7 Exponential and Logarithmic Functions

PA

M11.D.1.1.3 7.1 Graph Exponential Growth Functions

M11.D.1.1.3 7.2 Graph Exponential Decay Functions

M11.D.1.1.3 7.3 Use Functions Involving e

M11.D.1.1.3 7.4 Evaluate Logarithms and Graph Logarithmic Functions

7.5 Apply Properties of Logarithms

7.6 Solve Exponential and Logarithmic Equations

7.7 Write and Apply Exponential and Power Functions

Before

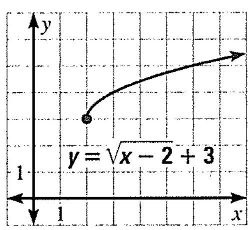
In previous chapters, you learned the following skills, which you'll use in Chapter 7: graphing functions, finding inverse functions, and writing functions.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement using the graph at the right.

1. The **domain** of the function is _?_.
2. The **range** of the function is _?_.
3. The **inverse** of the function is _?_.



SKILLS CHECK

Graph the function. State the domain and range. ***(Review p. 446 for 7.1–7.3.)***

4. $y = -2\sqrt{x} - 1$ 5. $y = \sqrt{x+3}$ 6. $y = \sqrt[3]{x-2} + 5$

Find the inverse of the function. ***(Review p. 438 for 7.4.)***

7. $y = 3x + 5$ 8. $y = -2x^3 + 1$ 9. $y = \frac{1}{2}x^2, x \geq 0$

Write a quadratic function in standard form for the parabola that passes through the given points. ***(Review p. 309 for 7.7.)***

10. $(0, -1), (1, 2), (3, 14)$ 11. $(3, 8), (4, 17), (7, 56)$ 12. $(-3, 9), (1, -7), (5, -55)$

52. **BICYCLE COSTS** You want to buy a bicycle that costs \$360. In order to pay for the bicycle, you save \$30 per week. How many weeks will it take to save enough money to buy the bicycle? *(p. 34)*

53. **CHARITABLE DONATIONS** The table below shows the amounts of money (in millions of dollars) received by a charitable organization during the first 6 years of its existence. Approximate the best-fitting line for the data. Then use the best-fitting line to predict the amount of money the organization will receive in the eighth year of its existence. *(p. 113)*

Year	1	2	3	4	5	6
Donations (millions of dollars)	1.71	2.3	2.78	3.22	3.69	4.28

54. **ICE SHOW** The attendance at an ice show was 9800 people. The tickets for the ice show were \$35 for lower-level seats and \$25 for upper-level seats. The total income from ticket sales was \$280,000. Use a linear system to find the numbers of lower-level and upper-level tickets sold for the ice show. *(p. 160)*

55. **CONCERT TICKETS** Tickets to a school's band concert are \$4 for students, \$8 for adults, and \$6 for senior citizens. At Friday night's concert, there were 140 students, 170 adults, and 55 senior citizens in attendance. At Saturday night's concert, there were 126 students, 188 adults, and 64 senior citizens in attendance. Organize this information using matrices. Then use matrix multiplication to find the income from ticket sales for Friday and Saturday nights' concerts. *(p. 195)*

56. **PHYSICAL SCIENCE** While standing at the edge of a cliff, you drop a rock from a height of 85 feet above the ground. Write an equation giving the height h (in feet) of the rock above the ground after t seconds. How long does it take for the rock to hit the ground? *(p. 266)*

57. **BASEBALL** Three points on the parabola formed by throwing a baseball are (0, 6), (20, 56), and (36, 24). Write a quadratic function that models the baseball's path. *(p. 309)*

58. **MANUFACTURING** At a factory, molten plastic is poured into molds to make toy blocks. Each mold is a rectangular prism with a height that is 3 inches greater than the length of each side of the square base. A machine pours 200 cubic inches of liquid plastic into each mold. What are the dimensions of a mold? *(p. 370)*

59. **PROFIT** Your friend starts a housekeeping business. The table below shows the profit (in dollars) of the business during the first 6 months of its existence. Use a graphing calculator to find a polynomial model for the data. Predict the profit in the ninth month. *(p. 393)*

Month	1	2	3	4	5	6
Profit (dollars)	2	4	18	50	106	192

60. **GEOMETRY** You have a beach ball that has a volume of approximately 7240 cubic inches. Find the radius of the beach ball. (*Hint:* Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere.) *(p. 414)*

Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 538. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing exponential and logarithmic functions**
2. **Solving exponential and logarithmic equations**
3. **Writing and applying exponential and power functions**

KEY VOCABULARY

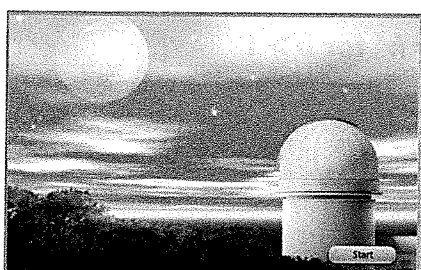
- exponential function, *p. 478*
- exponential growth function, *p. 478*
- growth factor, *p. 478*
- asymptote, *p. 478*
- exponential decay function, *p. 486*
- decay factor, *p. 486*
- natural base *e*, *p. 492*
- logarithm of *y* with base *b*, *p. 499*
- common logarithm, *p. 500*
- natural logarithm, *p. 500*
- exponential equation, *p. 515*
- logarithmic equation, *p. 517*

Why?

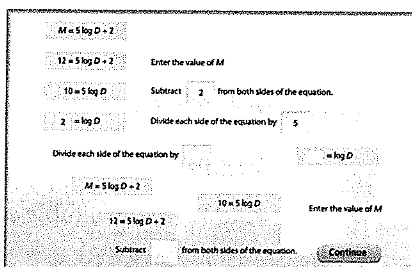
You can use exponential and logarithmic functions to model many scientific relationships. For example, you can use a logarithmic function to relate the size of a telescope lens and the ability of the telescope to see certain stars.

Animated Algebra

The animation illustrated below for Example 7 on page 519 helps you answer this question: How is the diameter of a telescope's objective lens related to the apparent magnitude of the dimmest star that can be seen with the telescope?



The magnitude of stars is a measure of their brightness as viewed from Earth.



Solve to find the diameter of a telescope that reveals stars of a given magnitude.

Animated Algebra at classzone.com

Other animations for Chapter 7: pages 480, 487, 502, and 538

7.1 Graph Exponential Growth Functions

PA M11.D.1.1.3 Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

Before You graphed polynomial and radical functions.

Now You will graph and use exponential growth functions.

Why? So you can model sports equipment costs, as in Ex. 40.

Key Vocabulary
- exponential function
- exponential growth function
- growth factor
- asymptote

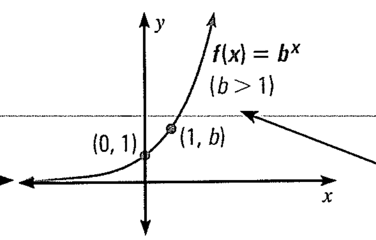
An **exponential function** has the form $y = ab^x$ where $a \neq 0$ and the base b is a positive number other than 1. If $a > 0$ and $b > 1$, then the function $y = ab^x$ is an **exponential growth function**, and b is called the **growth factor**. The simplest type of exponential growth function has the form $y = b^x$.

KEY CONCEPT *For Your Notebook*

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.

The x-axis is an asymptote of the graph. An asymptote is a line that a graph approaches more and more closely.



The graph rises from left to right, passing through the points (0, 1) and (1, b).

The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 Graph $y = b^x$ for $b > 1$

Graph $y = 2^x$.

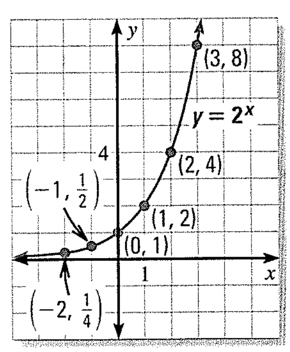
Solution

STEP 1 **Make** a table of values.

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

STEP 2 **Plot** the points from the table.

STEP 3 **Draw,** from *left* to *right*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.



The graph of a function $y = ab^x$ is a vertical stretch or shrink of the graph of $y = b^x$. The y-intercept of the graph of $y = ab^x$ occurs at $(0, a)$ rather than $(0, 1)$.

EXAMPLE 2 Graph $y = ab^x$ for $b > 1$

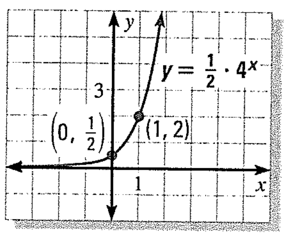
Graph the function.

a. $y = \frac{1}{2} \cdot 4^x$

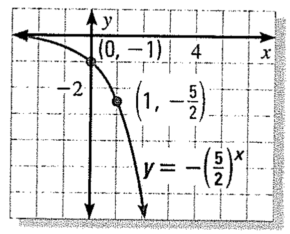
b. $y = -\left(\frac{5}{2}\right)^x$

Solution

a. Plot $\left(0, \frac{1}{2}\right)$ and $(1, 2)$. Then, from *left* to *right*, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the right.



b. Plot $(0, -1)$ and $\left(1, -\frac{5}{2}\right)$. Then, from *left* to *right*, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the right.



CLASSIFY FUNCTIONS
Note that the function in part (b) of Example 2 is not an exponential growth function because $a = -1 < 0$.

TRANSLATIONS To graph a function of the form $y = ab^{x-h} + k$, begin by sketching the graph of $y = ab^x$. Then translate the graph horizontally by h units and vertically by k units.

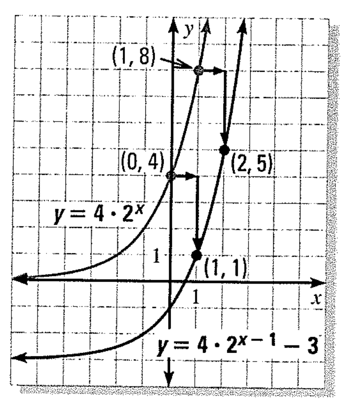
EXAMPLE 3 Graph $y = ab^{x-h} + k$ for $b > 1$

Graph $y = 4 \cdot 2^{x-1} - 3$. State the domain and range.

Solution

Begin by sketching the graph of $y = 4 \cdot 2^x$, which passes through $(0, 4)$ and $(1, 8)$. Then translate the graph right 1 unit and down 3 units to obtain the graph of $y = 4 \cdot 2^{x-1} - 3$.

The graph's asymptote is the line $y = -3$. The domain is all real numbers, and the range is $y > -3$.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function. State the domain and range.

1. $y = 4^x$

2. $y = \frac{1}{2} \cdot 3^x$

3. $f(x) = 3^{x+1} + 2$

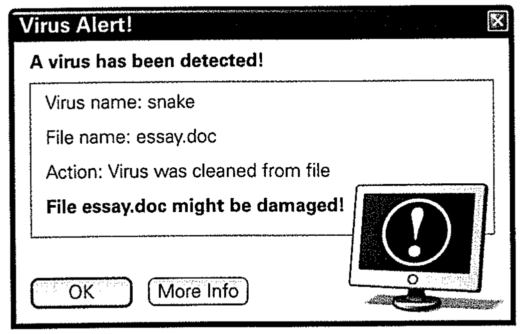
EXPONENTIAL GROWTH MODELS When a real-life quantity increases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$y = a(1 + r)^t$$

where a is the initial amount and r is the percent increase expressed as a decimal. Note that the quantity $1 + r$ is the growth factor.

EXAMPLE 4 Solve a multi-step problem

COMPUTERS In 1996, there were 2573 computer viruses and other computer security incidents. During the next 7 years, the number of incidents increased by about 92% each year.



- Write an exponential growth model giving the number n of incidents t years after 1996. About how many incidents were there in 2003?
- Graph the model.
- Use the graph to estimate the year when there were about 125,000 computer security incidents.

Solution

STEP 1 The initial amount is $a = 2573$ and the percent increase is $r = 0.92$. So, the exponential growth model is:

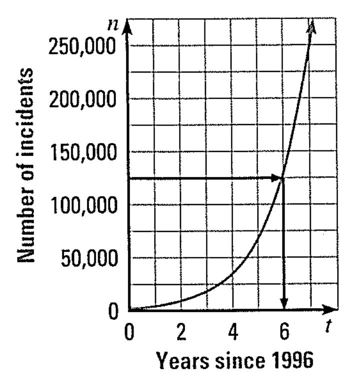
$n = a(1 + r)^t$ **Write exponential growth model.**

$= 2573(1 + 0.92)^t$ **Substitute 2573 for *a* and 0.92 for *r*.**

$= 2573(1.92)^t$ **Simplify.**

AVOID ERRORS
Notice that the percent increase and the growth factor are two different values. An increase of 92% corresponds to a growth factor of 1.92.

Using this model, you can estimate the number of incidents in 2003 ($t = 7$) to be $n = 2573(1.92)^7 \approx 247{,}485$.



STEP 2 The graph passes through the points (0, 2573) and (1, 4940.16). Plot a few other points. Then draw a smooth curve through the points.

STEP 3 Using the graph, you can estimate that the number of incidents was about 125,000 during 2002 ($t \approx 6$).

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✓ GUIDED PRACTICE for Example 4

4. **WHAT IF?** In Example 4, estimate the year in which there were about 250,000 computer security incidents.

5. In the exponential growth model $y = 527(1.39)^x$, identify the initial amount, the growth factor, and the percent increase.

COMPOUND INTEREST Exponential growth functions are used in real-life situations involving *compound interest.* Compound interest is interest paid on the initial investment, called the *principal,* and on previously earned interest. Interest paid only on the principal is called *simple interest.*

KEY CONCEPT *For Your Notebook*

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

EXAMPLE 5 Find the balance in an account

FINANCE You deposit \$4000 in an account that pays 2.92% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. Quarterly

b. Daily

Solution

a. With interest compounded quarterly, the balance after 1 year is:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	**Write compound interest formula.**
$= 4000\left(1 + \frac{0.0292}{4}\right)^{4 \cdot 1}$	$P = 4000, r = 0.0292, n = 4, t = 1$
$= 4000(1.0073)^4$	**Simplify.**
≈ 4118.09	**Use a calculator.**

▸ The balance at the end of 1 year is \$4118.09.

b. With interest compounded daily, the balance after 1 year is:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	**Write compound interest formula.**
$= 4000\left(1 + \frac{0.0292}{365}\right)^{365 \cdot 1}$	$P = 4000, r = 0.0292, n = 365, t = 1$
$= 4000(1.00008)^{365}$	**Simplify.**
≈ 4118.52	**Use a calculator.**

▸ The balance at the end of 1 year is \$4118.52.

✓ **GUIDED PRACTICE** for Example 5

6. **FINANCE** You deposit \$2000 in an account that pays 4% annual interest. Find the balance after 3 years if the interest is compounded daily.

7.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS on p. WS13 for Exs. 17, 29, and 37**

★ = **STANDARDIZED TEST PRACTICE Exs. 2, 24, 25, 32, 40, and 41**

◆ = **MULTIPLE REPRESENTATIONS Ex. 42**

SKILL PRACTICE

1. **VOCABULARY** In the exponential growth model $y = 2.4(1.5)^x$, identify the initial amount, the growth factor, and the percent increase.

2. ★ **WRITING** What is an asymptote?

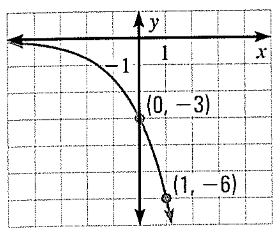
EXAMPLES 1 and 2 on pp. 478–479 for Exs. 3–14

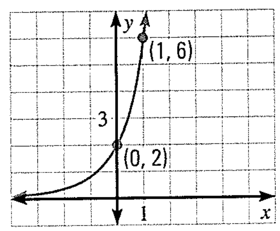
MATCHING GRAPHS Match the function with its graph.

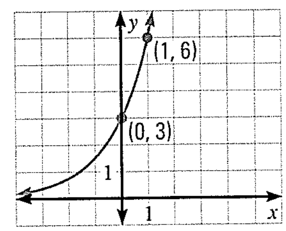
3. $y = 3 \cdot 2^x$

4. $y = -3 \cdot 2^x$

5. $y = 2 \cdot 3^x$

A.


B.


C.


GRAPHING FUNCTIONS Graph the function.

6. $y = 3^x$

7. $y = -2^x$

8. $f(x) = 5 \cdot 2^x$

9. $y = 5^x$

10. $y = 2 \cdot 4^x$

11. $g(x) = -(1.5)^x$

12. $y = 3\left(\frac{4}{3}\right)^x$

13. $y = \frac{1}{2} \cdot 3^x$

14. $h(x) = -2(2.5)^x$

EXAMPLE 3 on p. 479 for Exs. 15–24

TRANSLATING GRAPHS Graph the function. State the domain and range.

15. $y = -3 \cdot 2^{x+2}$

16. $y = 5 \cdot 4^x + 2$

17. $y = 2^{x+1} + 3$

18. $y = 3^{x-2} - 1$

19. $y = 2 \cdot 3^{x-2} - 1$

20. $y = -3 \cdot 4^{x-1} - 2$

21. $f(x) = 6 \cdot 2^{x-3} + 3$

22. $g(x) = 5 \cdot 3^{x+2} - 4$

23. $h(x) = -2 \cdot 5^{x-1} + 1$

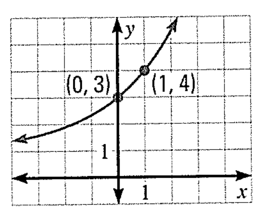
24. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $f(x) = 2(1.5)^x - 1$

Ⓑ $f(x) = 2(1.5)^x + 1$

Ⓒ $f(x) = 3(1.5)^x - 1$

Ⓓ $f(x) = 3(1.5)^x + 1$



25. ★ **MULTIPLE CHOICE** The student enrollment E of a high school was 1310 in 1998 and has increased by 10% per year since then. Which exponential growth model gives the school's student enrollment in terms of t, where t is the number of years since 1998?

Ⓐ $E = 0.1(1310)^t$

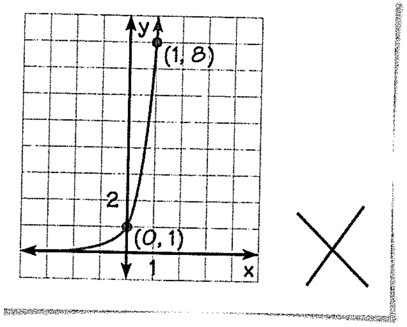
Ⓑ $E = 1310(0.1)^t$

Ⓒ $E = 1.1(1310)^t$

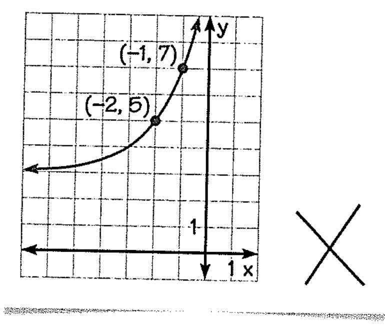
Ⓓ $E = 1310(1.1)^t$

ERROR ANALYSIS *Describe* **and correct the error in graphing the function.**

26. $y = 2 \cdot 4^x$



27. $y = 2^{x-3} + 3$



WRITING MODELS In Exercises 28–30, write an exponential growth model that describes the situation.

28. In 1992, 1219 monk parakeets were observed in the United States. For the next 11 years, about 12% more parakeets were observed each year.

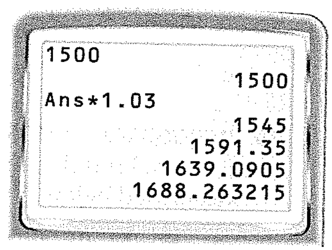
29. You deposit \$800 in an account that pays 2% annual interest compounded daily.

30. You purchase an antique table for \$450. The value of the table increases by 6% per year.

31. **GRAPHING CALCULATOR** You deposit \$1500 in a bank account that pays 3% annual interest compounded yearly.

    a. Type 1500 into a graphing calculator and press ENTER. Then enter the formula ANS * 1.03, as shown at the right. Press ENTER seven times to find your balance after 7 years.

    b. Find the number of years it takes for your balance to exceed \$2500.



32. ★ **OPEN-ENDED MATH** Write an exponential function of the form $y = ab^{x-h} + k$ whose graph has a y-intercept of 5 and an asymptote of $y = 2$.

33. **GRAPHING CALCULATOR** Consider the exponential growth function $y = ab^{x-h} + k$ where $a = 2$, $b = 5$, $h = -4$, and $k = 3$. Predict the effect on the function's graph of each change in a, b, h, or k described in parts (a)–(d). Use a graphing calculator to check your prediction.

    a. a changes to 1 b. b changes to 4 c. h changes to 3 d. k changes to -1

34. **CHALLENGE** Consider the exponential function $f(x) = ab^x$.

    a. Show that $\dfrac{f(x+1)}{f(x)} = b$.

    b. Use the result from part (a) to explain why there is no exponential function of the form $f(x) = ab^x$ whose graph passes through the points in the table below.

x	0	1	2	3	4
y	4	4	8	24	72

PROBLEM SOLVING

EXAMPLE 4
on p. 480
for Exs. 35–36

35. DVD PLAYERS From 1997 to 2002, the number n (in millions) of DVD players sold in the United States can be modeled by $n = 0.42(2.47)^t$ where t is the number of years since 1997.

a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function. Estimate the number of DVD players sold in 2001.

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36. INTERNET Each March from 1998 to 2003, a website recorded the number y of referrals it received from Internet search engines. The results can be modeled by $y = 2500(1.50)^t$ where t is the number of years since 1998.

a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function and state the domain and range. Estimate the number of referrals the website received from Internet search engines in March of 2002.

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EXAMPLE 5
on p. 481
for Exs. 37–38

37. ACCOUNT BALANCE You deposit \$2200 in a bank account. Find the balance after 4 years for each of the situations described below.

a. The account pays 3% annual interest compounded quarterly.

b. The account pays 2.25% annual interest compounded monthly.

c. The account pays 2% annual interest compounded daily.

38. DEPOSITING FUNDS You want to have \$3000 in your savings account after 3 years. Find the amount you should deposit for each of the situations described below.

a. The account pays 2.25% annual interest compounded quarterly.

b. The account pays 3.5% annual interest compounded monthly.

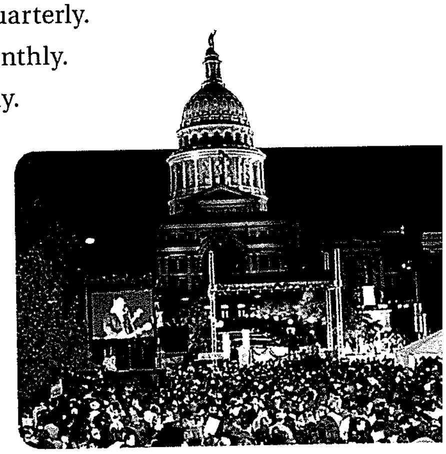
c. The account pays 4% annual interest compounded yearly.

39. MULTI-STEP PROBLEM In 1990, the population of Austin, Texas, was 494,290. During the next 10 years, the population increased by about 3% each year.

a. Write a model giving the population P (in thousands) of Austin t years after 1990. What was the population in 2000?

b. Graph the model and state the domain and range.

c. Estimate the year when the population was about 590,000.



Austin, Texas

40. ★ SHORT RESPONSE At an online auction, the opening bid for a pair of in-line skates is \$50. The price of the skates increases by 10.5% per bid during the next 6 bids.

a. Write a model giving the price p (in dollars) of the skates after n bids.

b. What was the price after 5 bids? According to the model, what will the price be after 100 bids? Is this predicted price reasonable? *Explain.*

41. ★ **EXTENDED RESPONSE** In 2000, the average price of a football ticket for a Minnesota Viking's game was $48.28. During the next 4 years, the price increased an average of 6% each year.

a. Write a model giving the average price p (in dollars) of a ticket t years after 2000.

b. Graph the model. Estimate the year when the average price of a ticket was about $60.

c. *Explain* how you can use the graph of $p(t)$ to determine the minimum and maximum t-values in the domain for which the function gives meaningful results.

42. ◆ **MULTIPLE REPRESENTATIONS** In 1977, there were 41 breeding pairs of bald eagles in Maryland. Over the next 24 years, the number of breeding pairs increased by about 8.9% each year.

a. **Writing an Equation** Write a model giving the number n of breeding pairs of bald eagles in Maryland t years after 1977.

b. **Making a Table** Make a table of values for the model.

c. **Drawing a Graph** Graph the model.

d. **Using a Graph** About how many breeding pairs of bald eagles were in Maryland in 2001?

43. **REASONING** Is investing $3000 at 6% annual interest and $3000 at 8% annual interest equivalent to investing $6000 (the total of the two principals) at 7% annual interest (the average of the two interest rates)? *Explain.*

44. **CHALLENGE** The yearly cost for residents to attend a state university has increased from $5200 to $9000 in the last 5 years.

a. To the nearest tenth of a percent, what has been the average annual growth rate in cost?

b. If this growth rate continues, what will the cost be in 5 more years?

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TEST PRACTICE at classzone.com

45. What is the effect on the graph of the equation $y = x^2 - 2$ when it is changed to $y = x^2 + 8$?

Ⓐ The graph is translated 10 units up.

Ⓑ The graph is translated 10 units down.

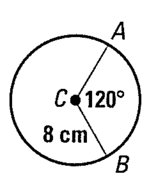
Ⓒ The graph is translated 10 units to the right.

Ⓓ The graph is translated 10 units to the left.

46. What is the approximate length of arc AB?

Ⓐ 5.3 cm　　Ⓑ 8.4 cm

Ⓒ 16.8 cm　　Ⓓ 33.5 cm



7.2 Graph Exponential Decay Functions

PA M11.D.1.1.3 Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

Before You graphed and used exponential growth functions.

Now You will graph and use exponential decay functions.

Why? So you can model depreciation, as in Ex. 31.

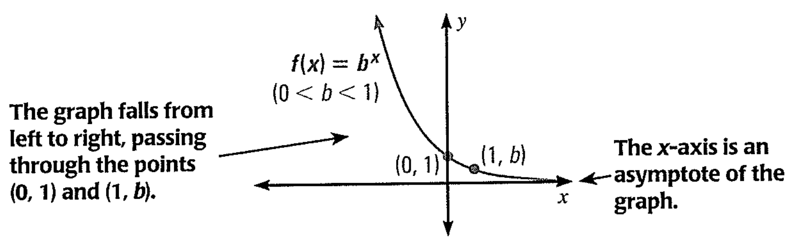
Key Vocabulary
- **exponential decay function**
- **decay factor**

In Lesson 7.1 you studied exponential growth functions. In this lesson, you will study **exponential decay functions**, which have the form $y = ab^x$ where $a > 0$ and $0 < b < 1$. The base b of an exponential decay function is called the **decay factor**.

KEY CONCEPT *For Your Notebook*

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 Graph $y = b^x$ for $0 < b < 1$

Graph $y = \left(\frac{1}{2}\right)^x$.

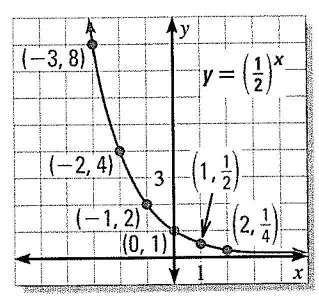
Solution

STEP 1 **Make** a table of values.

x	-3	-2	-1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

STEP 2 **Plot** the points from the table.

STEP 3 **Draw,** from *right* to *left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



TRANSFORMATIONS Recall from Lesson 7.1 that the graph of a function $y = ab^x$ is a vertical stretch or shrink of the graph of $y = b^x$, and the graph of $y = ab^{x-h} + k$ is a translation of the graph of $y = ab^x$.

EXAMPLE 2 Graph $y = ab^x$ for $0 < b < 1$

CLASSIFY FUNCTIONS
Note that the function in part (b) of Example 2 is not an exponential decay function because $a = -3 < 0$.

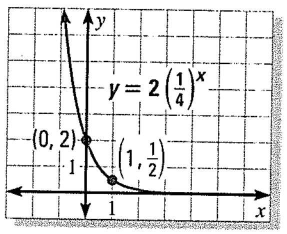
Graph the function.

a. $y = 2\left(\frac{1}{4}\right)^x$

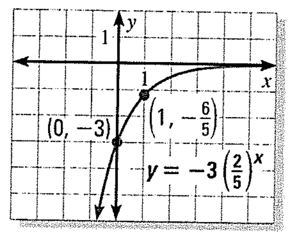
b. $y = -3\left(\frac{2}{5}\right)^x$

Solution

a. Plot (0, 2) and $\left(1, \frac{1}{2}\right)$. Then, from *right* to *left*, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the left.



b. Plot (0, −3) and $\left(1, -\frac{6}{5}\right)$. Then, from *right* to *left*, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the left.



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✓ GUIDED PRACTICE for Examples 1 and 2

Graph the function.

1. $y = \left(\frac{2}{3}\right)^x$

2. $y = -2\left(\frac{3}{4}\right)^x$

3. $f(x) = 4\left(\frac{1}{5}\right)^x$

EXAMPLE 3 Graph $y = ab^{x-h} + k$ for $0 < b < 1$

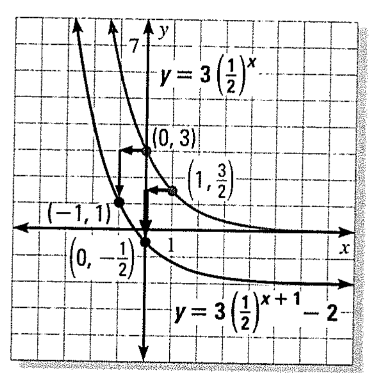
Graph $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$. State the domain and range.

Solution

Begin by sketching the graph of $y = 3\left(\frac{1}{2}\right)^x$, which passes through (0, 3) and $\left(1, \frac{3}{2}\right)$.

Then translate the graph left 1 unit and down 2 units. Notice that the translated graph passes through (−1, 1) and $\left(0, -\frac{1}{2}\right)$.

The graph's asymptote is the line $y = -2$. The domain is all real numbers, and the range is $y > -2$.



EXPONENTIAL DECAY MODELS When a real-life quantity decreases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$y = a(1 - r)^t$$

where a is the initial amount and r is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.

EXAMPLE 4 Solve a multi-step problem

SNOWMOBILES A new snowmobile costs $4200. The value of the snowmobile decreases by 10% each year.

- Write an exponential decay model giving the snowmobile's value y (in dollars) after t years. Estimate the value after 3 years.
- Graph the model.
- Use the graph to estimate when the value of the snowmobile will be $2500.

Solution

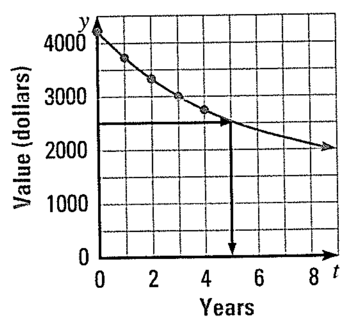
STEP 1 The initial amount is $a = 4200$ and the percent decrease is $r = 0.10$. So, the exponential decay model is:

$y = a(1 - r)^t$	**Write exponential decay model.**
$= \mathbf{4200}(1 - \mathbf{0.10})^t$	**Substitute 4200 for *a* and 0.10 for *r*.**
$= 4200(0.90)^t$	**Simplify.**

When $t = 3$, the snowmobile's value is $y = 4200(0.90)^3 = \$3061.80$.

AVOID ERRORS
Notice that the percent decrease, 10%, tells you how much value the snowmobile *loses* each year. The decay factor, 0.90, tells you what fraction of the snowmobile's value *remains* each year.

STEP 2 The graph passes through the points (0, 4200) and (1, 3780). It has the t-axis as an asymptote. Plot a few other points. Then draw a smooth curve through the points.



STEP 3 Using the graph, you can estimate that the value of the snowmobile will be $2500 after about 5 years.

✓ GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

4. $y = \left(\frac{1}{4}\right)^{x-1} + 1$

5. $y = 5\left(\frac{2}{3}\right)^{x+1} - 2$

6. $g(x) = -3\left(\frac{3}{4}\right)^{x-5} + 4$

7. **WHAT IF?** In Example 4, suppose the value of the snowmobile decreases by 20% each year. Write and graph an equation to model this situation. Use the graph to estimate when the value of the snowmobile will be $2500.

8. **SNOWMOBILE** The value of a snowmobile has been decreasing by 7% each year since it was new. After 3 years, the value is $3000. Find the original cost of the snowmobile.

7.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 9, 19, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 27, 28, 33, and 35

SKILL PRACTICE

1. **VOCABULARY** In the exponential decay model $y = 1250(0.85)^t$, identify the initial amount, the decay factor, and the percent decrease.

2. ★ **WRITING** *Explain* how to tell whether the function $y = b^x$ represents exponential growth or exponential decay.

CLASSIFYING FUNCTIONS **Tell whether the function represents *exponential growth* or *exponential decay*.**

3. $f(x) = 3\left(\frac{3}{4}\right)^x$ 4. $f(x) = 4\left(\frac{5}{2}\right)^x$ 5. $f(x) = \frac{2}{7} \cdot 4^x$ 6. $f(x) = 25(0.25)^x$

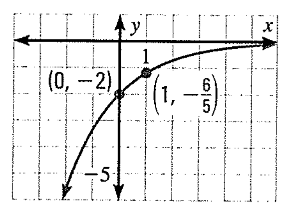
EXAMPLES 1 and 2
on pp. 486–487 for Exs. 7–15

GRAPHING FUNCTIONS **Graph the function.**

7. $y = \left(\frac{1}{4}\right)^x$ 8. $y = \left(\frac{1}{3}\right)^x$ 9. $f(x) = 2\left(\frac{1}{5}\right)^x$ 10. $y = -(0.2)^x$

11. $y = -4\left(\frac{1}{3}\right)^x$ 12. $g(x) = 2(0.75)^x$ 13. $y = \left(\frac{3}{5}\right)^x$ 14. $h(x) = -3\left(\frac{3}{8}\right)^x$

15. ★ **MULTIPLE CHOICE** The graph of which function is shown?



Ⓐ $y = 2\left(-\frac{3}{5}\right)^x$ Ⓑ $y = -2\left(\frac{3}{5}\right)^x$

Ⓒ $y = -2\left(\frac{2}{5}\right)^x$ Ⓓ $y = 2\left(-\frac{2}{5}\right)^x$

EXAMPLE 3
on p. 487 for Exs. 16–25

TRANSLATING GRAPHS **Graph the function. State the domain and range.**

16. $y = \left(\frac{1}{3}\right)^x + 1$ 17. $y = -\left(\frac{1}{2}\right)^{x-1}$ 18. $y = 2\left(\frac{1}{3}\right)^{x+1} - 3$

19. $y = \left(\frac{2}{3}\right)^{x-4} - 1$ 20. $y = 3(0.25)^x + 3$ 21. $y = \left(\frac{1}{3}\right)^{x-2} + 2$

22. $f(x) = -3\left(\frac{1}{4}\right)^{x-1}$ 23. $g(x) = 6\left(\frac{1}{2}\right)^{x+5} - 2$ 24. $h(x) = 4\left(\frac{1}{2}\right)^{x+1}$

25. **GRAPHING CALCULATOR** Consider the exponential decay function $y = ab^{x-h} + k$ where $a = 3$, $b = 0.4$, $h = 2$, and $k = -1$. Predict the effect on the function's graph of each change in a, b, h, or k described in parts (a)–(d). Use a graphing calculator to check your prediction.

   a. a changes to 4 b. b changes to 0.2

   c. h changes to 5 d. k changes to 3

26. **ERROR ANALYSIS** You invest \$500 in the stock of a company. The value of the stock decreases 2% each year. *Describe* and correct the error in writing a model for the value of the stock after t years.

$$y = \left(\begin{array}{c}\text{Initial}\\ \text{amount}\end{array}\right)\left(\begin{array}{c}\text{Decay}\\ \text{factor}\end{array}\right)^t$$

$$y = 500(0.02)^t$$

27. ★ **MULTIPLE CHOICE** What is the asymptote of the graph of $y = \left(\frac{1}{2}\right)^{x-2} + 3$?

Ⓐ $y = -3$ Ⓑ $y = -2$ Ⓒ $y = 2$ Ⓓ $y = 3$

28. ★ **OPEN-ENDED MATH** Write an exponential function whose graph lies between the graphs of $y = (0.5)^x$ and $y = (0.25)^x + 3$.

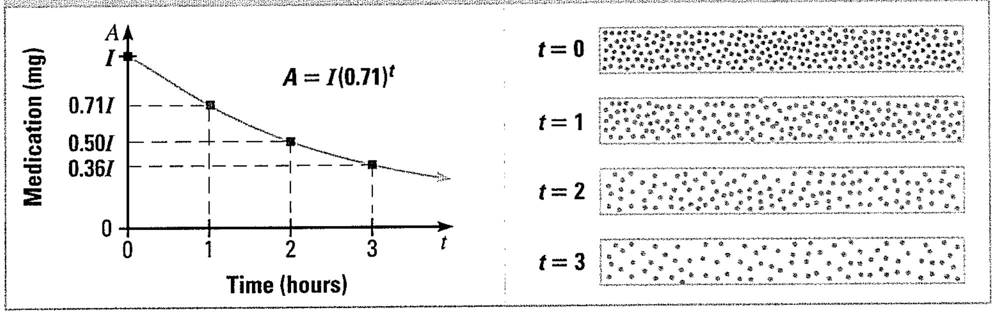
29. **CHALLENGE** Do $f(x) = 5(4)^{-x}$ and $g(x) = 5(0.25)^x$ represent the same function? *Justify* your answer.

PROBLEM SOLVING

EXAMPLE 4 on p. 488 for Exs. 30–31

30. **MEDICINE** When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.

Amount of Ibuprofen in Bloodstream



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

a. Dosage: 200 mg Time: 1.5 hours
b. Dosage: 325 mg Time: 3.5 hours
c. Dosage: 400 mg Time: 5 hours

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31. **BIKE COSTS** You buy a new mountain bike for \$200. The value of the bike decreases by 25% each year.

a. Write a model giving the mountain bike's value y (in dollars) after t years. Use the model to estimate the value of the bike after 3 years.

b. Graph the model.

c. Estimate when the value of the bike will be \$100.

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32. **DEPRECIATION** The table shows the amount d that a boat depreciates during each year t since it was new. Show that the ratio of depreciation amounts for consecutive years is constant. Then write an equation that gives d as a function of t.

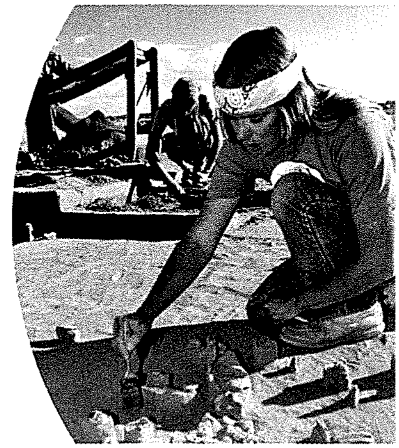
Year, t	1	2	3	4	5
Depreciation, d	\$1906	\$1832	\$1762	\$1692	\$1627

33. ★ **SHORT RESPONSE** The value of a car can be modeled by the equation $y = 24{,}000(0.845)^t$ where t is the number of years since the car was purchased.

a. Graph the model. Estimate when the value of the car will be \$10,000.

b. Use the model to predict the value of the car after 50 years. Is this a reasonable value? *Explain.*

34. **MULTI-STEP PROBLEM** When a plant or animal dies, it stops acquiring carbon-14 from the atmosphere. Carbon-14 decays over time with a half-life of about 5730 years. The percent P of the original amount of carbon-14 that remains in a sample after t years is given by this equation:



$$P = 100\left(\frac{1}{2}\right)^{t/5730}$$

a. What percent of the original carbon-14 remains in a sample after 2500 years? 5000 years? 10,000 years?

b. Graph the model.

c. An archaeologist found a bison bone that contained about 37% of the carbon-14 present when the bison died. Use the graph to estimate the age of the bone when it was found.

35. ★ **EXTENDED RESPONSE** The number E of eggs a Leghorn chicken produces per year can be modeled by the equation $E = 179.2(0.89)^{w/52}$ where w is the age (in weeks) of the chicken and $w \geq 22$.

a. Interpret Identify the decay factor and the percent decrease.

b. Graph Graph the model.

c. Estimate Estimate the egg production of a chicken that is 2.5 years old.

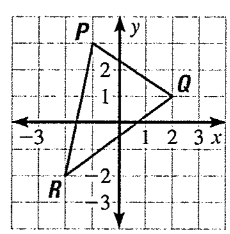
d. Reasoning *Explain* how you can rewrite the given equation so that time is measured in years rather than in weeks.

36. **CHALLENGE** You buy a new stereo for \$1300 and are able to sell it 4 years later for \$275. Assume that the resale value of the stereo decays exponentially with time. Write an equation giving the stereo's resale value V (in dollars) as a function of the time t (in years) since you bought it.

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

37. If $\triangle PQR$ is translated to the left 3 units and down 2 units, in which quadrant will the image of point Q appear?



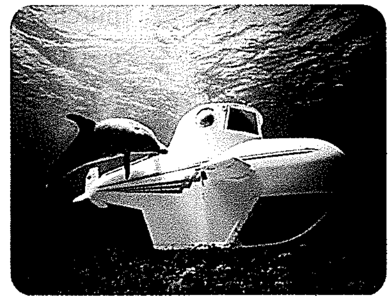
(A) Quadrant I (B) Quadrant II

(C) Quadrant III (D) Quadrant IV

38. This year's price for a certain laptop computer is 16.7% lower than last year's price of \$960. Approximately what percent of this year's price for the computer is last year's price?

(A) 83.3% (B) 85.0% (C) 116.7% (D) 120.0%

7.3 Use Functions Involving e



PA M11.D.1.1.3 Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

Before You studied exponential growth and decay functions.

Now You will study functions involving the natural base e.

Why? So you can model visibility underwater, as in Ex. 59.

Key Vocabulary
- **natural base e**

The history of mathematics is marked by the discovery of special numbers such as π and i. Another special number is denoted by the letter e. The number is called the **natural base e** or the *Euler number* after its discoverer, Leonhard Euler (1707–1783). The expression $\left(1 + \frac{1}{n}\right)^n$ approaches e as n increases.

n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

KEY CONCEPT *For Your Notebook*

The Natural Base e

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718281828$.

EXAMPLE 1 Simplify natural base expressions

REVIEW EXPONENTS
For help with properties of exponents, see p. 330.

Simplify the expression.

a. $e^2 \cdot e^5 = e^{2+5}$
$= e^7$

b. $\frac{12e^4}{3e^3} = 4e^{4-3}$
$= 4e$

c. $(5e^{-3x})^2 = 5^2(e^{-3x})^2$
$= 25e^{-6x} = \frac{25}{e^{6x}}$

EXAMPLE 2 Evaluate natural base expressions

Use a calculator to evaluate the expression.

Expression	Keystrokes	Display
a. e^4	2nd [e^x] 4) ENTER	54.59815003
b. $e^{-0.09}$	2nd [e^x] (–) .09) ENTER	0.9139311853

✓ GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $e^7 \cdot e^4$
2. $2e^{-3} \cdot 6e^5$
3. $\dfrac{24e^8}{4e^5}$
4. $(10e^{-4x})^3$
5. Use a calculator to evaluate $e^{3/4}$.

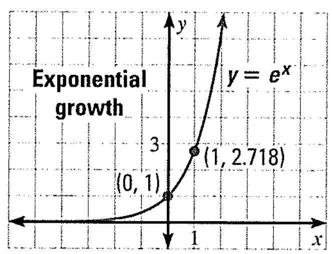
KEY CONCEPT *For Your Notebook*

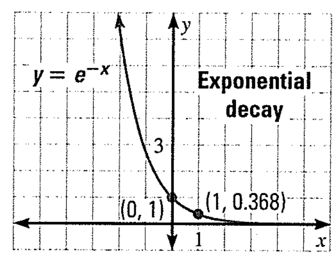
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function.*

- If $a > 0$ and $r > 0$, the function is an exponential growth function.
- If $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.





EXAMPLE 3 Graph natural base functions

Graph the function. State the domain and range.

a. $y = 3e^{0.25x}$

b. $y = e^{-0.75(x-2)} + 1$

Solution

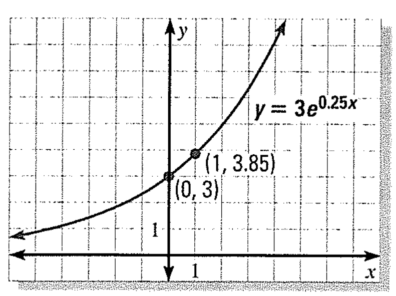
ANOTHER WAY
You can also write the function from part (a) in the form $y = ab^x$ in order to graph it:

$y = 3e^{0.25x}$

$y = 3(e^{0.25})^x$

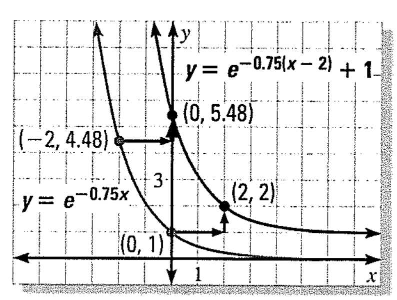
$y \approx 3(1.28)^x$

a. Because $a = 3$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Plot the points (0, 3) and (1, 3.85) and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

b. $a = 1$ is positive and $r = -0.75$ is negative, so the function is an exponential decay function. Translate the graph of $y = e^{-0.75x}$ right 2 units and up 1 unit.



The domain is all real numbers, and the range is $y > 1$.

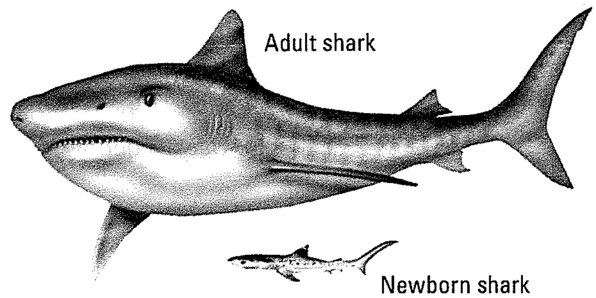
EXAMPLE 4 Solve a multi-step problem

BIOLOGY The length ℓ (in centimeters) of a tiger shark can be modeled by the function

$$\ell = 337 - 276e^{-0.178t}$$

where t is the shark's age (in years).

- Graph the model.
- Use the graph to estimate the length of a tiger shark that is 3 years old.



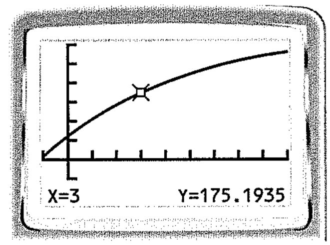
INTERPRET VARIABLES
On a graphing calculator, enter the function $\ell = 337 - 276e^{-0.178t}$ using the variables x and y, as shown below:
$y = 337 - 276e^{-0.178x}$

Solution

STEP 1 **Graph** the model, as shown.

STEP 2 **Use** the *trace* feature to determine that $\ell \approx 175$ when $t = 3$.

▶ The length of a 3-year-old tiger shark is about 175 centimeters.



✓ **GUIDED PRACTICE** for Examples 3 and 4

Graph the function. State the domain and range.

6. $y = 2e^{0.5x}$

7. $f(x) = \frac{1}{2}e^{-x} + 1$

8. $y = 1.5e^{0.25(x-1)} - 2$

9. WHAT IF? In Example 4, use the given function to estimate the length of a tiger shark that is 5 years old.

CONTINUOUSLY COMPOUNDED INTEREST In Lesson 7.1, you learned that the balance of an account earning compound interest is given by this formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

As the frequency n of compounding approaches positive infinity, the compound interest formula approximates the following formula.

KEY CONCEPT *For Your Notebook*

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

EXAMPLE 5 Model continuously compounded interest

FINANCE You deposit \$4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

Solution

Use the formula for continuously compounded interest.

$A = Pe^{rt}$	**Write formula.**
$= 4000e^{0.06(1)}$	**Substitute 4000 for *P*, 0.06 for *r*, and 1 for *t*.**
≈ 4247.35	**Use a calculator.**

▶ The balance at the end of 1 year is \$4247.35.

✓ GUIDED PRACTICE for Example 5

10. **FINANCE** You deposit \$2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time.

    a. 2 years b. 5 years c. 7.5 years

11. **FINANCE** Find the amount of interest earned in parts (a)–(c) of Exercise 10.

7.3 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 5, 35, and 57
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 52, 53, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The number _?_ is an irrational number approximately equal to 2.71828.

2. ★ **WRITING** Tell whether the function $f(x) = \frac{1}{3}e^{4x}$ is an example of *exponential growth* or *exponential decay. Explain.*

EXAMPLE 1 on p. 492 for Exs. 3–18

SIMPLIFYING EXPRESSIONS Simplify the expression.

3. $e^3 \cdot e^4$
4. $e^{-2} \cdot e^6$
5. $(2e^{3x})^3$
6. $(2e^{-2})^{-4}$
7. $(3e^{5x})^{-1}$
8. $e^x \cdot e^{-3x} \cdot e^4$
9. $\sqrt{9e^6}$
10. $e^x \cdot 5e^{x+3}$
11. $\frac{3e}{e^x}$
12. $\frac{4e^x}{e^{4x}}$
13. $\sqrt[3]{8e^{9x}}$
14. $\frac{6e^{4x}}{8e}$

15. ★ **MULTIPLE CHOICE** What is the simplified form of $(4e^{2x})^3$?

    (A) $4e^{6x}$ (B) $4e^{8x}$ (C) $64e^{6x}$ (D) $64e^{8x}$

16. ★ **MULTIPLE CHOICE** What is the simplified form of $\sqrt{\frac{4(27e^{13}x)}{3e^7x^{-3}}}$?

    (A) $6e^{10}x$ (B) $6e^6x^4$ (C) $\frac{6e^3}{x^2}$ (D) $6e^3x^2$

ERROR ANALYSIS *Describe* and correct the error in simplifying the expression.

17. $(3e^{5x})^2 = 3e^{(5x)(2)}$
$= 3e^{10x}$

18. $\dfrac{e^{6x}}{e^{-2x}} = e^{6x-2x}$
$= e^{4x}$

EXAMPLE 2 on p. 492 for Exs. 19–30

EVALUATING EXPRESSIONS Use a calculator to evaluate the expression.

19. e^3 **20.** $e^{-3/4}$ **21.** $e^{2.2}$ **22.** $e^{1/2}$

23. $e^{-2/5}$ **24.** $e^{4.3}$ **25.** e^7 **26.** e^{-4}

27. $2e^{-0.3}$ **28.** $5e^{2/3}$ **29.** $-6e^{2.4}$ **30.** $0.4e^{4.1}$

GROWTH OR DECAY Tell whether the function is an example of *exponential growth* or *exponential decay.*

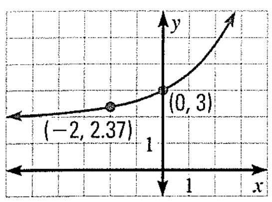
31. $f(x) = 3e^{-x}$ **32.** $f(x) = \frac{1}{3}e^{4x}$ **33.** $f(x) = e^{-4x}$ **34.** $f(x) = \frac{3}{5}e^{x}$

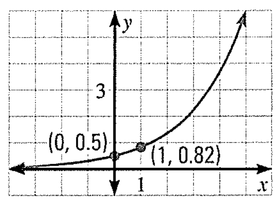
35. $f(x) = \frac{1}{4}e^{-5x}$ **36.** $f(x) = e^{3x}$ **37.** $f(x) = 2e^{4x}$ **38.** $f(x) = 4e^{-2x}$

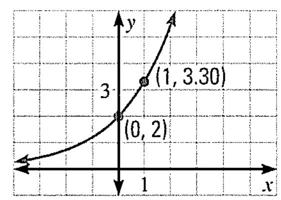
EXAMPLE 3 on p. 493 for Exs. 39–50

MATCHING GRAPHS Match the function with its graph.

39. $y = 0.5e^{0.5x}$ **40.** $y = 2e^{0.5x}$ **41.** $y = e^{0.5x} + 2$

A.


B.


C.


GRAPHING FUNCTIONS Graph the function. State the domain and range.

42. $y = e^{-2x}$ **43.** $y = 3e^{x}$ **44.** $y = 0.5e^{x}$

45. $y = 2e^{-3x} - 1$ **46.** $y = 2.5e^{-0.5x} + 2$ **47.** $y = 0.6e^{x-2}$

48. $f(x) = \frac{1}{2}e^{x+3} - 2$ **49.** $g(x) = \frac{4}{3}e^{x-1} + 1$ **50.** $h(x) = e^{-2(x+1)} - 3$

51. **GRAPHING CALCULATOR** Use the *table* feature of a graphing calculator to find the value of n for which $\left(1 + \frac{1}{n}\right)^n$ gives the value of e correct to 9 decimal places. *Explain* the process you used to find your answer.

52. ★ **SHORT RESPONSE** Can e be expressed as a ratio of two integers? *Explain* your reasoning.

53. ★ **OPEN-ENDED MATH** Find values of a, b, r, and q such that $f(x) = ae^{rx}$ and $g(x) = be^{qx}$ are exponential *decay* functions and $\frac{f(x)}{g(x)}$ is an exponential *growth* function.

54. **CHALLENGE** *Explain* why $A = P\left(1 + \frac{r}{n}\right)^{nt}$ approximates $A = Pe^{rt}$ as n approaches positive infinity. $\left(\textit{Hint:} \text{ Let } m = \frac{n}{r}.\right)$

PROBLEM SOLVING

EXAMPLE 4
on p. 494
for Exs. 55–56

55. **CAMERA PHONES** The number of camera phones shipped globally can be modeled by the function $y = 1.28e^{1.31x}$ where x is the number of years since 1997 and y is the number of camera phones shipped (in millions). How many camera phones were shipped in 2002?

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56. **BIOLOGY** Scientists used traps to study the Formosan subterranean termite population in New Orleans. The mean number y of termites collected annually can be modeled by $y = 738e^{0.345t}$ where t is the number of years since 1989. What was the mean number of termites collected in 1999?

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EXAMPLE 5
on p. 495
for Exs. 57–58

57. **FINANCE** You deposit \$2000 in an account that pays 4% annual interest compounded continuously. What is the balance after 5 years?

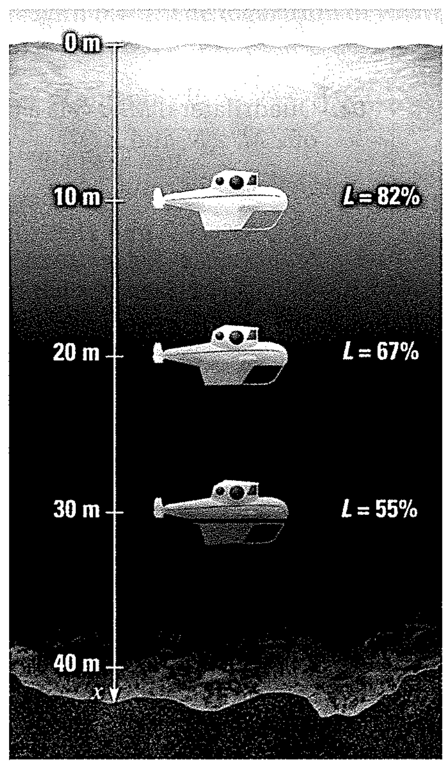
58. **FINANCE** You deposit \$800 in an account that pays 2.65% annual interest compounded continuously. What is the balance after 12.5 years?

59. **MULTI-STEP PROBLEM** The percent L of surface light that filters down through bodies of water can be modeled by the exponential function $L(x) = 100e^{kx}$ where k is a measure of the murkiness of the water and x is the depth below the surface (in meters).

    a. A recreational submersible is traveling in clear water with a k-value of about -0.02. Write and graph an equation giving the percent of surface light that filters down through clear water as a function of depth.

    b. Use your graph to estimate the percent of surface light available at a depth of 40 meters.

    c. Use your graph to estimate how deep the submersible can descend in clear water before only 50% of surface light is available.



60. ★ **EXTENDED RESPONSE** The growth of the bacteria *mycobacterium tuberculosis* can be modeled by the function $P(t) = P_0e^{0.116t}$ where $P(t)$ is the population after t hours and P_0 is the population when $t = 0$.

    a. **Model** At 1:00 P.M., there are 30 *mycobacterium tuberculosis* bacteria in a sample. Write a function for the number of bacteria after 1:00 P.M.

    b. **Graph** Graph the function from part (a).

    c. **Estimate** What is the population at 5:00 P.M.?

    d. **Reasoning** *Describe* how to find the population at 3:45 P.M.

PROBLEM SOLVING

EXAMPLE 5 on p. 509 for Exs. 69–72

69. CONVERSATION Three groups of people are having separate conversations in a room. The sound of each conversation has an intensity of 1.4×10^{-5} watts per square meter. What is the decibel level of the combined conversations in the room?

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70. PARKING GARAGE The sound made by each of five cars in a parking garage has an intensity of 3.2×10^{-4} watts per square meter. What is the decibel level of the sound made by all five cars in the parking garage?

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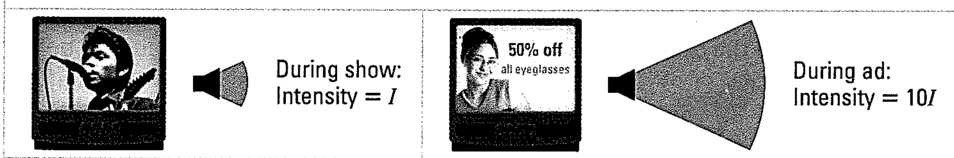
71. ★ **SHORT RESPONSE** The intensity of the sound TV ads make is ten times as great as the intensity for an average TV show. How many decibels louder is a TV ad? *Justify* your answer using properties of logarithms.

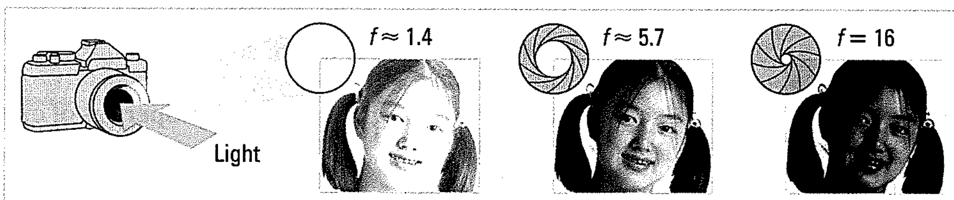
Intensity of Television Sound



72. BIOLOGY The loudest animal on Earth is the blue whale. It can produce a sound with an intensity of $10^{6.8}$ watts per square meter. The loudest sound a human can make has an intensity of $10^{0.8}$ watts per square meter. *Compare* the decibel levels of the sounds made by a blue whale and a human.

73. ★ **EXTENDED RESPONSE** The f-stops on a 35 millimeter camera control the amount of light that enters the camera. Let s be a measure of the amount of light that strikes the film and let f be the f-stop. Then s and f are related by the equation:

$$s = \log_2 f^2$$



a. Use Properties Expand the expression for s.

b. Calculate The table shows the first eight f-stops on a 35 millimeter camera. Copy and complete the table. *Describe* the pattern you observe.

f	1.414	2.000	2.828	4.000	5.657	8.000	11.314	16.000
s	?	?	?	?	?	?	?	?

c. Reasoning Many 35 millimeter cameras have nine f-stops. What do you think the ninth f-stop is? *Explain* your reasoning.

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

74. **CHALLENGE** Under certain conditions, the wind speed s (in knots) at an altitude of h meters above a grassy plain can be modeled by this function:

$$s(h) = 2 \ln (100h)$$

a. By what factor does the wind speed increase when the altitude doubles?

b. Show that the given function can be written in terms of common logarithms as $s(h) = \frac{2}{\log e}(\log h + 2)$.

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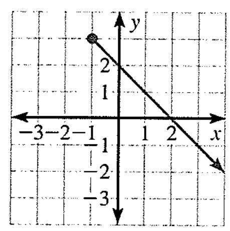
75. Which of the following is *not* an example of a Pythagorean triple?

Ⓐ 8, 15, 17 Ⓑ 48, 64, 80 Ⓒ 7, 23, 25 Ⓓ 11, 60, 61

76. Which inequality best describes the range of the function whose graph is shown?

Ⓐ $y \leq -1$ Ⓑ $y \leq 3$

Ⓒ $y \geq -1$ Ⓓ $y \geq 3$



QUIZ for Lessons 7.4–7.5

Evaluate the logarithm without using a calculator. *(p. 499)*

1. $\log_4 16$
2. $\log_5 1$
3. $\log_8 8$
4. $\log_{1/2} 32$

Graph the function. State the domain and range. *(p. 499)*

5. $y = \log_2 x$
6. $y = \ln x + 2$
7. $y = \log_3 (x + 4) - 1$

Expand the expression. *(p. 507)*

8. $\log_2 5x$
9. $\log_5 x^7$
10. $\ln 5xy^3$
11. $\log_3 \frac{6y^4}{x^8}$

Condense the expression. *(p. 507)*

12. $\log_3 5 - \log_3 20$
13. $\ln 6 + \ln 4x$
14. $\log_6 5 + 3 \log_6 2$
15. $4 \ln x - 5 \ln x$

Use the change-of-base formula to evaluate the logarithm. *(p. 507)*

16. $\log_3 10$
17. $\log_7 14$
18. $\log_5 24$
19. $\log_8 40$

20. **SOUND INTENSITY** The sound of an alarm clock has an intensity of $I = 10^{-4}$ watts per square meter. Use the model $L(I) = 10 \log \frac{I}{I_0}$, where $I_0 = 10^{-12}$ watts per square meter, to find the alarm clock's loudness $L(I)$. *(p. 507)*

When it is not convenient to write each side of an exponential equation using the same base, you can solve the equation by taking a logarithm of each side.

ANOTHER WAY
For an alternative method for solving the problem in Example 2, turn to page 523 for the **Problem Solving Workshop**.

EXAMPLE 2 Take a logarithm of each side

Solve $4^x = 11$.

$4^x = 11$	Write original equation.
$\log_4 4^x = \log_4 11$	Take $\log_4$ of each side.
$x = \log_4 11$	$\log_b b^x = x$
$x = \dfrac{\log 11}{\log 4}$	Change-of-base formula
$x \approx 1.73$	Use a calculator.

▸ The solution is about 1.73. Check this in the original equation.

NEWTON'S LAW OF COOLING An important application of exponential equations is *Newton's law of cooling.* This law states that for a cooling substance with initial temperature T_0, the temperature T after t minutes can be modeled by

$$T = (T_0 - T_R)e^{-rt} + T_R$$

where T_R is the surrounding temperature and r is the substance's cooling rate.

EXAMPLE 3 Use an exponential model

CARS You are driving on a hot day when your car overheats and stops running. It overheats at 280°F and can be driven again at 230°F. If $r = 0.0048$ and it is 80°F outside, how long (in minutes) do you have to wait until you can continue driving?

Solution

$T = (T_0 - T_R)e^{-rt} + T_R$	Newton's law of cooling
$230 = (280 - 80)e^{-0.0048t} + 80$	Substitute for T, T_0, T_R, and r.
$150 = 200e^{-0.0048t}$	Subtract 80 from each side.
$0.75 = e^{-0.0048t}$	Divide each side by 200.
$\ln 0.75 = \ln e^{-0.0048t}$	Take natural log of each side.
$-0.2877 \approx -0.0048t$	$\ln e^x = \log_e e^x = x$
$60 \approx t$	Divide each side by -0.0048.

▸ You have to wait about 60 minutes until you can continue driving.

✓ GUIDED PRACTICE for Examples 2 and 3

Solve the equation.

4. $2^x = 5$ **5.** $7^{9x} = 15$ **6.** $4e^{-0.3x} - 7 = 13$

SOLVING LOGARITHMIC EQUATIONS **Logarithmic equations** are equations that involve logarithms of variable expressions. You can use the following property to solve some types of logarithmic equations.

KEY CONCEPT ***For Your Notebook***

Property of Equality for Logarithmic Equations

Algebra If b, x, and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$.

EXAMPLE 4 Solve a logarithmic equation

Solve $\log_5 (4x - 7) = \log_5 (x + 5)$.

$\log_5 (4x - 7) = \log_5 (x + 5)$	**Write original equation.**
$4x - 7 = x + 5$	**Property of equality for logarithmic equations**
$3x - 7 = 5$	**Subtract x from each side.**
$3x = 12$	**Add 7 to each side.**
$x = 4$	**Divide each side by 3.**

▶ The solution is 4.

CHECK Check the solution by substituting it into the original equation.

$\log_5 (4x - 7) = \log_5 (x + 5)$	**Write original equation.**
$\log_5 (4 \cdot 4 - 7) \stackrel{?}{=} \log_5 (4 + 5)$	**Substitute 4 for x.**
$\log_5 9 = \log_5 9$ ✓	**Solution checks.**

EXPONENTIATING TO SOLVE EQUATIONS The property of equality for exponential equations on page 515 implies that if you are given an equation $x = y$, then you can *exponentiate* each side to obtain an equation of the form $b^x = b^y$. This technique is useful for solving some logarithmic equations.

EXAMPLE 5 Exponentiate each side of an equation

Solve $\log_4 (5x - 1) = 3$.

$\log_4 (5x - 1) = 3$	**Write original equation.**
$4^{\log_4 (5x - 1)} = 4^3$	**Exponentiate each side using base 4.**
$5x - 1 = 64$	$b^{\log_b x} = x$
$5x = 65$	**Add 1 to each side.**
$x = 13$	**Divide each side by 5.**

▶ The solution is 13.

CHECK $\log_4 (5x - 1) = \log_4 (5 \cdot 13 - 1) = \log_4 64$

Because $4^3 = 64$, $\log_4 64 = 3$. ✓

EXAMPLE 4 on p. 531 for Exs. 15–22

WRITING POWER FUNCTIONS Write a power function $y = ax^b$ whose graph passes through the given points.

15. (4, 3), (8, 15) **16.** (5, 9), (8, 34) **17.** (2, 3), (6, 12) **18.** (3, 14), (9, 44)

19. (4, 8), (8, 30) **20.** (5, 10), (12, 81) **21.** (4, 6.2), (7, 23) **22.** (3.1, 5), (6.8, 9.7)

EXAMPLE 5 on p. 532 for Exs. 23–26

FINDING POWER MODELS Use the given points (x, y) to draw a scatter plot of the points $(\ln x, \ln y)$. Then find a power model for the data.

23. (1, 0.6), (2, 4.1), (3, 12.4), (4, 27), (5, 49.5) **24.** (1, 1.5), (2, 4.8), (3, 9.5), (4, 15.4), (5, 22.3)

25. (1, 2.5), (2, 3.7), (3, 4.7), (4, 5.5), (5, 6.2) **26.** (1, 0.81), (2, 0.99), (3, 1.11), (4, 1.21), (5, 1.29)

27. ★ MULTIPLE CHOICE Which equation is equivalent to $\log y = 2x + 1$?

Ⓐ $y = 10(100)^x$ Ⓑ $y = 10^x$ Ⓒ $y = e^{2x+1}$ Ⓓ $y = e^2$

ERROR ANALYSIS *Describe* and correct the error in writing y as a function of x.

28.

$\ln y = 2x + 1$

$y = e^{2x+1}$

$y = e^{2x} + e^1$

$y = (e^2)^x + e$

$y = 7.39^x + 2.72$

✗

29.

$\ln y = 3 \ln x - 2$

$\ln y = \ln 3x - 2$

$y = e^{\ln 3x - 2}$

$y = e^{\ln 3x} \cdot e^{-2}$

$y = (3x)(0.135) = 0.405x$

✗

30. CHALLENGE Take the natural logarithm of both sides of the equations $y = ab^x$ and $y = ax^b$. What are the slope and y-intercept of the line relating x and $\ln y$ for $y = ab^x$? of the line relating $\ln x$ and $\ln y$ for $y = ax^b$?

PROBLEM SOLVING

GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

EXAMPLES 2, 3, 5, and 6 on pp. 530–533 for Exs. 31–35

31. BIOLOGY Scientists use the circumference of an animal's femur to estimate the animal's weight. The table shows the femur circumference C (in millimeters) and the weight W (in kilograms) for several animals.

Animal	Giraffe	Polar bear	Lion	Squirrel	Otter
C (mm)	173	135	93.5	13	28
W (kg)	710	448	143	0.399	9.68

a. Draw a scatter plot of the data pairs $(\ln C, \ln W)$.

b. Find a power model for the original data.

c. Predict the weight of a cheetah if the circumference of its femur is 68.7 millimeters.

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32. **ASTRONOMY** The table shows the mean distance x from the sun (in astronomical units) and the period y (in years) of six planets. Draw a scatter plot of the data pairs $(\ln x, \ln y)$. Find a power model for the original data.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
x	0.387	0.723	1.000	1.524	5.203	9.539
y	0.241	0.615	1.000	1.881	11.862	29.458

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33. ★ **SHORT RESPONSE** The table shows the numbers of business and non-business users of instant messaging for the years 1998–2004.

Years since 1997	1	2	3	4	5	6	7
Business users (in millions)	1	2	5	7	20	40	80
Non-business users (in millions)	55	97	140	160	195	235	260

a. Find an exponential model for the number of business users over time.

b. *Explain* how to tell whether a linear, exponential, or power function best models the number of non-business users over time. Then find the best-fitting model.

34. **MULTI-STEP PROBLEM** The boiling point of water increases with atmospheric pressure. At sea level, where the atmospheric pressure is about 760 millimeters of mercury, water boils at 100°C. The table shows the boiling point T of water (in degrees Celsius) for several different values of atmospheric pressure P (in millimeters of mercury).

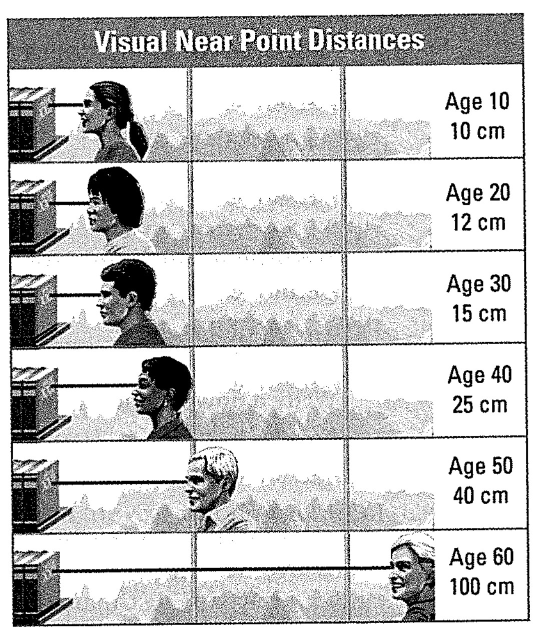
P	T
149	60
234	70
355	80
526	90
760	100
1075	110

a. Graph Draw a scatter plot of the data pairs $(\ln P, \ln T)$.

b. Model Find a power model for the original data.

c. Predict When the atmospheric pressure is 620 millimeters of mercury, at what temperature does water boil?

35. ★ **EXTENDED RESPONSE** Your visual *near point* is the closest point at which your eyes can see an object distinctly. Your near point moves farther away from you as you grow older. The diagram shows the near point y (in centimeters) at age x (in years).



a. Graph Draw a scatter plot of the data pairs $(x, \ln y)$.

b. Graph Draw a scatter plot of the data pairs $(\ln x, \ln y)$.

c. Interpret Based on your scatter plots, does an exponential function or a power function best fit the original data? *Explain* your reasoning.

d. Model Based on your answer for part (c), write a model for the original data. Use your model to predict the near point for an 80-year-old person.

7 CHAPTER SUMMARY

Animated Algebra
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Electronic Function Library

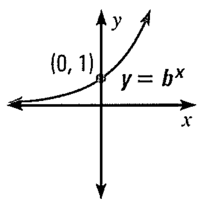
BIG IDEAS

For Your Notebook

Big Idea 1: Graphing Exponential and Logarithmic Functions

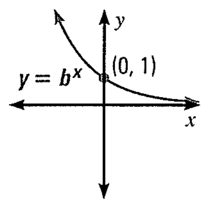
Parent functions for exponential functions have the form $y = b^x$. Parent functions for logarithmic functions have the form $y = \log_b x$.

Exponential Growth



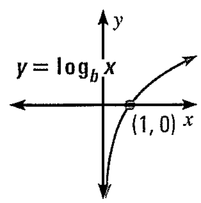
$b > 1$

Exponential Decay

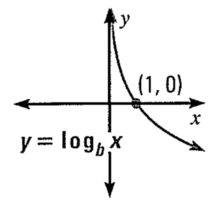


$0 < b < 1$

Logarithmic Functions



$b > 1$



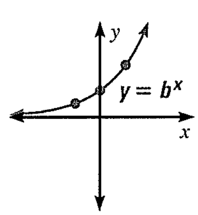
$0 < b < 1$

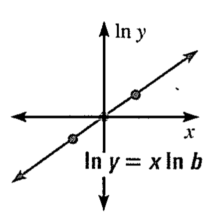
Big Idea 2: Solving Exponential and Logarithmic Equations

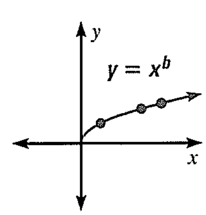
Solving an Exponential Equation	Solving a Logarithmic Equation
If each side can be written using the same base, equate exponents. $3^{x+1} = 9^x$; $3^{x+1} = (3^2)^x$; $x + 1 = 2x$; $1 = x$	If the equation has the form $\log_b x = \log_b y$, use the fact that $x = y$. $\log_2 (4x - 2) = \log_2 3x$; $4x - 2 = 3x$; $x = 2$
If each side cannot be written using the same base, take a logarithm of each side. $6^x = 15$; $\log_6 6^x = \log_6 15$; $x = \frac{\log 15}{\log 6} \approx 1.511$	If a logarithm is set equal to a constant, exponentiate each side. $\log_5 (x + 1) = 2$; $x + 1 = 5^2$; $x = 24$

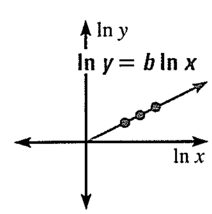
Big Idea 3: Writing and Applying Exponential and Power Functions

Write an Exponential Model	Write a Power Model
An exponential model fits a set of data pairs (x, y) if a linear model fits the set of data pairs $(x, \ln y)$.	A power model fits a set of data pairs (x, y) if a linear model fits the set of data pairs $(\ln x, \ln y)$.









7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- exponential function, *p. 478*
- exponential growth function, *p. 478*
- growth factor, *p. 478*
- asymptote, *p. 478*
- exponential decay function, *p. 486*
- decay factor, *p. 486*
- natural base *e*, *p. 492*
- logarithm of *y* with base *b*, *p. 499*
- common logarithm, *p. 500*
- natural logarithm, *p. 500*
- exponential equation, *p. 515*
- logarithmic equation, *p. 517*

VOCABULARY EXERCISES

1. What is the asymptote of the graph of the function $y = -2\left(\frac{1}{4}\right)^{x+1} + 5$?
2. Identify the decay factor in the model $y = 7.2(0.89)^x$.
3. **WRITING** *Explain* the meaning of $\log_b y$.
4. Copy and complete: A logarithm with base *e* is called a(n) __?__ logarithm.
5. Is $y = (1.4)^x$ an *exponential function* or a *power function*? *Explain.*

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

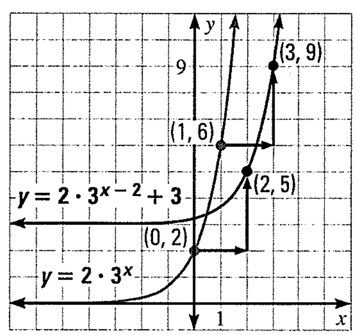
7.1 Graph Exponential Growth Functions *pp. 478–485*

EXAMPLE

Graph $y = 2 \cdot 3^{x-2} + 3$. State the domain and range.

Begin by sketching the graph of $y = 2 \cdot 3^x$, which passes through (0, 2) and (1, 6). Then translate the graph right 2 units and up 3 units. Notice that the translated graph passes through (2, 5) and (3, 9).

The graph's asymptote is the line $y = 3$. The domain is all real numbers, and the range is $y > 3$.



EXERCISES

EXAMPLES 1, 2, 3, and 5 on pp. 478–481 for Exs. 6–9

Graph the function. State the domain and range.

6. $y = 5^x$
7. $y = 3(2.5)^x$
8. $f(x) = -3 \cdot 4^{x+1} - 2$
9. **FINANCE** You deposit $1500 in an account that pays 7% annual interest compounded daily. Find the balance after 2 years.

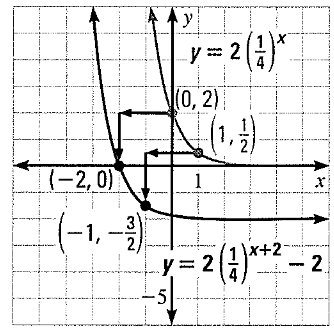
7 CHAPTER REVIEW

7.2 Graph Exponential Decay Functions

pp. 486–491

EXAMPLE

Graph $y = 2\left(\frac{1}{4}\right)^{x+2} - 2$. State the domain and range.



Begin by sketching the graph of $y = 2\left(\frac{1}{4}\right)^x$, which passes through (0, 2) and $\left(1, \frac{1}{2}\right)$. Then translate the graph left 2 units and down 2 units. Notice that the translated graph passes through $(-2, 0)$ and $\left(-1, -\frac{3}{2}\right)$.

The graph's asymptote is the line $y = -2$. The domain is all real numbers, and the range is $y > -2$.

EXAMPLES 1, 2, and 3 on pp. 486–487 for Exs. 10–12

EXERCISES

Graph the function. State the domain and range.

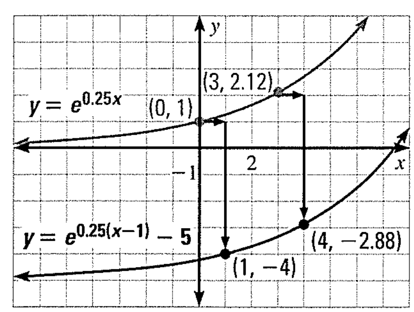
10. $y = \left(\frac{1}{8}\right)^x$
11. $y = \left(\frac{1}{3}\right)^x - 4$
12. $f(x) = 2(0.8)^{x-1} + 3$

7.3 Use Functions Involving *e*

pp. 492–498

EXAMPLE

Graph $y = e^{0.25(x-1)} - 5$. State the domain and range.



Because $a = 1$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Begin by sketching the graph of $y = e^{0.25x}$. Translate the graph right 1 unit and down 5 units.

The domain is all real numbers, and the range is $y > -5$.

EXAMPLES 3 and 5 on pp. 493–495 for Exs. 13–16

EXERCISES

Graph the function. State the domain and range.

13. $y = 2e^{-x}$
14. $y = e^{x-2}$
15. $f(x) = e^{-0.4(x+2)} + 6$

16. **PHYSIOLOGY** Nitrogen-13 is a radioactive isotope of nitrogen used in a physiological test called positron emission tomograph (PET). A typical PET scan begins with 6.9 picograms of nitrogen-13 (1 picogram = 10^{-12} grams). The number N of picograms of nitrogen-13 remaining after t minutes can be modeled by $N = 6.9e^{-0.0695t}$. How many picograms of nitrogen-13 remain after 10 minutes?

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Chapter Review Practice

7.4 Find Logarithms and Graph Logarithmic Functions *pp. 499–505*

EXAMPLE

Evaluate the logarithm.

a. $\log_5 625$ b. $\log 0.001$ c. $\log_{125} 5$ d. $\log_2 \frac{1}{64}$

To help you find the value of $\log_b y$, ask yourself what power of b gives you y.

a. 5 to what power gives 625?
$5^4 = 625$, so $\log_5 625 = \mathbf{4}$.

b. 10 to what power gives 0.001?
$10^{-3} = 0.001$, so $\log 0.001 = \mathbf{-3}$.

c. 125 to what power gives 5?
$125^{1/3} = 5$, so $\log_{125} 5 = \mathbf{\frac{1}{3}}$.

d. 2 to what power gives $\frac{1}{64}$?
$2^{-6} = \frac{1}{64}$, so $\log_2 \frac{1}{64} = \mathbf{-6}$.

EXERCISES

EXAMPLES 2, 4, 7, and 8 on pp. 500–503 for Exs. 17–24

Evaluate the logarithm without using a calculator.

17. $\log_3 243$ **18.** $\log_7 1$ **19.** $\log_{1/6} 216$ **20.** $\log_{125} \frac{1}{5}$

Graph the function. State the domain and range.

21. $y = \log_{1/6} x$ **22.** $y = \log_3 x - 4$ **23.** $f(x) = \ln (x - 1) + 3$

24. BIOLOGY Researchers have found that after 25 years of age, the average size of the pupil in a person's eye decreases. The relationship between pupil diameter d (in millimeters) and age a (in years) can be modeled by $d = -2.1158 \ln a + 13.669$. What is the average diameter of a pupil for a person 25 years old? 50 years old?

7.5 Apply Properties of Logarithms *pp. 507–513*

EXAMPLES

Expand the expression.

$$\log_5 \frac{6x}{y^3} = \log_5 6x - \log_5 y^3$$
$$= \log_5 6 + \log_5 x - \log_5 y^3$$
$$= \log_5 6 + \log_5 x - 3 \log_5 y$$

Condense the expression.

$$3 \log_3 8 - \log_3 16 = \log_3 8^3 - \log_3 16$$
$$= \log_3 \frac{8^3}{16}$$
$$= \log_3 32$$

EXERCISES

EXAMPLES 2 and 3 on p. 508 for Exs. 25–31

Expand the expression.

25. $\log_8 3xy$ **26.** $\ln 10x^3y$ **27.** $\log \frac{8}{y^4}$ **28.** $\ln \frac{3y}{x^5}$

Condense the expression.

29. $3 \log_7 4 + \log_7 6$ **30.** $\ln 12 - 2 \ln x$ **31.** $2 \ln 3 + 5 \ln 2 - \ln 8$

MULTIPLE CHOICE QUESTIONS

If you have difficulty solving a multiple choice problem directly, you may be able to use another approach. First, eliminate as many wrong answers as you can. Then, make an educated guess from among the remaining choices.

PROBLEM 1

Which exponential function has a graph that passes through the points (2, −12) and (4, −48)?

A $y = 3 \cdot 2^x$ B $y = -\sqrt{3} \cdot 2^x$ C $y = -\frac{4}{3} \cdot 3^x$ D $y = -3 \cdot 2^x$

METHOD 1

SOLVE DIRECTLY Substitute the coordinates of the two points into $y = ab^x$ and solve the resulting system.

STEP 1 **Substitute** the coordinates of the points into $y = ab^x$.

$-12 = ab^2$ Substitute (2, −12).

$-48 = ab^4$ Substitute (4, −48).

STEP 2 **Solve** the first equation for a.

$\frac{-12}{b^2} = a$ Divide each side by b^2.

STEP 3 **Substitute** $\frac{-12}{b^2}$ for a in the second equation and solve for b.

$-48 = \left(\frac{-12}{b^2}\right)b^4$ Substitute.

$-48 = -12b^2$ Simplify.

$4 = b^2$ Divide each side by −12.

$2 = b$ Take the positive square root because $b > 0$.

STEP 4 **Substitute** the value of b into $a = \frac{-12}{b^2}$ to find the value of a.

$$a = \frac{-12}{b^2} = \frac{-12}{2^2} = \frac{-12}{4} = -3$$

The equation is $y = -3 \cdot 2^x$.

▸ The correct answer is D.

METHOD 2

ELIMINATE CHOICES Another method is to check whether both of the points are solutions of the equations given in the answer choices.

Substitute the coordinates of the points into the equation in each answer choice. You can stop as soon as you realize that one of the points is not a solution.

Choice A: $y = 3 \cdot 2^x$

$-12 \stackrel{?}{=} 3 \cdot 2^2$

$-12 \neq 12$

Choice B: $y = -\sqrt{3} \cdot 2^x$

$-12 \stackrel{?}{=} -\sqrt{3} \cdot 2^2$

$-12 \neq -4\sqrt{3}$

Choice C: $y = -\frac{4}{3} \cdot 3^x$ $y = -\frac{4}{3} \cdot 3^x$

$-12 \stackrel{?}{=} -\frac{4}{3} \cdot 3^2$ $-48 \stackrel{?}{=} -\frac{4}{3} \cdot 3^4$

$-12 = -12$ ✓ $-48 \neq -108$

Choice D: $y = -3 \cdot 2^x$ $y = -3 \cdot 2^x$

$-12 \stackrel{?}{=} -3 \cdot 2^2$ $-48 \stackrel{?}{=} -3 \cdot 2^4$

$-12 = -12$ ✓ $-48 = -48$ ✓

▸ The correct answer is D.

PROBLEM 2

You buy a new personal computer for \$1600. It is estimated that the computer's value will decrease by 50% each year. After about how many years will the computer be worth \$250?

A $1\frac{1}{3}$ years B 2 years C $2\frac{2}{3}$ years D 3 years

METHOD 1

SOLVE DIRECTLY Write and solve an equation to find the time it takes for the computer to depreciate to \$250.

Let y be the value (in dollars) of the computer t years after the purchase. An exponential decay model for the value is:

$$y = a(1 - r)^t$$
$$250 = 1600(1 - 0.5)^t$$
$$0.156 \approx (0.5)^t$$
$$\log_{0.5} 0.156 = \log_{0.5} (0.5)^t$$
$$t \approx \frac{\log 0.156}{\log 0.5} \approx 2.68 \text{ years}$$

The computer will be worth \$250 after about $2.68 \approx 2\frac{2}{3}$ years.

▶ The correct answer is C.

METHOD 2

ELIMINATE CHOICES Use estimation to find how long it will take for the computer to depreciate to \$250.

The computer depreciates by 50% each year.

After 0 years, it is worth \$1600.

After 1 year, it will be worth 0.5(\$1600) = \$800.

After 2 years, it will be worth 0.5(\$800) = \$400.

After 3 years, it will be worth 0.5(\$400) = \$200.

The computer will be worth \$250 at some time *between* 2 and 3 years after purchase. So, you can eliminate choices A, B, and D.

▶ The correct answer is C.

PRACTICE

Explain **why you can eliminate the highlighted answer choice.**

1. Which power function has a graph that passes through the points (−2, −16) and (1, 2)?

   A $y = \frac{1}{2}x^5$ B $y = 2x^5$ C ✗ $y = 2x^{1/2}$ D $y = 2x^3$

2. For which equation is 4 a solution?

   A ✗ $e^{2x} + 1 = 17$ B $\log_2 (x + 2) = \log_2 2x$
   C $3^{x-2} + 1 = 10$ D $\ln (x + 2) + \ln x = 1$

3. What is the domain of the function $y = -5 \cdot 2^{x+2}$?

   A All real numbers B ✗ All real numbers except −2
   C All real numbers less than 0 D All real numbers greater than −5

TEST PREPARATION

MULTIPLE CHOICE

1. In 1999, the tuition for one year at Harvard University was \$22,054. During the next 4 years, the tuition increased by 4.25% each year. Which model represents the situation?

A $y = 22{,}054(0.0425)^t$

B $y = 22{,}054(0.425)^t$

C $y = 22{,}054(1.0425)^t$

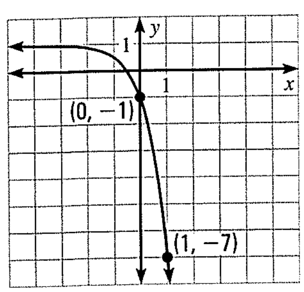
D $y = 22{,}054(1.425)^t$

2. You deposit \$1000 in an account that pays 3% annual interest. How much more interest is earned after 2 years if the interest is compounded daily than if the interest is compounded monthly?

A \$.07 B \$1.27

C \$61.76 D \$1061.76

3. The graph of which function is shown?



A $y = 2 \cdot 4^x + 1$ B $y = -2 \cdot 4^x + 1$

C $y = 2 \cdot 4^x - 1$ D $y = -2 \cdot 4^x - 1$

4. Which function can be obtained by translating the graph of $y = \log_3 (x + 2) - 4$ left 1 unit?

A $y = \log_3 (x + 2) - 5$

B $y = \log_3 (x + 2) - 3$

C $y = \log_3 (x + 1) - 4$

D $y = \log_3 (x + 3) - 4$

5. Which expression is equivalent to $3 \log x + \log 3$?

A $\log 9x$ B $4 \log 3x$

C $\log 3x^3$ D $\log (x^3 + 3)$

6. The population of the United States is expected to increase by 0.9% each year from 2003 to 2014. The U.S. population was about 290 million in 2003. To the nearest million, what is the projected population for 2010?

A 295 million

B 309 million

C 574 million

D 4891 million

7. Which expression is equivalent to $\sqrt{100e^{6x}}$?

A $10e^{3x}$ B $7e^x + 3e^{2x}$

C $(2e^x)^3$ D $10e^{\sqrt{6x}}$

8. Which function is an exponential decay function?

A $y = 2 \cdot 5^x$ B $y = -2 \cdot 5^x$

C $y = 2e^x$ D $y = 2(0.5)^x$

9. Which function is the inverse of $y = e^{2x} - 3$?

A $y = 2 \ln (x + 3)$ B $y = 2 \ln (x - 3)$

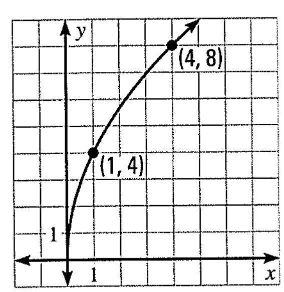
C $y = \dfrac{\ln (x + 3)}{2}$ D $y = \dfrac{\ln (x - 3)}{2}$

10. The graph of $f(x) = ab^x$ passes through the points (0, 8) and (2, 2). What is the value of $f(5)$?

A $\frac{1}{4}$ B 2

C 8 D 40

11. The graph of which function is shown?



A $y = 4x^{0.25}$ B $y = 4x^{0.5}$

C $y = \frac{1}{2}x^2$ D $y = 2x^{1/3}$

MULTIPLE CHOICE

12. What is the solution of the equation $\log_3 (5x + 3) = 5$?

    A $\frac{2}{5}$ B $\frac{4}{15}$ C $\frac{12}{5}$ D 48

13. What is the solution set of the equation $\log(x + 3) + \log x = 1$?

    A $\{2, -5\}$ B $\{2\}$ C $\{5\}$ D $\{2, 5\}$

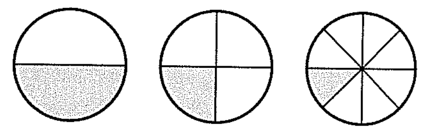
14. The model $y = 7.7e^{0.14x}$ gives the number y (in thousands per cubic centimeter) of bacteria in a liquid culture after x hours. To the nearest tenth of an hour, after how many hours will there be 50,000 bacteria per cubic centimeter?

    A 1.87 hr B 13.4 hr
    C 46.4 hr D 62.7 hr

15. What is the solution to the equation $9^{2x+1} = 3^{5x-1}$?

    A $\frac{3}{8}$ B $\frac{2}{3}$ C 2 D 3

16. The diagram below shows the first three stages of a sequence. What fraction of the circle is shaded in the nth stage?



Stage 1 Stage 2 Stage 3

    A $\frac{1}{2n}$ B $2n$ C 2^n D 2^{-n}

OPEN-ENDED

17. A movie grosses \$37 million in its first week of release. The weekly gross y decreases by 30% each week.

    A. Write an exponential decay model for the weekly gross in week x.

    B. What is a reasonable domain for this situation? *Explain.*

18. The table shows the number of transistors per integrated circuit for computers introduced in various years.

    A. Find an exponential model for the original data.

    B. Use your model to predict the number of transistors per integrated circuit in 2008.

    C. In 1965, Gordon Moore made an observation that became known as Moore's law. Moore's law states that the number of transistors per integrated circuit would double about every 18 months. According to your model, does Moore's law hold? *Explain* your reasoning.

Year	Transistors
1974	6,000
1979	29,000
1982	134,000
1985	275,000
1989	1,200,000
1993	3,100,000
1997	7,500,000
1999	9,500,000
2000	42,000,000
2004	125,000,000

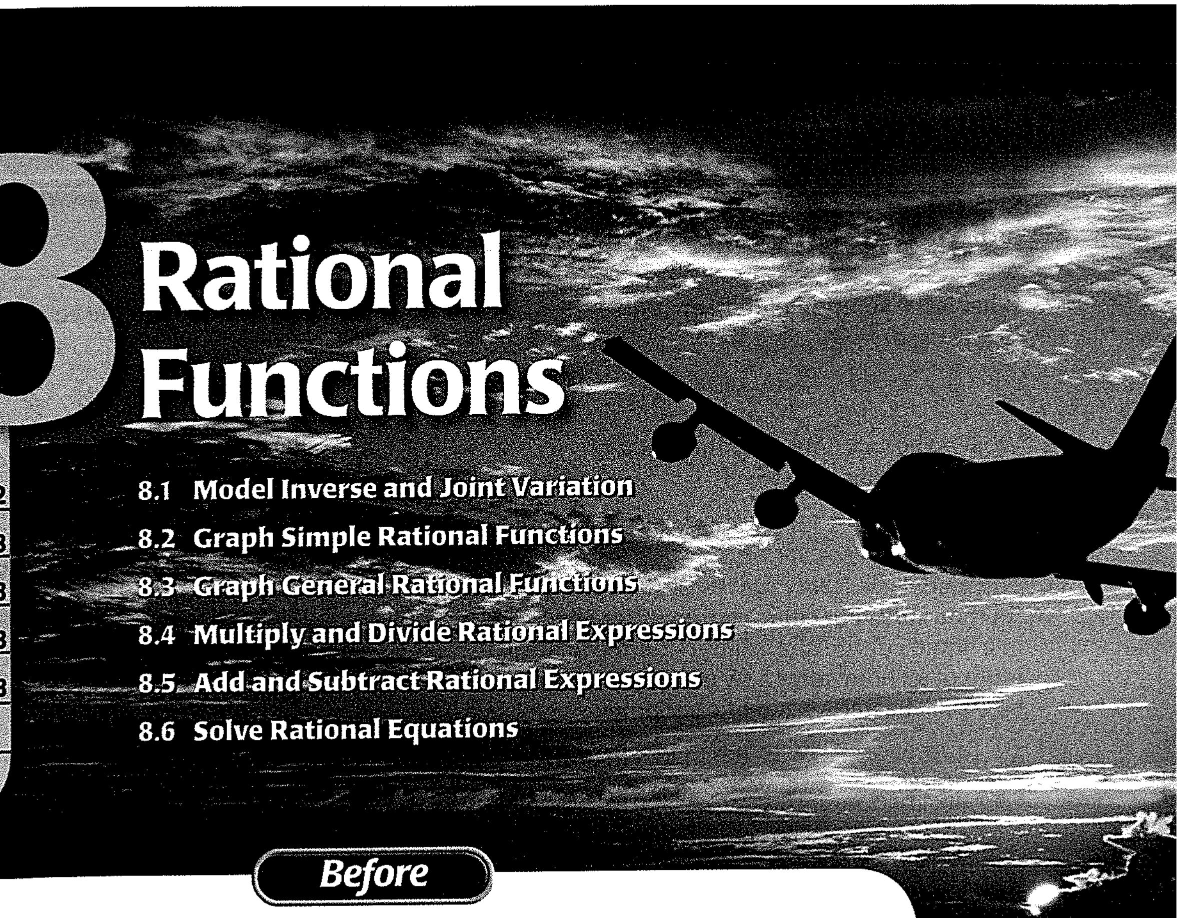
19. For a temperature of 60°F and a height of h feet above sea level, the air pressure P (in pounds per square inch) can be modeled by the equation $P = 14.7e^{-0.0004h}$, and the air density D (in pounds per cubic foot) can be modeled by the equation $D = 0.0761e^{-0.0004h}$.

    A. What is the air density at 10,000 feet above sea level?

    B. To the nearest foot, at what height is the air pressure 12 pounds per square inch?

    C. What is the relationship between air pressure and air density at 60°F? *Explain* your reasoning.

    D. One rule of thumb states that air pressure decreases by about 1% for every 80 meter increase in altitude. Do you agree with this? *Explain.*



8 Rational Functions

PA

M11.A.2.1.2 8.1 Model Inverse and Joint Variation

M11.D.1.1.3 8.2 Graph Simple Rational Functions

M11.D.1.1.3 8.3 Graph General Rational Functions

M11.D.2.2.3 8.4 Multiply and Divide Rational Expressions

M11.D.2.2.3 8.5 Add and Subtract Rational Expressions

8.6 Solve Rational Equations

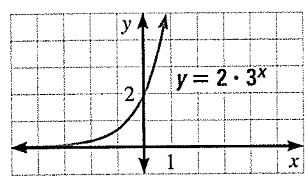
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: writing direct variation equations, factoring polynomials, and performing polynomial operations.

Prerequisite Skills

VOCABULARY CHECK

1. The **asymptote** of the graph at the right is _?_.
2. Two variables x and y show **direct variation** provided _?_ where a is a nonzero constant.
3. An **extraneous solution** of a transformed equation is not an actual _?_ of the original equation.



SKILLS CHECK

The variables x and y vary directly. Write an equation that relates x and y. Then find the value of y when $x = -2$. ***(Review p. 107 for 8.1.)***

4. $x = 2, y = 8$
5. $x = -1, y = 4$
6. $x = 12, y = 2$

Factor the polynomial completely. ***(Review pp. 252, 353 for 8.4, 8.5.)***

7. $x^2 - 11x - 26$
8. $2x^3 - 4x^2 + 2x$
9. $6x^4 - 4x^3 - 24x + 16$

Perform the indicated operation. ***(Review p. 346 for 8.4, 8.5.)***

10. $(3x^2 - 6) + (7x^2 - x)$
11. $(-2x^2 + 6) - (x^2 - x)$
12. $(x + 2)(x - 9)^2$