)^2 - 5$ *(p. 245)* **18.** $y > x^2 + 2x + 1$ *(p. 300)*

19. $y = x^3 - 2$ *(p. 337)* **20.** $y = 3(x + 2)(x - 1)^2$ *(p. 387)* **21.** $y = -\sqrt{x - 2} + 4$ *(p. 446)*

Solve the system of linear equations using any method.

22. $2x + 5y = 1$ *(p. 160)*
$3x - 2y = 30$

23. $3x - y = -9$ *(p. 160)*
$4x + 3y = 14$

24. $2x + 3y = 47$ *(p. 178)*
$7x - 8y = -2$
$2x - y + 3z = -19$

Write the expression as a complex number in standard form. *(p. 275)*

25. $(4 - 2i) + (5 + i)$ **26.** $(3 + 4i) - (7 + 2i)$ **27.** $(4 - 2i)(6 + 5i)$

Write the quadratic function in vertex form by completing the square. *(p. 284)*

28. $y = x^2 + 6x + 16$ **29.** $y = -x^2 + 12x - 46$ **30.** $y = 2x^2 - 4x + 7$

Simplify the expression. Assume all variables are positive.

31. $(2x^3y^2)^3$ *(p. 330)* **32.** $(x^8)^{-3/4}$ *(p. 420)* **33.** $\frac{x^3y^{-4}}{x^{-4}y^{-5}}$ *(p. 330)* **34.** $\left(\frac{x^2y^{1/3}}{x^{1/4}y}\right)^2$ *(p. 420)*

Perform the indicated operation.

35. $(x^2 + 11x - 9) + (4x^2 - 5x - 7)$ *(p. 346)* **36.** $(x^3 + 3x - 10) - (2x^3 + 3x^2 + 8x)$ *(p. 346)*

37. $(2x - 5)(x^2 + 4x - 7)$ *(p. 346)* **38.** $(x^3 - 10x^2 + 33x - 28) \div (x - 5)$ *(p. 362)*

Factor the polynomial completely. *(p. 353)*

39. $x^4 - 3x^2 - 40$ **40.** $x^3 - 125$ **41.** $x^3 - 6x^2 - 9x + 54$

Let $f(x) = 2x - 6$ and $g(x) = 5x + 1$. Perform the indicated operation and state the domain. *(p. 428)*

42. $f(x) + g(x)$ **43.** $f(x) \cdot g(x)$ **44.** $f(g(x))$ **45.** $g(f(x))$

Find the inverse of the function. *(p. 438)*

46. $f(x) = 4x + 6$ **47.** $f(x) = \frac{3}{7}x + 7$ **48.** $f(x) = \frac{1}{3}x - \frac{2}{3}$

49. $f(x) = \frac{x^3 - 5}{6}$ **50.** $f(x) = \sqrt[3]{\frac{2x + 7}{3}}$ **51.** $f(x) = -\frac{8}{9}x^5 + 2$