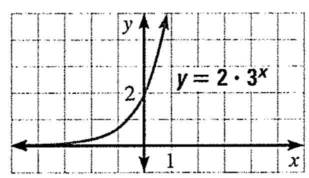
8 Rational Functions

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: writing direct variation equations, factoring polynomials, and performing polynomial operations.

Prerequisite Skills

VOCABULARY CHECK



1. The **asymptote** of the graph at the right is _?_.
2. Two variables x and y show **direct variation** provided _?_ where a is a nonzero constant.
3. An **extraneous solution** of a transformed equation is not an actual _?_ of the original equation.

SKILLS CHECK

The variables x and y vary directly. Write an equation that relates x and y. Then find the value of y when $x = -2$. *(Review p. 107 for 8.1.)*

4. $x = 2, y = 8$ **5.** $x = -1, y = 4$ **6.** $x = 12, y = 2$

Factor the polynomial completely. *(Review pp. 252, 353 for 8.4, 8.5.)*

7. $x^2 - 11x - 26$ **8.** $2x^3 - 4x^2 + 2x$ **9.** $6x^4 - 4x^3 - 24x + 16$

Perform the indicated operation. *(Review p. 346 for 8.4, 8.5.)*

10. $(3x^2 - 6) + (7x^2 - x)$ **11.** $(-2x^2 + 6) - (x^2 - x)$ **12.** $(x + 2)(x - 9)^2$

MULTIPLE CHOICE

12. What is the solution of the equation $\log_3 (5x + 3) = 5$?

A $\frac{2}{5}$ B $\frac{4}{15}$ C $\frac{12}{5}$ D 48

13. What is the solution set of the equation $\log(x + 3) + \log x = 1$?

A $\{2, -5\}$ B $\{2\}$ C $\{5\}$ D $\{2, 5\}$

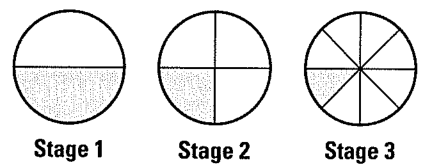
14. The model $y = 7.7e^{0.14x}$ gives the number y (in thousands per cubic centimeter) of bacteria in a liquid culture after x hours. To the nearest tenth of an hour, after how many hours will there be 50,000 bacteria per cubic centimeter?

A 1.87 hr B 13.4 hr
C 46.4 hr D 62.7 hr

15. What is the solution to the equation $9^{2x+1} = 3^{5x-1}$?

A $\frac{3}{8}$ B $\frac{2}{3}$ C 2 D 3

16. The diagram below shows the first three stages of a sequence. What fraction of the circle is shaded in the nth stage?



A $\frac{1}{2n}$ B $2n$ C 2^n D 2^{-n}

OPEN-ENDED

17. A movie grosses \$37 million in its first week of release. The weekly gross y decreases by 30% each week.

A. Write an exponential decay model for the weekly gross in week x.

B. What is a reasonable domain for this situation? *Explain.*

18. The table shows the number of transistors per integrated circuit for computers introduced in various years.

A. Find an exponential model for the original data.

B. Use your model to predict the number of transistors per integrated circuit in 2008.

C. In 1965, Gordon Moore made an observation that became known as Moore's law. Moore's law states that the number of transistors per integrated circuit would double about every 18 months. According to your model, does Moore's law hold? *Explain* your reasoning.

Year	Transistors
1974	6,000
1979	29,000
1982	134,000
1985	275,000
1989	1,200,000
1993	3,100,000
1997	7,500,000
1999	9,500,000
2000	42,000,000
2004	125,000,000

19. For a temperature of 60°F and a height of h feet above sea level, the air pressure P (in pounds per square inch) can be modeled by the equation $P = 14.7e^{-0.0004h}$, and the air density D (in pounds per cubic foot) can be modeled by the equation $D = 0.0761e^{-0.0004h}$.

A. What is the air density at 10,000 feet above sea level?

B. To the nearest foot, at what height is the air pressure 12 pounds per square inch?

C. What is the relationship between air pressure and air density at 60°F? *Explain* your reasoning.

D. One rule of thumb states that air pressure decreases by about 1% for every 80 meter increase in altitude. Do you agree with this? *Explain.*

Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 602. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing rational functions**
2. **Performing operations with rational expressions**
3. **Solving rational equations**

KEY VOCABULARY

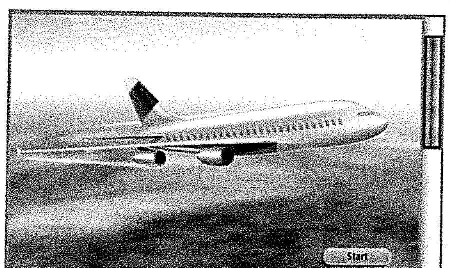
- inverse variation, *p. 551*
- constant of variation, *p. 551*
- joint variation, *p. 553*
- rational function, *p. 558*
- simplified form of a rational expression, *p. 573*
- complex fraction, *p. 584*
- cross multiplying, *p. 589*

Why?

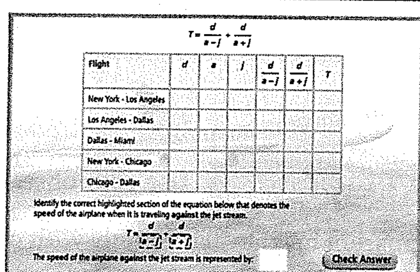
You can use rational functions to model real-life situations. For example, you can model the time it takes to travel across the United States and back in an airplane.

Animated Algebra

The animation illustrated below for Exercise 41 on page 587 helps you answer this question: How does the time required to fly from New York to Los Angeles and back depend on the speeds of the airplane and the jet stream?



The winds of the jet stream affect the overall speed of an airplane.



Examine how flying with the wind or against the wind affects the flight time.

Animated Algebra at classzone.com

Other animations for Chapter 8: pages 554, 559, 568, and 602

8.1 Investigating Inverse Variation

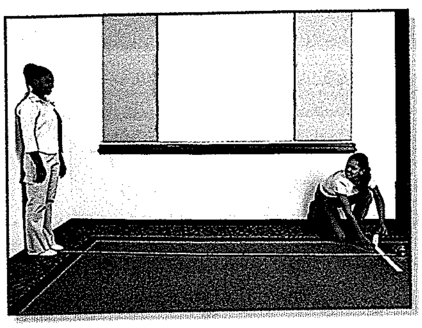
MATERIALS • tape measure or meter stick • centimeter ruler • masking tape

QUESTION **How can you model data that show inverse variation?**

EXPLORE **Collect and record data**

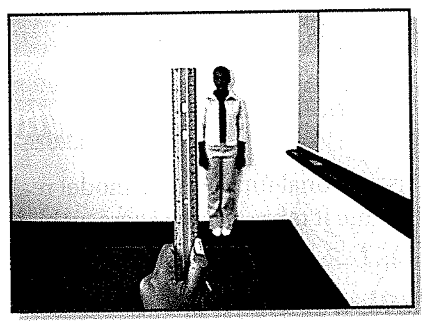
STEP 1 ***Mark distances***

Work with a partner. Have your partner stand against a wall. Place the end of the tape measure against the wall between your partner's feet. Use tape to mark off distances from 3 meters to 9 meters away from the wall.



STEP 2 ***Measure apparent height***

Face your partner, with your toes touching the 3 meter mark. Hold a centimeter ruler at arm's length and line up the "0" end of the ruler with the top of your partner's head. Measure the apparent height of your partner to the nearest centimeter.



STEP 3 ***Repeat for other distances***

Repeat Step 2 for each marked distance and record your results in a table like the one shown.

Distance (m), x	3	4	5	6	7	8	9
Apparent height (cm), y	?	?	?	?	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Does apparent height vary directly with distance? *Justify* your answer mathematically.
2. Find the product $x \cdot y$ for each ordered pair in the table. What do you notice?
3. Based on your results from Exercise 2, write an equation relating distance and apparent height.
4. Use your equation to predict your partner's apparent height at an unmeasured distance. Then test your prediction by measuring your partner's apparent height at that distance. How close was your prediction?

8.1 Model Inverse and Joint Variation

PA M11.A.2.1.2 Solve problems using direct and inverse proportions.

Before You wrote and used direct variation models.

Now You will use inverse variation and joint variation models.

Why? So you can model music frequencies, as in Ex. 40.

Key Vocabulary
- **inverse variation**
- **constant of variation**
- **joint variation**

You have learned that two variables x and y show direct variation if $y = ax$ for some nonzero constant a. Another type of variation is called *inverse variation.*

KEY CONCEPT *For Your Notebook*

Inverse Variation

Two variables x and y show **inverse variation** if they are related as follows:

$$y = \frac{a}{x},\ a \neq 0$$

The constant a is the **constant of variation**, and y is said to *vary inversely* with x.

EXAMPLE 1 Classify direct and inverse variation

Tell whether x and y show *direct variation, inverse variation,* or *neither.*

Given Equation	Rewritten Equation	Type of Variation
a. $xy = 7$	$y = \frac{7}{x}$	Inverse
b. $y = x + 3$		Neither
c. $\frac{y}{4} = x$	$y = 4x$	Direct

REVIEW DIRECT VARIATION
The equation in part (b) does not show direct variation because $y = x + 3$ is not of the form $y = ax$.

EXAMPLE 2 Write an inverse variation equation

The variables x and y vary inversely, and $y = 7$ when $x = 4$. Write an equation that relates x and y. Then find y when $x = -2$.

$y = \frac{a}{x}$ **Write general equation for inverse variation.**

$7 = \frac{a}{4}$ **Substitute 7 for y and 4 for x.**

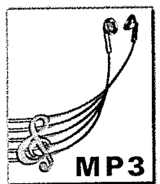
$28 = a$ **Solve for a.**

▶ The inverse variation equation is $y = \frac{28}{x}$. When $x = -2$, $y = \frac{28}{-2} = -14$.

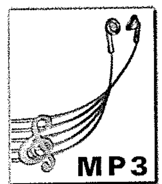
EXAMPLE 3 Write an inverse variation model

MP3 PLAYERS The number of songs that can be stored on an MP3 player varies inversely with the average size of a song. A certain MP3 player can store 2500 songs when the average size of a song is 4 megabytes (MB).

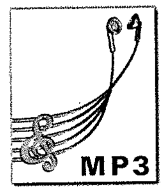
- Write a model that gives the number n of songs that will fit on the MP3 player as a function of the average song size s (in megabytes).
- Make a table showing the number of songs that will fit on the MP3 player if the average size of a song is 2 MB, 2.5 MB, 3 MB, and 5 MB as shown below. What happens to the number of songs as the average song size increases?



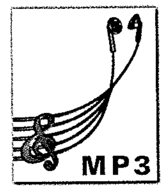
2 MB



2.5 MB



3 MB



5 MB

Solution

STEP 1 **Write** an inverse variation model.

$n = \frac{a}{s}$ **Write general equation for inverse variation.**

$2500 = \frac{a}{4}$ **Substitute 2500 for *n* and 4 for *s*.**

$10{,}000 = a$ **Solve for *a*.**

▶ A model is $n = \frac{10{,}000}{s}$.

STEP 2 **Make** a table of values.

Average size of song (MB), *s*	2	2.5	3	5
Number of songs, *n*	5000	4000	3333	2000

▶ From the table, you can see that the number of songs that will fit on the MP3 player decreases as the average song size increases.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

1. $3x = y$
2. $xy = 0.75$
3. $y = x - 5$

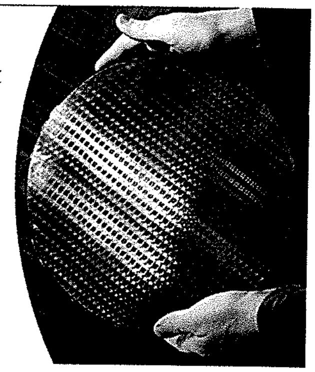
The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 2$.

4. $x = 4, y = 3$
5. $x = 8, y = -1$
6. $x = \frac{1}{2}, y = 12$

7. **WHAT IF?** In Example 3, what is a model for the MP3 player if it stores 3000 songs when the average song size is 5 MB?

CHECKING FOR INVERSE VARIATION The general equation $y = \frac{a}{x}$ for inverse variation can be rewritten as $xy = a$. This tells you that a set of data pairs (x, y) shows inverse variation if the products xy are constant or approximately constant.

EXAMPLE 4 Check data for inverse variation



COMPUTER CHIPS The table compares the area A (in square millimeters) of a computer chip with the number c of chips that can be obtained from a silicon wafer.

- Write a model that gives c as a function of A.
- Predict the number of chips per wafer when the area of a chip is 81 square millimeters.

Area (mm^2), A	58	62	66	70
Number of chips, c	448	424	392	376

AVOID ERRORS
To check data pairs (x, y) for *direct* variation, you find the *quotients* $\frac{y}{x}$. However, to check data pairs for *inverse* variation, you find the *products xy*.

Solution

STEP 1 **Calculate** the product $A \cdot c$ for each data pair in the table.

$58(448) = 25{,}984$

$62(424) = 26{,}288$

$66(392) = 25{,}872$

$70(376) = 26{,}320$

Each product is approximately equal to 26,000. So, the data show inverse variation. A model relating A and c is:

$$A \cdot c = 26{,}000, \text{ or } c = \frac{26{,}000}{A}$$

STEP 2 **Make** a prediction. The number of chips per wafer for a chip with an area of 81 square millimeters is $c = \frac{26{,}000}{81} \approx 321$.

✓ GUIDED PRACTICE for Example 4

8. **WHAT IF?** In Example 4, predict the number of chips per wafer when the area of each chip is 79 square millimeters.

KEY CONCEPT *For Your Notebook*

Joint Variation

Joint variation occurs when a quantity varies directly with *the product of two or more* other quantities. In the equations below, a is a nonzero constant.

$z = axy$ **z varies jointly with x and y.**

$p = aqrs$ **p varies jointly with q, r, and s.**

EXAMPLE 5 Write a joint variation equation

The variable z varies jointly with x and y. Also, $z = -75$ when $x = 3$ and $y = -5$. Write an equation that relates x, y, and z. Then find z when $x = 2$ and $y = 6$.

Solution

STEP 1 **Write** a general joint variation equation.

$z = axy$

STEP 2 **Use** the given values of z, x, and y to find the constant of variation a.

$-75 = a(3)(-5)$	**Substitute −75 for z, 3 for x, and −5 for y.**
$-75 = -15a$	**Simplify.**
$5 = a$	**Solve for a.**

STEP 3 **Rewrite** the joint variation equation with the value of a from Step 2.

$z = 5xy$

STEP 4 **Calculate** z when $x = 2$ and $y = 6$ using substitution.

$z = 5xy = 5(2)(6) = 60$

EXAMPLE 6 Compare different types of variation

Write an equation for the given relationship.

Relationship	**Equation**
a. y varies inversely with x.	$y = \frac{a}{x}$
b. z varies jointly with x, y, and r.	$z = axyr$
c. y varies inversely with the square of x.	$y = \frac{a}{x^2}$
d. z varies directly with y and inversely with x.	$z = \frac{ay}{x}$
e. x varies jointly with t and r and inversely with s.	$x = \frac{atr}{s}$

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Examples 5 and 6

The variable z varies jointly with x and y. Use the given values to write an equation relating x, y, and z. Then find z when $x = -2$ and $y = 5$.

9. $x = 1, y = 2, z = 7$

10. $x = 4, y = -3, z = 24$

11. $x = -2, y = 6, z = 18$

12. $x = -6, y = -4, z = 56$

Write an equation for the given relationship.

13. x varies inversely with y and directly with w.

14. p varies jointly with q and r and inversely with s.

8.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS14 for Exs. 15, 21, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 30, 35, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If z varies directly with the product of x and y, then z is said to vary __?__ with x and y.

2. ★ **WRITING** *Describe* how to tell whether a set of data pairs (x, y) shows inverse variation.

EXAMPLE 1 on p. 551 for Exs. 3–11

DETERMINING VARIATION Tell whether x and y show *direct variation, inverse variation,* or *neither.*

3. $xy = \frac{1}{5}$
4. $y = x + 4$
5. $\frac{y}{x} = 8$
6. $4x = y$
7. $y = \frac{2}{x}$
8. $x + y = 6$
9. $8y = x$
10. $xy = 12$

11. ★ **MULTIPLE CHOICE** Which equation represents inverse variation?

(A) $y = 4x$ (B) $y = x - 1$ (C) $xy = 5$ (D) $\frac{y}{7} = x$

EXAMPLE 2 on p. 551 for Exs. 12–19

USING INVERSE VARIATION The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 3$.

12. $x = 5, y = -4$
13. $x = 1, y = 9$
14. $x = -3, y = 8$
15. $x = 7, y = 2$
16. $x = \frac{3}{4}, y = 28$
17. $x = -4, y = -\frac{5}{4}$
18. $x = -12, y = -\frac{1}{6}$
19. $x = \frac{5}{3}, y = -7$

EXAMPLE 4 on p. 553 for Exs. 20–23

INTERPRETING DATA Determine whether x and y show *direct variation, inverse variation,* or *neither.*

20.

x	y
1.5	40
2.5	24
4	15
7.5	8
10	6

21.

x	y
12	132
18	198
23	253
29	319
34	374

22.

x	y
4	16
5	11
6.2	10
7	9
11	6

23.

x	y
4	21
6	14
8	10.5
8.4	10
12	7

EXAMPLE 5 on p. 554 for Exs. 24–30

USING JOINT VARIATION Write an equation relating x, y, and z given that z varies jointly with x and y. Then find z when $x = -4$ and $y = 5$.

24. $x = 2, y = -6, z = 24$
25. $x = 8, y = 6, z = 12$
26. $x = -\frac{1}{4}, y = -3, z = 15$
27. $x = 6, y = -7, z = -3$
28. $x = 9, y = -2, z = 6$
29. $x = 5, y = -3, z = 75$

30. ★ **MULTIPLE CHOICE** Suppose z varies jointly with x and y, and $z = -36$ when $x = -3$ and $y = -4$. What is the constant of variation?

(A) -3 (B) -2 (C) 3 (D) 12

EXAMPLE 6
on p. 554
for Exs. 31–33

WRITING EQUATIONS Write an equation for the given relationship.

31. x varies directly with y and inversely with z.

32. y varies jointly with x and the square of z.

33. w varies inversely with y and jointly with x and z.

34. **ERROR ANALYSIS** A variable z varies jointly with x and the cube of y and inversely with the square root of w. *Describe* and correct the error in writing an equation relating the variables.

$$z = \frac{a\sqrt{w}}{xy^3}$$

35. ★ **OPEN-ENDED MATH** Let $f(x)$ represent a direct variation function, $g(x)$ represent an inverse variation function, and $h(x)$ be the sum of $f(x)$ and $g(x)$. Write possible functions $f(x)$ and $g(x)$ so that $h(2) = 5$.

36. **CHALLENGE** Suppose x varies inversely with y and y varies inversely with z. How does x vary with z? *Justify* your answer algebraically.

PROBLEM SOLVING

EXAMPLES 3 and 4
on pp. 552–553
for Exs. 37–39

37. **DIGITAL CAMERAS** The number n of photos your digital camera can store varies inversely with the average size s (in megapixels) of the photos. Your digital camera can store 54 photos when the average photo size is 1.92 megapixels. Write a model that gives n as a function of s. How many photos can your camera store when the average photo size is 3.87 megapixels?

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38. **ELECTRONICS** The table below compares the current I (in milliamps) with the resistance R (in ohms) for several electrical circuits. Write a model that gives R as a function of I. Then predict R when $I = 34$ milliamps.

Current (milliamps), *I*	7.4	8.9	12.1	17.9
Resistance (ohms), *R*	1200	1000	750	500

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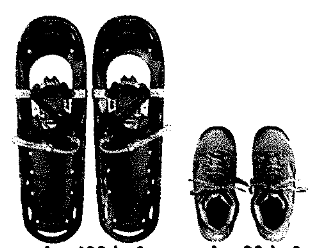
39. **SNOWSHOES** When you stand on snow, the average pressure P (in pounds per square inch) that you exert on the snow varies inversely with the total area A (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives P as a function of A. Then find the pressure if you wear the boots shown.



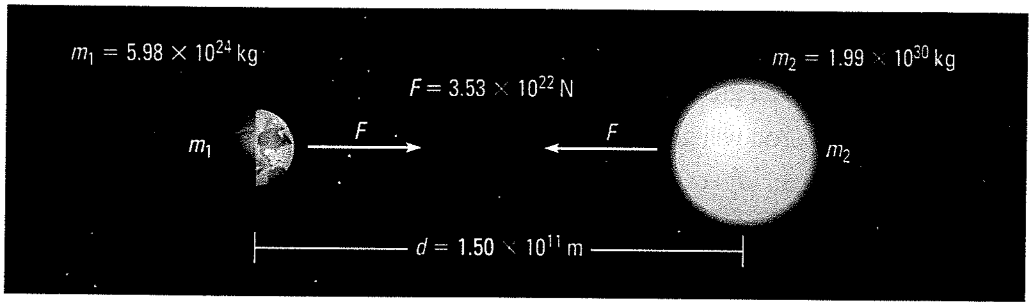
$A = 400$ in.^{2} $A = 60$ in.^{2}

40. **MULTI-STEP PROBLEM** A piano string's frequency f (in hertz) varies directly with the square root of the string's tension T (in Newtons) and inversely with both the string's length L and diameter d (each in centimeters).

   a. The middle C note has a frequency of 262 Hz. The string producing this note has a tension of 670 N, a length of 62 cm, and a diameter of 0.1025 cm. Write an equation relating f, T, L, and d.

   b. Find the frequency of the note produced by a string with a tension of 1629 N, a length of 201.6 cm, and a diameter of 0.49 cm.

41. ★ **EXTENDED RESPONSE** The *law of universal gravitation* states that the gravitational force F (in Newtons) between two objects varies jointly with their masses m_1 and m_2 (in kilograms) and inversely with the square of the distance d (in meters) between the two objects. The constant of variation is denoted by G and is called the *universal gravitational constant.*

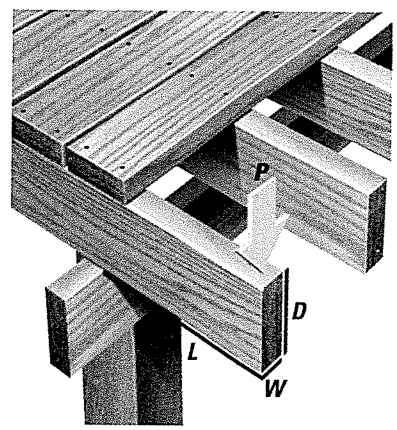


a. **Model** Write an equation that gives F in terms of m_1, m_2, d and G.

b. **Approximate** Use the information above about Earth and the Sun to approximate the universal gravitational constant G.

c. **Reasoning** *Explain* what happens to the gravitational force as the masses of the two objects increase and the distance between them is held constant. *Explain* what happens to the gravitational force as the masses of the two objects are held constant and the distance between them increases.

42. **CHALLENGE** The load P (in pounds) that can be safely supported by a horizontal beam varies jointly with the beam's width W and the square of its depth D, and inversely with its unsupported length L.



a. How does P change when the width and length of the beam are doubled?

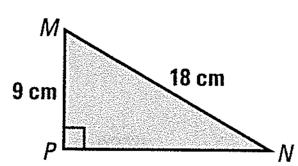
b. How does P change when the width and depth of the beam are doubled?

c. How does P change when all three dimensions are doubled?

d. *Describe* several ways a beam can be modified if the safe load it is required to support is increased by a factor of 4.

PA **PENNSYLVANIA MIXED REVIEW** **TEST PRACTICE** at classzone.com

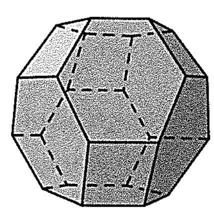
43. What is the approximate area of $\triangle MNP$?



Ⓐ 40.5 cm^2 Ⓑ 57.3 cm^2

Ⓒ 70.1 cm^2 Ⓓ 114.6 cm^2

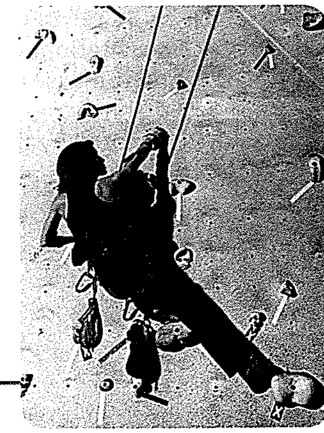
44. The solid at the right has 14 faces: 8 hexagons and 6 squares. How many vertices does the solid have?



Ⓐ 24 Ⓑ 32

Ⓒ 44 Ⓓ 88

8.2 Graph Simple Rational Functions



PA M11.D.1.1.3 Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

Before You graphed polynomial functions.

Now You will graph rational functions.

Why? So you can find average monthly costs, as in Ex. 38.

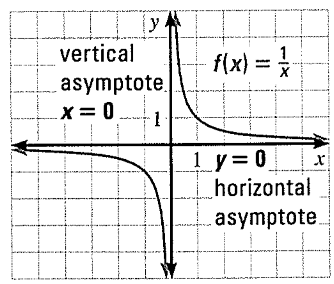
Key Vocabulary
- **rational function**
- **domain,** *p. 72*
- **range,** *p. 72*
- **asymptote,** *p. 478*

A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when $a = 1$ is shown below.

KEY CONCEPT *For Your Notebook*

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called *branches*. The domain and range are all nonzero real numbers.



Any function of the form $g(x) = \frac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.

EXAMPLE 1 **Graph a rational function of the form $y = \frac{a}{x}$**

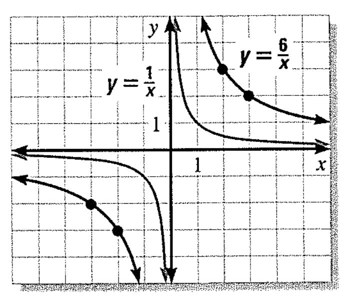
Graph the function $y = \frac{6}{x}$. Compare the graph with the graph of $y = \frac{1}{x}$.

Solution

STEP 1 **Draw** the asymptotes $x = 0$ and $y = 0$.

STEP 2 **Plot** points to the left and to the right of the vertical asymptote, such as $(-3, -2)$, $(-2, -3)$, $(2, 3)$, and $(3, 2)$.

STEP 3 **Draw** the branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



INTERPRET TRANSFORMATIONS

The graph of $y = \frac{6}{x}$ is a vertical stretch of the graph of $y = \frac{1}{x}$ by a factor of 6.

The graph of $y = \frac{6}{x}$ lies farther from the axes than the graph of $y = \frac{1}{x}$. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

KEY CONCEPT *For Your Notebook*

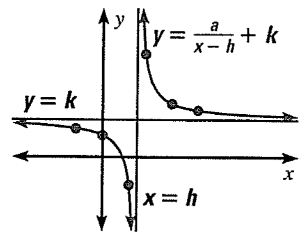
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x - h} + k$, follow these steps:

STEP 1 **Draw** the asymptotes $x = h$ and $y = k$.

STEP 2 **Plot** points to the left and to the right of the vertical asymptote.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



EXAMPLE 2 Graph a rational function of the form $y = \frac{a}{x - h} + k$

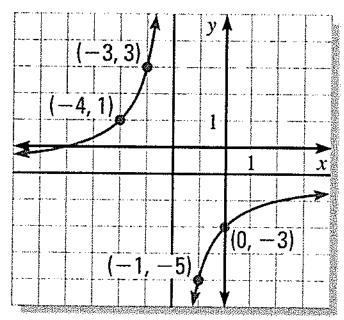
Graph $y = \frac{-4}{x + 2} - 1$. State the domain and range.

Solution

STEP 1 **Draw** the asymptotes $x = -2$ and $y = -1$.

STEP 2 **Plot** points to the left of the vertical asymptote, such as $(-3, 3)$ and $(-4, 1)$, and points to the right, such as $(-1, -5)$ and $(0, -3)$.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



The domain is all real numbers except -2, and the range is all real numbers except -1.

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INTERPRET TRANSFORMATIONS

The graph of $y = \frac{-4}{x + 2} - 1$ is the graph of $y = \frac{-4}{x}$ translated left 2 units and down 1 unit.

✓ GUIDED PRACTICE for Examples 1 and 2

Graph the function. State the domain and range.

1. $f(x) = \frac{-4}{x}$

2. $y = \frac{8}{x} - 5$

3. $y = \frac{1}{x - 3} + 2$

OTHER RATIONAL FUNCTIONS All rational functions of the form $y = \frac{ax + b}{cx + d}$ also have graphs that are hyperbolas.

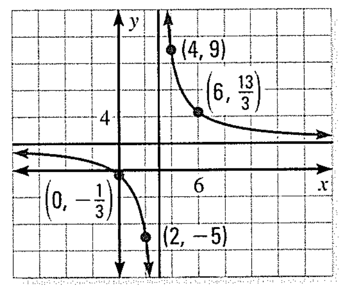
- The vertical asymptote of the graph is the line $x = -\frac{d}{c}$, because the function is undefined when the denominator $cx + d$ is zero.
- The horizontal asymptote is the line $y = \frac{a}{c}$.

EXAMPLE 3 Graph a rational function of the form $y = \frac{ax + b}{cx + d}$

Graph $y = \frac{2x + 1}{x - 3}$. State the domain and range.

Solution

STEP 1 **Draw** the asymptotes. Solve $x - 3 = 0$ for x to find the vertical asymptote $x = 3$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{2}{1} = 2$.



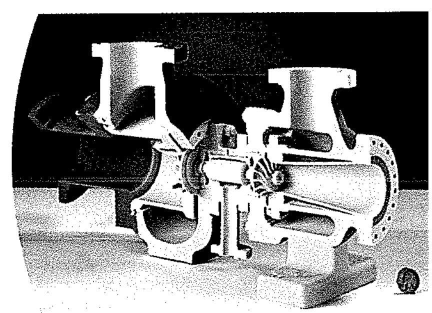
STEP 2 **Plot** points to the left of the vertical asymptote, such as $(2, -5)$ and $\left(0, -\frac{1}{3}\right)$, and points to the right, such as $(4, 9)$ and $\left(6, \frac{13}{3}\right)$.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

▶ The domain is all real numbers except 3.
The range is all real numbers except 2.

EXAMPLE 4 Solve a multi-step problem

3-D MODELING A 3-D printer builds up layers of material to make three-dimensional models. Each deposited layer bonds to the layer below it. A company decides to make small display models of engine components using a 3-D printer. The printer costs \$24,000. The material for each model costs \$300.



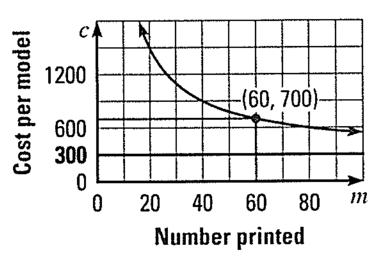
- Write an equation that gives the average cost per model as a function of the number of models printed.
- Graph the function. Use the graph to estimate how many models must be printed for the average cost per model to fall to \$700.
- What happens to the average cost as more models are printed?

Solution

STEP 1 **Write** a function. Let c be the average cost and m be the number of models printed.

$$c = \frac{\text{Unit cost} \cdot \text{Number printed} + \text{Cost of printer}}{\text{Number printed}} = \frac{300m + 24{,}000}{m}$$

STEP 2 **Graph** the function. The asymptotes are the lines $m = 0$ and $c = 300$. The average cost falls to \$700 per model after 60 models are printed.



STEP 3 **Interpret** the graph. As more models are printed, the average cost per model approaches \$300.

DRAW GRAPHS
Because the number of models and average cost cannot be negative, graph only the branch of the hyperbola that lies in the first quadrant.

GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

4. $y = \dfrac{x-1}{x+3}$

5. $y = \dfrac{2x+1}{4x-2}$

6. $f(x) = \dfrac{-3x+2}{-x-1}$

7. **WHAT IF?** In Example 4, how do the function and graph change if the cost of the 3-D printer is $21,000?

8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS14 for Exs. 5, 21, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 35, 40, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The function $y = \dfrac{7}{x+4} + 3$ has a(n) _?_ of all real numbers except 3 and a(n) _?_ of all real numbers except −4.

2. ★ **WRITING** Is $f(x) = \dfrac{-3x+5}{2^x+1}$ a rational function? *Explain* your answer.

EXAMPLE 1 on p. 558 for Exs. 3–10

GRAPHING FUNCTIONS **Graph the function. Compare the graph with the graph of $y = \dfrac{1}{x}$.**

3. $y = \dfrac{3}{x}$

4. $y = \dfrac{10}{x}$

5. $y = \dfrac{-5}{x}$

6. $y = \dfrac{-0.5}{x}$

7. $y = \dfrac{0.1}{x}$

8. $f(x) = \dfrac{15}{x}$

9. $g(x) = \dfrac{-6}{x}$

10. $h(x) = \dfrac{-3}{x}$

EXAMPLE 2 on p. 559 for Exs. 11–23

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

11. $y = \dfrac{4}{x} + 3$

12. $y = \dfrac{3}{x} - 2$

13. $y = \dfrac{6}{x-1}$

14. $f(x) = \dfrac{1}{x+2}$

15. $y = \dfrac{-5}{x} - 7$

16. $y = \dfrac{-6}{x} + 4$

17. $y = \dfrac{-3}{x+2}$

18. $g(x) = \dfrac{-2}{x-7}$

19. $y = \dfrac{-4}{x+4} + 3$

20. $y = \dfrac{10}{x+7} - 5$

21. $y = \dfrac{-3}{x-4} - 1$

22. $h(x) = \dfrac{11}{x-9} + 9$

23. ★ **MULTIPLE CHOICE** What are the asymptotes of the graph of $y = \dfrac{3}{x+8} - 3$?

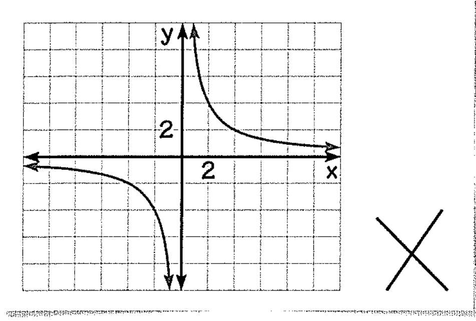
(A) $x = 8, y = 3$ (B) $x = 8, y = -3$ (C) $x = -8, y = 3$ (D) $x = -8, y = -3$

24. **GRAPHING CALCULATOR** Consider the function $y = \dfrac{a}{x-h} + k$ where $a = 1$, $h = 3$, and $k = -2$. Predict the effect on the functions graph of each change in a, h, or k described in parts (a)–(c). Use a graphing calculator to check your prediction by graphing the original and revised functions in the same coordinate plane.

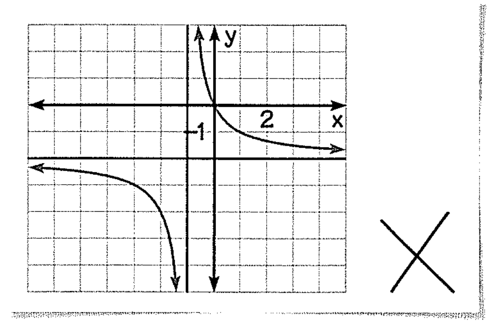
a. a changes to −3 b. h changes to −1 c. k changes to 2

ERROR ANALYSIS *Describe* and correct the error in the graph.

25. $y = \frac{-8}{x}$



26. $y = \frac{2}{x-1} - 2$



EXAMPLE 3 on p. 560 for Exs. 27–34

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

27. $y = \frac{x+4}{x-3}$
28. $y = \frac{x-1}{x+5}$
29. $y = \frac{x+6}{4x-8}$
30. $y = \frac{8x+3}{2x-6}$
31. $y = \frac{-5x+2}{4x+5}$
32. $f(x) = \frac{6x-1}{3x-1}$
33. $g(x) = \frac{5x}{2x+3}$
34. $h(x) = \frac{5x+3}{-x+10}$

35. ★ **OPEN-ENDED MATH** Write a rational function such that the domain is all real numbers except −8 and the range is all real numbers except 3.

36. **CHALLENGE** Show that the equation $f(x) = \frac{a}{x-h} + k$ represents a rational function by writing the right side as a quotient of polynomials.

PROBLEM SOLVING

EXAMPLE 4 on p. 560 for Exs. 37–38

37. **INTERNET SERVICE** An Internet service provider charges a \$50 installation fee and a monthly fee of \$43. Write and graph an equation that gives the average cost per month as a function of the number of months of service. After how many months will the average cost be \$53?

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38. **ROCK CLIMBING GYM** To join a rock climbing gym, you must pay an initial fee of \$100 and a monthly fee of \$59. Write and graph an equation that gives the average cost per month as a function of the number of months of membership. After how many months will the average cost be \$69?

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39. ◆ **MULTIPLE REPRESENTATIONS** The time t (in seconds) it takes for sound to travel 1 kilometer can be modeled by $t = \frac{1000}{0.6T + 331}$ where T is the air temperature (in degrees Celsius).

a. **Evaluating a Function** How long does it take for sound to travel 5 kilometers when the air temperature is 25°C? *Explain.*

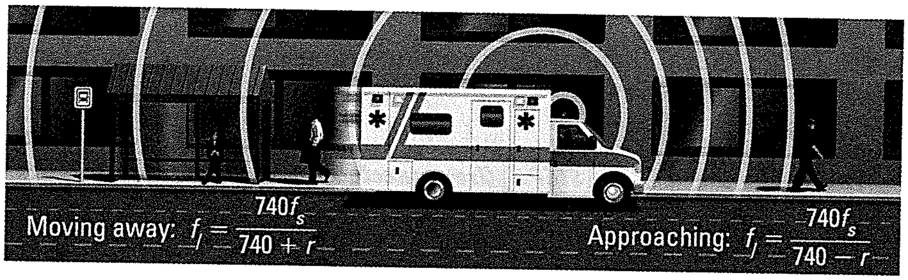
b. **Drawing a Graph** Suppose you are 1 kilometer from a lightning strike, and it takes 3 seconds to hear the thunder. Graph the given function, and use the graph to estimate the air temperature.

40. ★ **SHORT RESPONSE** A business is studying the cost to remove a pollutant from the ground at its site. The function $y = \frac{15x}{1.1 - x}$ models the estimated cost y (in thousands of dollars) to remove x percent (expressed as a decimal) of the pollutant.

a. Graph the function. *Describe* a reasonable domain and range.

b. How much does it cost to remove 20% of the pollutant? 40% of the pollutant? 80% of the pollutant? Does doubling the percent of the pollutant removed double the cost? *Explain.*

41. ★ **EXTENDED RESPONSE** The *Doppler effect* occurs when the source of a sound is moving relative to a listener, so that the frequency f_l (in hertz) heard by the listener is different from the frequency f_s (in hertz) at the source. The frequency heard depends on whether the sound source is approaching or moving away from the listener. In both equations below, r is the speed (in miles per hour) of the sound source.



a. An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies you hear when the ambulance is approaching and when the ambulance is moving away.

b. Graph the equations from part (a) using the domain $0 \le r \le 60$.

c. For any speed r, how does the frequency heard for an approaching sound source compare with the frequency heard when the source moves away?

42. **CHALLENGE** A sailboat travels at a speed of 10 knots for 3 hours. It then uses a motor for power, which increases its speed to 15 knots. Write and graph an equation giving the boat's average speed s (in knots) for the entire trip as a function of the time t (in hours) that it uses the motor for power.

PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

43. On Monday, Anna reads one quarter of a novel. On Tuesday, she reads one third of the remaining pages. On Wednesday, she reads one quarter of the remaining pages. On Thursday, she reads the remaining 105 pages. How many pages does the novel have?

Ⓐ 219 Ⓑ 280 Ⓒ 340 Ⓓ 420

44. Which equation best describes the relationship between x and y shown in the table?

x	0.2	0.5	0.8	1.1
y	0.16	1	2.56	4.84

Ⓐ $y = 4x$ Ⓑ $x = 4y$ Ⓒ $x = 4y^2$ Ⓓ $y = 4x^2$

EXAMPLE 2 Graph a rational function ($m = n$)

REVIEW ZEROS OF FUNCTIONS
For help with finding zeros of functions, see p. 252.

Graph $y = \dfrac{2x^2}{x^2 - 9}$.

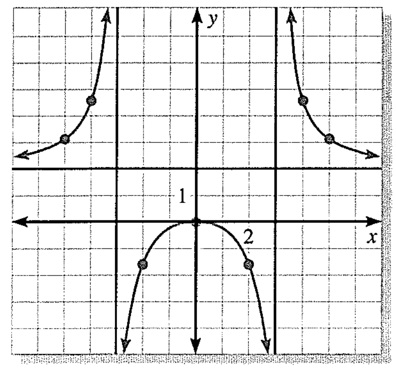
Solution

The zero of the numerator $2x^2$ is 0, so 0 is an x-intercept. The zeros of the denominator $x^2 - 9$ are ± 3, so $x = 3$ and $x = -3$ are vertical asymptotes.

The numerator and denominator have the same degree, so the horizontal asymptote is $y = \dfrac{a_m}{b_n} = \dfrac{2}{1} = 2$.

Plot points between and beyond the vertical asymptotes.

	x	y
To the left of $x = -3$	−5	3.1
	−4	4.6
Between $x = -3$ and $x = 3$	−2	−1.6
	0	0
	2	−1.6
To the right of $x = 3$	4	4.6
	5	3.1



EXAMPLE 3 Graph a rational function ($m > n$)

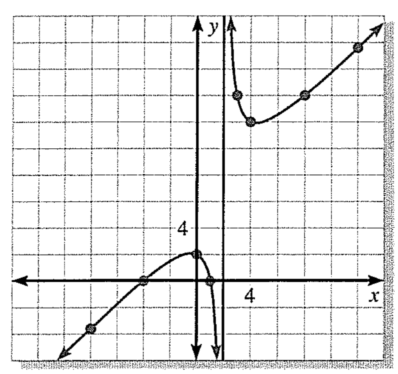
Graph $y = \dfrac{x^2 + 3x - 4}{x - 2}$.

Solution

The numerator factors as $(x + 4)(x - 1)$, so the x-intercepts are −4 and 1. The zero of the denominator $x - 2$ is 2, so $x = 2$ is a vertical asymptote.

The degree of the numerator, 2, is greater than the degree of the denominator, 1, so the graph has no horizontal asymptote. The graph has the same end behavior as the graph of $y = x^{2-1} = x$. Plot points on each side of the vertical asymptote.

	x	y
To the left of $x = 2$	−8	−3.6
	−4	0
	0	2
	1	0
To the right of $x = 2$	3	14
	4	12
	8	14
	12	17.6



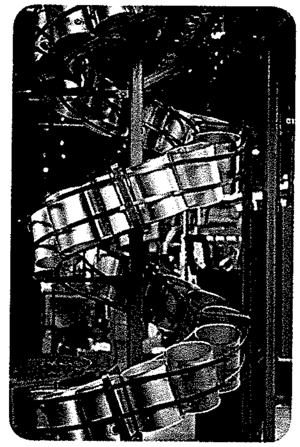
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Graph the function.

1. $y = \dfrac{4}{x^2 + 2}$
2. $y = \dfrac{3x^2}{x^2 - 1}$
3. $f(x) = \dfrac{x^2 - 5}{x^2 + 1}$
4. $y = \dfrac{x^2 - 2x - 3}{x - 4}$

EXAMPLE 4 Solve a multi-step problem

MANUFACTURING A food manufacturer wants to find the most efficient packaging for a can of soup with a volume of 342 cubic centimeters. Find the dimensions of the can that has this volume and uses the least amount of material possible.



Solution

STEP 1 **Write** an equation that gives the height h of the soup can in terms of its radius r. Use the formula for the volume of a cylinder and the fact that the soup can's volume is 342 cubic centimeters.

$V = \pi r^2 h$ **Formula for volume of cylinder**

$342 = \pi r^2 h$ **Substitute 342 for *V*.**

$\dfrac{342}{\pi r^2} = h$ **Solve for *h*.**

STEP 2 **Write** a function that gives the surface area S of the soup can in terms of only its radius r.

$S = 2\pi r^2 + 2\pi r h$ **Formula for surface area of cylinder**

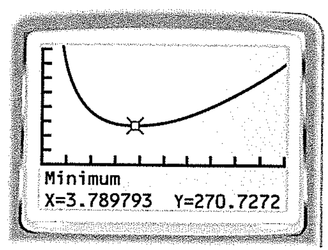
$= 2\pi r^2 + 2\pi r\left(\dfrac{342}{\pi r^2}\right)$ **Substitute $\frac{342}{\pi r^2}$ for *h*.**

$= 2\pi r^2 + \dfrac{684}{r}$ **Simplify.**

INTERPRET FUNCTIONS
The function for the surface area is a rational function because it can be written as a quotient of polynomials:

$S = \dfrac{2\pi r^3 + 684}{r}$

STEP 3 **Graph** the function for the surface area S using a graphing calculator. Then use the *minimum* feature to find the minimum value of S.



You get a minimum value of about 271, which occurs when $r \approx 3.79$ and $h \approx \dfrac{342}{\pi(3.79)^2} \approx 7.58$.

▶ So, the soup can using the least amount of material has a radius of about 3.79 centimeters and a height of about 7.58 centimeters. Notice that the height and the diameter are equal for this can.

✓ GUIDED PRACTICE for Example 4

5. **WHAT IF?** In Example 4, suppose the manufacturer wants to find the most efficient packaging for a soup can with a volume of 544 cubic centimeters. Find the dimensions of this can.

8.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 7, 15, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 14, 24, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The graph of a rational function f has no _?_ when the degree of the function's numerator is greater than the degree of its denominator.

2. ★ **WRITING** Let $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials with no common factors other than ± 1. *Describe* how to find the x-intercepts and the vertical asymptotes of the graph of f.

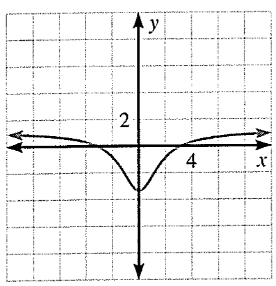
EXAMPLES 1, 2, and 3 on pp. 565–566 for Exs. 3–23

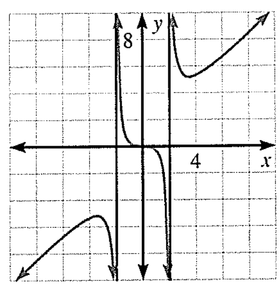
MATCHING GRAPHS **Match the function with its graph.**

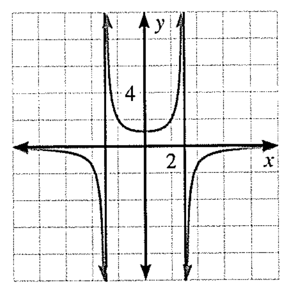
3. $y = \dfrac{-10}{x^2 - 9}$

4. $y = \dfrac{x^2 - 10}{x^2 + 3}$

5. $y = \dfrac{x^3}{x^2 - 4}$

A.


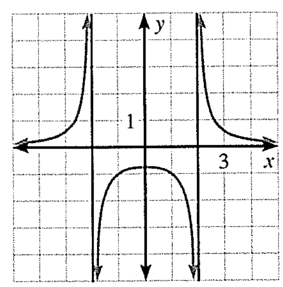
B.


C.


6. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $y = \dfrac{3}{x^2 - 4}$ Ⓑ $y = \dfrac{3x^2}{x^2 - 4}$

Ⓒ $y = \dfrac{x^2 - 4}{3x^2}$ Ⓓ $y = \dfrac{x^3}{x^2 - 4}$



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ANALYZING GRAPHS **Identify the x-intercept(s) and vertical asymptote(s) of the graph of the function.**

7. $y = \dfrac{5}{x^2 - 1}$

8. $y = \dfrac{x + 1}{x^2 + 5}$

9. $f(x) = \dfrac{x^2 + 9}{x^2 - 2x - 15}$

10. $y = \dfrac{x^2 - 7x - 60}{x + 3}$

11. $y = \dfrac{x^3 + 27}{3x^2 + x}$

12. $g(x) = \dfrac{2x^2 - 3x - 20}{x^2 + 1}$

13. **ERROR ANALYSIS** *Describe* and correct the error in finding the vertical asymptote(s) of $f(x) = \dfrac{x - 2}{x^2 - 8x + 7}$.

The vertical asymptote occurs at the zero of the numerator $x - 2$. So, the vertical asymptote is $x = 2$.

14. ★ **MULTIPLE CHOICE** What is the horizontal asymptote of the graph of the function $y = \frac{4x^2 - 21x + 5}{x^2 - 12}$?

Ⓐ $y = 0$ Ⓑ $y = \frac{1}{4}$ Ⓒ $y = 4$ Ⓓ $y = 4x$

GRAPHING FUNCTIONS Graph the function.

15. $y = \frac{2x}{x^2 - 1}$

16. $y = \frac{8}{x^2 - x - 6}$

17. $f(x) = \frac{x^2 - 9}{2x^2 + 1}$

18. $y = \frac{x - 4}{x^2 - 3x}$

19. $y = \frac{x^2 + 11x + 18}{2x + 1}$

20. $g(x) = \frac{x^3 - 8}{6 - x^2}$

21. $y = \frac{x^2 + 3}{2x^3}$

22. $y = \frac{x^2 - 5x - 36}{3x}$

23. $h(x) = \frac{3x^2 + 10x - 8}{x^2 + 4}$

24. ★ **OPEN-ENDED MATH** Write two different rational functions whose graphs have the same end behavior as the graph of $y = 3x^2$.

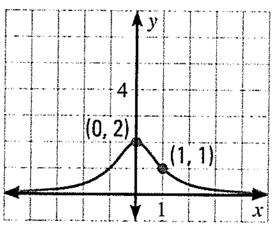
GRAPHING CALCULATOR Use a graphing calculator to find the range of the rational function.

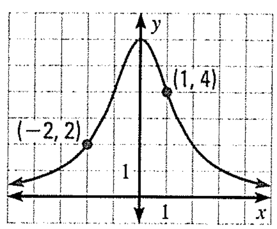
25. $y = \frac{15}{x^2 + 2}$

26. $y = \frac{3x^2}{x^2 - 9}$

27. $y = \frac{x^2 - 2x}{2x + 3}$

CHALLENGE The graph of a function of the form $f(x) = \frac{a}{x^2 + b}$ is shown. Find the values of a and b.

28.


29.


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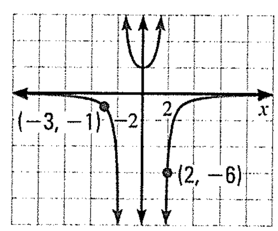

PROBLEM SOLVING

EXAMPLE 4
on p. 567
for Exs. 31–32

GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

31. **AGRICULTURE** A farmer makes cylindrical bales of hay that have a volume of 100 cubic feet. Each bale is to be wrapped in plastic to keep the hay dry.

a. Using the formula for the volume of a cylinder, write an equation that gives the length ℓ of a bale in terms of the radius r.

b. Write a function that gives the surface area of a bale in terms of only the radius r.

c. Find the dimensions of a bale that has the given volume and uses the least amount of plastic possible when the bale is wrapped.

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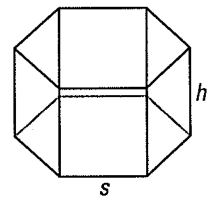
32. **AQUARIUM DESIGN** A manufacturer is designing an aquarium whose base is a regular hexagon. The aquarium should have a volume of 24 cubic feet and use the least amount of material possible. Let s be the length (in feet) of a side of the base, and let h be the height (in feet).



a. Write an equation that gives h in terms of s. (*Hint:* The volume of the aquarium is given by $V = \frac{3\sqrt{3}}{2}s^2h$.)

b. Find the dimensions s and h that minimize the amount of material used. (*Hint:* The surface area of the aquarium is given by $S = \frac{3\sqrt{3}}{2}s^2 + 6sh$.)

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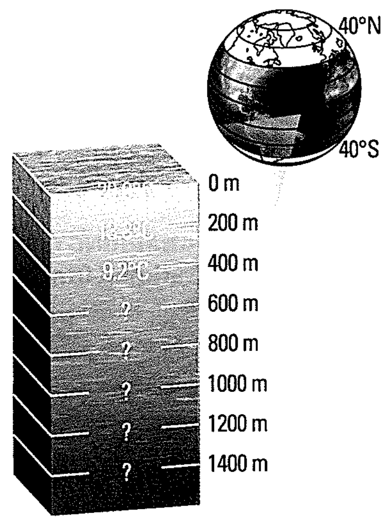
33. ◆ **MULTIPLE REPRESENTATIONS** The mean temperature T (in degrees Celsius) of the Atlantic Ocean between latitudes 40°N and 40°S can be modeled by



$$T = \frac{17{,}800d + 20{,}000}{3d^2 + 740d + 1000}$$

where d is the depth (in meters).

a. **Making a Table** Make a table of values showing the mean temperature for depths from 1000 meters to 1300 meters in 50 meter intervals.

b. **Using a Graph** Graph the model. Use your graph to estimate the depth at which the mean temperature is 4°C.

34. **MULTI-STEP PROBLEM** From 1993 to 2002, the number n (in billions) of shares of stock sold on the New York Stock Exchange can be modeled by

$$n = \frac{1054t + 6204}{-6.62t + 100}$$

where t is the number of years since 1993.

a. Graph the model.

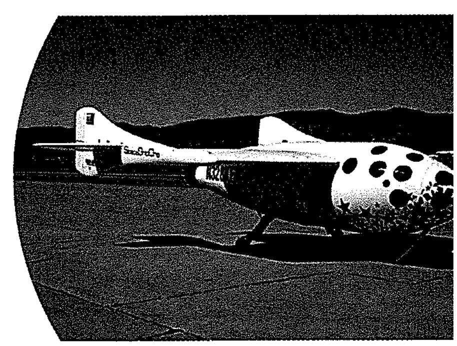
b. *Describe* the general trends shown by the graph.

c. Estimate the year when the number of shares of stock sold was first greater than 100 billion.

35. ★ **EXTENDED RESPONSE** The acceleration due to gravity g (in meters per second squared) changes as altitude changes and is given by the function

$$g = \frac{3.99 \times 10^{14}}{h^2 + (1.28 \times 10^7)h + (4.07 \times 10^{13})}$$

where h is the altitude (in meters) above sea level.



This spacecraft reached an altitude of 112 km in 2004.

a. **Graph** Graph the function.

b. **Apply** A mountaineer is climbing to a height of 8000 meters. What is the value of g at this altitude?

c. **Apply** A spacecraft reaches an altitude of 112 kilometers above Earth. What is the value of g at this altitude?

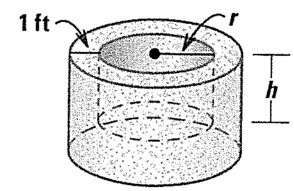
d. **Explain** *Describe* what happens to the value of g as altitude increases.

36. **CHALLENGE** You need to build a cylindrical water tank using 100 cubic feet of concrete. The sides and the base of the tank must be 1 foot thick.

a. Write an equation that gives the tank's inner height h in terms of its inner radius r.

b. Write an equation that gives the volume V of water that the tank can hold as a function of r.

c. Graph the equation from part (b). What values of r and h maximize the tank's capacity?



PA PENNSYLVANIA MIXED REVIEW

TEST PRACTICE at classzone.com

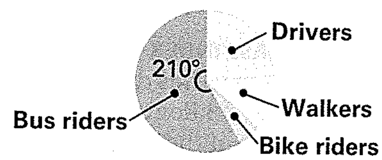
37. Doris plants a 75 square foot rectangular garden. She uses 36 feet of fencing to enclose the garden. What are the approximate dimensions of the garden?

Ⓐ 5.6 ft by 13.4 ft Ⓑ 5.7 ft by 12.3 ft Ⓒ 6.0 ft by 12.0 ft Ⓓ 6.6 ft by 11.4 ft

38. The circle graph represents 840 students. The red section of the circle graph represents the number of students who ride a bus to school everyday. How many students ride a bus to school everyday?

Ⓐ 176 Ⓑ 350

Ⓒ 490 Ⓓ 513



QUIZ for Lessons 8.1–8.3

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = -4$. *(p. 551)*

1. $x = 8, y = 3$
2. $x = 2, y = -9$
3. $x = -5, y = \frac{8}{3}$
4. $x = -\frac{1}{4}, y = -32$

Graph the function.

5. $y = \frac{3}{2x}$ *(p. 558)*
6. $y = \frac{4}{x-2} + 1$ *(p. 558)*
7. $f(x) = \frac{-2x}{3x-6}$ *(p. 558)*
8. $y = \frac{-8}{x^2-1}$ *(p. 565)*
9. $y = \frac{x^2-6}{x^2+2}$ *(p. 565)*
10. $g(x) = \frac{x^3-8}{2x^2}$ *(p. 565)*

11. **SOFTBALL** A pitcher throws 16 strikes in her first 38 pitches. The table shows how the pitcher's strike percentage changes if she throws x consecutive strikes after the first 38 pitches. Write a rational function for the strike percentage in terms of x. Graph the function. How many consecutive strikes must the pitcher throw to reach a strike percentage of 0.60? *(p. 558)*

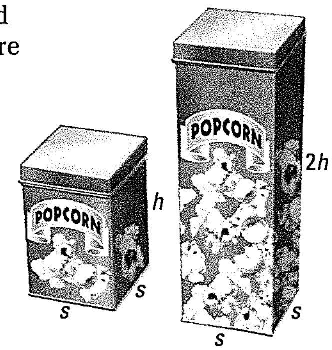
x	Total strikes	Total pitches	Strike percentage
0	16	38	0.42
5	21	43	0.49
10	26	48	0.54
x	$x + 16$	$x + 38$	?

EFFICIENCY Manufacturers often package their products in a way that uses the least amount of packaging material. One measure of the efficiency of a package is the ratio of its surface area to its volume. The smaller the ratio, the more efficient the packaging.

EXAMPLE 2 Solve a multi-step problem

PACKAGING A company makes a tin to hold flavored popcorn. The tin is a rectangular prism with a square base. The company is designing a new tin with the same base and twice the height of the old tin.

- Find the surface area and volume of each tin.
- Calculate the ratio of surface area to volume for each tin.
- What do the ratios tell you about the efficiencies of the two tins?



Solution

	Old tin	New tin	
STEP 1	$S = 2s^2 + 4sh$	$S = 2s^2 + 4s(2h)$	**Find surface area, S.**
		$= 2s^2 + 8sh$	
	$V = s^2h$	$V = s^2(2h)$	**Find volume, V.**
		$= 2s^2h$	
STEP 2	$\frac{S}{V} = \frac{2s^2 + 4sh}{s^2h}$	$\frac{S}{V} = \frac{2s^2 + 8sh}{2s^2h}$	**Write ratio of S to V.**
	$= \frac{\cancel{s}(2s + 4h)}{\cancel{s}(sh)}$	$= \frac{\cancel{2s}(s + 4h)}{\cancel{2s}(sh)}$	**Divide out common factor.**
	$= \frac{2s + 4h}{sh}$	$= \frac{s + 4h}{sh}$	**Simplified form**

STEP 3 $\frac{2s + 4h}{sh} > \frac{s + 4h}{sh}$ because the left side of the inequality has a greater numerator than the right side and both have the same (positive) denominator. The ratio of surface area to volume is *greater* for the old tin than for the new tin. So, the old tin is *less* efficient than the new tin.

✓ GUIDED PRACTICE for Examples 1 and 2

Simplify the expression, if possible.

1. $\frac{2(x + 1)}{(x + 1)(x + 3)}$
2. $\frac{40x + 20}{10x + 30}$
3. $\frac{4}{x(x + 2)}$
4. $\frac{x + 4}{x^2 - 16}$
5. $\frac{x^2 - 2x - 3}{x^2 - x - 6}$
6. $\frac{2x^2 + 10x}{3x^2 + 16x + 5}$
7. **WHAT IF?** In Example 2, suppose the new popcorn tin is the same height as the old tin but has a base with sides twice as long. What is the ratio of surface area to volume for this tin?

KEY CONCEPT *For Your Notebook*

Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form.

Let a, b, c, and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ **Simplify $\frac{ac}{bd}$ if possible.**

Example $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \cdot 3 \cdot \cancel{x} \cdot x^2 \cdot \cancel{y^3}}{\cancel{10} \cdot 2 \cdot \cancel{x} \cdot \cancel{y^3}} = \frac{3x^2}{2}$

EXAMPLE 3 Standardized Test Practice

ANOTHER WAY

In Example 3, you can also first simplify each fraction, then multiply, and finally simplify the result:

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$$
$$= \frac{4x^2}{y} \cdot \frac{7x^4y^2}{4}$$
$$= \frac{\cancel{4} \cdot 7 \cdot x^6 \cdot \cancel{y} \cdot y}{\cancel{4} \cdot \cancel{y}}$$
$$= 7x^6y$$

What is a simplified form of $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$?

Ⓐ $\frac{5}{2}x^6y$ Ⓑ $7x^6y$ Ⓒ $7x^{11}y$ Ⓓ $7x^7y^{4/3}$

Solution

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y} = \frac{56x^7y^4}{8xy^3}$$ **Multiply numerators and denominators.**

$$= \frac{\cancel{8} \cdot 7 \cdot \cancel{x} \cdot x^6 \cdot \cancel{y^3} \cdot y}{\cancel{8} \cdot \cancel{x} \cdot \cancel{y^3}}$$ **Factor and divide out common factors.**

$$= 7x^6y$$ **Simplified form**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Multiply rational expressions

Multiply: $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$

$$\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x} = \frac{3x(1 - x)}{(x - 1)(x + 5)} \cdot \frac{(x + 5)(x - 4)}{3x}$$ **Factor numerators and denominators.**

$$= \frac{3x(1 - x)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$$ **Multiply numerators and denominators.**

$$= \frac{3x(-1)(x - 1)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$$ **Rewrite $1 - x$ as $(-1)(x - 1)$.**

$$= \frac{\cancel{3x}(-1)\cancel{(x - 1)}\cancel{(x + 5)}(x - 4)}{\cancel{(x - 1)}\cancel{(x + 5)}\cancel{(3x)}}$$ **Divide out common factors.**

$$= (-1)(x - 4)$$ **Simplify.**

$$= -x + 4$$ **Multiply.**

EXAMPLE 5 Multiply a rational expression by a polynomial

Multiply: $\dfrac{x+2}{x^3-27} \cdot (x^2+3x+9)$

$$\frac{x+2}{x^3-27} \cdot (x^2+3x+9) = \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1}$$ **Write polynomial as a rational expression.**

$$= \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$ **Factor denominator.**

$$= \frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}}$$ **Divide out common factors.**

$$= \frac{x+2}{x-3}$$ **Simplified form**

✓ GUIDED PRACTICE for Examples 3, 4, and 5

Multiply the expressions. Simplify the result.

8. $\dfrac{3x^5y^2}{8xy} \cdot \dfrac{6xy^2}{9x^3y}$

9. $\dfrac{2x^2-10x}{x^2-25} \cdot \dfrac{x+3}{2x^2}$

10. $\dfrac{x+5}{x^3-1} \cdot (x^2+x+1)$

KEY CONCEPT *For Your Notebook*

Dividing Rational Expressions

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression.

Let a, b, c, and d be expressions with $b \neq 0$, $c \neq 0$ and $d \neq 0$.

Property $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}$ **Simplify $\frac{ad}{bc}$ if possible.**

Examples $\dfrac{2}{5} \div \dfrac{7}{3} = \dfrac{2}{5} \cdot \dfrac{3}{7} = \dfrac{6}{35}$

$$\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}$$

EXAMPLE 6 Divide rational expressions

Divide: $\dfrac{7x}{2x-10} \div \dfrac{x^2-6x}{x^2-11x+30}$

$$\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30} = \frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x}$$ **Multiply by reciprocal.**

$$= \frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)}$$ **Factor.**

$$= \frac{7\cancel{x}\cancel{(x-5)}\cancel{(x-6)}}{2\cancel{(x-5)}\cancel{(x)}\cancel{(x-6)}}$$ **Divide out common factors.**

$$= \frac{7}{2}$$ **Simplified form**

EXAMPLE 7 Divide a rational expression by a polynomial

Divide: $\dfrac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

$$\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x}$$ **Multiply by reciprocal.**

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)}$$ **Factor.**

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}}$$ **Divide out common factors.**

$$= \frac{2x - 3}{4x^3}$$ **Simplified form**

✓ GUIDED PRACTICE for Examples 6 and 7

Divide the expressions. Simplify the result.

11. $\dfrac{4x}{5x - 20} \div \dfrac{x^2 - 2x}{x^2 - 6x + 8}$

12. $\dfrac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

8.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 7, 25, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 20, 23, 50, and 52

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To divide one rational expression by another, multiply the first rational expression by the _?_ of the second rational expression.

2. ★ **WRITING** How do you know when a rational expression is simplified?

EXAMPLE 1 on p. 573 for Exs. 3–20

REASONING Match the rational expression with its simplified form.

3. $\dfrac{x^2 - 9x + 14}{x^2 - 5x - 14}$

4. $\dfrac{x^2 - 4}{x^2 + 9x + 14}$

5. $\dfrac{x^2 + 5x - 14}{x^2 - 4x + 4}$

A. $\dfrac{x - 2}{x + 7}$

B. $\dfrac{x - 2}{x + 2}$

C. $\dfrac{x + 7}{x - 2}$

SIMPLIFYING Simplify the rational expression, if possible.

6. $\dfrac{4x^2}{20x^2 - 12x}$

7. $\dfrac{x^2 - x - 20}{x^2 + 2x - 15}$

8. $\dfrac{x^2 + 2x - 24}{x^2 + 7x + 6}$

9. $\dfrac{x^2 - 11x + 24}{x^2 - 3x - 40}$

10. $\dfrac{x^2 + 4x + 4}{x^2 - 5x + 4}$

11. $\dfrac{2x^2 + 2x - 4}{x^2 - 5x - 14}$

12. $\dfrac{x - 4}{x^3 - 64}$

13. $\dfrac{x^2 - 36}{x^2 + 12x + 36}$

14. $\dfrac{3x^3 + 6x^2 + 12x}{x^3 - 8}$

15. $\dfrac{8x^2 + 10x - 3}{6x^2 + 13x + 6}$

16. $\dfrac{5x^2 + 18x - 8}{10x^2 - x - 2}$

17. $\dfrac{x^3 - 5x^2 - 3x + 15}{x^2 - 8x + 15}$

ERROR ANALYSIS *Describe* **and correct the error in simplifying the rational expression.**

18. $\dfrac{\cancel{x^2}+16x-80}{\cancel{x^2}-16} = \dfrac{16x-80}{-16} = -x+5$

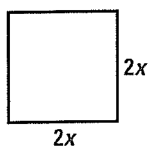
19. $\dfrac{x^2+\overset{2}{\cancel{16}}x+\overset{3}{\cancel{48}}}{x^2+\underset{1}{\cancel{8}}x+\underset{1}{\cancel{16}}} = \dfrac{x^2+2x+3}{x^2+x+1}$

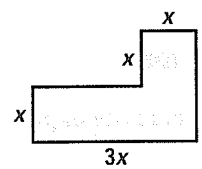
20. ★ MULTIPLE CHOICE Which rational expression is in simplified form?

Ⓐ $\dfrac{x^2-x-6}{x^2+3x+2}$ Ⓑ $\dfrac{x^2+6x+8}{x^2+2x-3}$ Ⓒ $\dfrac{x^2-6x+9}{x^2-2x-3}$ Ⓓ $\dfrac{x^2+3x-4}{x^2+x-2}$

EXAMPLE 2 on p. 574 for Exs. 21–23

GEOMETRY **A farmer wants to fence in the field shown. Write a simplified rational expression for the ratio of the field's perimeter to its area.**

21. 

22. 

23. ★ SHORT RESPONSE Which of the fields in Exercises 21 and 22 has the lower fencing cost per unit of area? *Explain.*

EXAMPLES 3, 4, and 5 on pp. 575–576 for Exs. 24–33

MULTIPLYING **Multiply the expressions. Simplify the result.**

24. $\dfrac{5x^3y}{x^2y^2} \cdot \dfrac{y^3}{15x^2}$

25. $\dfrac{48x^5y^3}{y^4} \cdot \dfrac{x^2y}{6x^3y^2}$

26. $\dfrac{x(x-3)}{x-2} \cdot \dfrac{(x+3)(x-2)}{x}$

27. $\dfrac{4(x+5)}{x^2} \cdot \dfrac{x(x+1)}{2(x+5)}$

28. $\dfrac{3x-12}{x+5} \cdot \dfrac{x+6}{2x-8}$

29. $\dfrac{x+5}{4x-16} \cdot \dfrac{2x^2-32}{x^2-25}$

30. $\dfrac{x^2+3x-4}{x^2+4x+4} \cdot \dfrac{2x^2+4x}{x^2-4x+3}$

31. $\dfrac{x^2-3x-10}{x^2-2x-15} \cdot (x^2+10x+21)$

32. $\dfrac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

33. $\dfrac{4x^2+20x}{x^3+4x^2} \cdot (x^2+8x+16)$

EXAMPLES 6 and 7 on pp. 576–577 for Exs. 34–43

DIVIDING **Divide the expressions. Simplify the result.**

34. $\dfrac{5x^2y^3}{x^7} \div \dfrac{30xy^4}{y^3}$

35. $\dfrac{8x^2y^2z}{xz^3} \div \dfrac{10xy}{x^4z}$

36. $\dfrac{(x+3)(x-2)}{x(x+1)} \div \dfrac{x+3}{x}$

37. $\dfrac{8x^2}{x+4} \div \dfrac{x}{2(x-4)}$

38. $\dfrac{x^2-6x-27}{2x^2+2x} \div \dfrac{x^2-14x+45}{x^2}$

39. $\dfrac{x^2-4x-5}{x+5} \div (x^2+6x+5)$

40. $\dfrac{3x^2+13x+4}{x^2-4} \div \dfrac{4x+16}{x+2}$

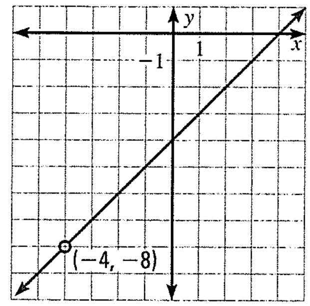
41. $\dfrac{x^2-x-2}{x^2+4x-5} \div \dfrac{x-2}{5x+25}$

42. $\dfrac{x^2-8x+15}{x^2+4x} \div (x^2-x-20)$

43. $\dfrac{x^2+12x+32}{6x+42} \div \dfrac{x^2+4x}{x^2-49}$

POINT DISCONTINUITY **In Exercises 44–46, use the following information.**

The graph of a rational function can have a hole in it, called a *point discontinuity*, where the function is undefined. An example is shown below.



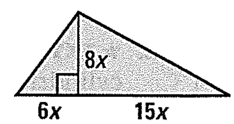
$$y = \frac{x^2 - 16}{x + 4} = \frac{\cancel{(x + 4)}(x - 4)}{\cancel{x + 4}} = x - 4$$

The graph of $y = \frac{x^2 - 16}{x + 4}$ is the same as the graph of $y = x - 4$ except that there is a hole at $(-4, -8)$ because the rational function is not defined when $x = -4$.

Graph the rational function. Use an open circle for a point discontinuity.

44. $y = \frac{x^2 + 10x + 21}{x + 3}$ **45.** $y = \frac{x^2 - 36}{x - 6}$ **46.** $y = \frac{2x^2 - x - 10}{x + 2}$

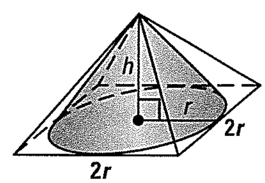
47. CHALLENGE Find the ratio of the perimeter to the area of the triangle shown at the right.



PROBLEM SOLVING

EXAMPLE 2 on p. 574 for Exs. 48, 50–52

48. GEOMETRY Find the ratio of the volume of the square pyramid to the volume of the inscribed cone. Write your answer in simplified form.



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49. ENTERTAINMENT From 1992 to 2002, the gross ticket sales S (in millions of dollars) to Broadway shows and the total attendance A (in millions) at the shows can be modeled by

$$S = \frac{-6420t + 292{,}000}{6.02t^2 - 125t + 1000} \quad \text{and} \quad A = \frac{-407t + 7220}{5.92t^2 - 131t + 1000}$$

where t is the number of years since 1992. Write a model for the *average* dollar amount a person paid per ticket as a function of the year. What was the average amount a person paid per ticket in 1999?

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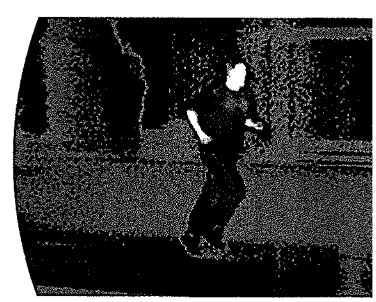
50. ★ SHORT RESPONSE Almost all of the energy generated by a long-distance runner is released in the form of heat. For a runner with height H and speed V, the rate h_g of heat generated and the rate h_r of heat released can be modeled by $h_g = k_1H^3V^2$ and $h_r = k_2H^2$ where k_1 and k_2 are constants.

a. Write the ratio of heat generated to heat released. Simplify the expression.

b. When the ratio of heat generated to heat released equals 1, how is speed related to height? Does a taller or shorter runner have the advantage? *Explain.*



Thermogram of runner

8.5 Add and Subtract Rational Expressions

PA M11.D.2.2.3 Simplify algebraic fractions.

Before You multiplied and divided rational expressions.

Now You will add and subtract rational expressions.

Why? So you can determine monthly car loan payments, as in Ex. 43.

Key Vocabulary
- **complex fraction**

As with numerical fractions, the procedure used to add (or subtract) two rational expressions depends upon whether the expressions have *like* or *unlike* denominators.

KEY CONCEPT *For Your Notebook*

Adding or Subtracting with Like Denominators

To add (or subtract) rational expressions with *like* denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

Let a, b, and c be expressions with $c \neq 0$.

	Addition	Subtraction
Properties	$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$
Examples	$\frac{3x}{5x^2} + \frac{7}{5x^2} = \frac{3x+7}{5x^2}$	$\frac{9x^3}{x+1} - \frac{x^2}{x+1} = \frac{9x^3-x^2}{x+1}$

EXAMPLE 1 Add or subtract with like denominators

Perform the indicated operation.

a. $\frac{7}{4x} + \frac{3}{4x}$ **b.** $\frac{2x}{x+6} - \frac{5}{x+6}$

Solution

a. $\frac{7}{4x} + \frac{3}{4x} = \frac{7+3}{4x} = \frac{10}{4x} = \frac{5}{2x}$ **Add numerators and simplify result.**

b. $\frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$ **Subtract numerators.**

✓ **GUIDED PRACTICE** for Example 1

Perform the indicated operation and simplify.

1. $\frac{7}{12x} - \frac{5}{12x}$ **2.** $\frac{2}{3x^2} + \frac{1}{3x^2}$ **3.** $\frac{4x}{x-2} - \frac{x}{x-2}$ **4.** $\frac{2x^2}{x^2+1} + \frac{2}{x^2+1}$

KEY CONCEPT *For Your Notebook*

Adding or Subtracting with Unlike Denominators

To add (or subtract) two rational expressions with *unlike* denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying their denominators, as shown above. However, if you use the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, you may have less simplifying to do.

EXAMPLE 2 Find a least common multiple (LCM)

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

Solution

STEP 1 **Factor** each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

STEP 2 **Form** the LCM by writing each factor to the highest power it occurs in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

EXAMPLE 3 Add with unlike denominators

Add: $\dfrac{7}{9x^2} + \dfrac{x}{3x^2 + 3x}$

REVIEW LCDS

For help with finding least common denominators, see p. 986.

Solution

To find the LCD, factor each denominator and write each factor to the highest power it occurs. Note that $9x^2 = 3^2x^2$ and $3x^2 + 3x = 3x(x + 1)$, so the LCD is $3^2x^2(x + 1) = 9x^2(x + 1)$.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)}$$ **Factor second denominator.**

$$= \frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x}$$ **LCD is $9x^2(x + 1)$.**

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x^2}{9x^2(x + 1)}$$ **Multiply.**

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)}$$ **Add numerators.**

✓ GUIDED PRACTICE for Examples 1 and 2

Solve the equation by cross multiplying. Check your solution(s).

1. $\frac{3}{5x} = \frac{2}{x-7}$

2. $\frac{-4}{x+3} = \frac{5}{x-3}$

3. $\frac{1}{2x+5} = \frac{x}{11x+8}$

4. **WHAT IF?** In Example 2, suppose you have 10 ounces of jewelry silver. How much pure silver must be mixed with the jewelry silver to make sterling silver?

USING LCDS When a rational equation is not expressed as a proportion, you can solve it by multiplying each side of the equation by the least common denominator of each rational expression.

★ EXAMPLE 3 Standardized Test Practice

ELIMINATE CHOICES
You can eliminate choice D because it yields a positive value on the left side of the equation and a negative value on the right side.

What is the solution of $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$?

Ⓐ -10 Ⓑ -8 Ⓒ -4 Ⓓ 6

Solution

$\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$	**Write original equation.**
$4x\left(\frac{5}{x} + \frac{7}{4}\right) = 4x\left(-\frac{9}{x}\right)$	**Multiply each side by the LCD, $4x$.**
$20 + 7x = -36$	**Simplify.**
$7x = -56$	**Subtract 20 from each side.**
$x = -8$	**Divide each side by 7.**

▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Solve a rational equation with two solutions

Solve: $1 - \frac{8}{x-5} = \frac{3}{x}$

$1 - \frac{8}{x-5} = \frac{3}{x}$	**Write original equation.**
$x(x-5)\left(1 - \frac{8}{x-5}\right) = x(x-5) \cdot \frac{3}{x}$	**Multiply each side by the LCD, $x(x-5)$.**
$x(x-5) - 8x = 3(x-5)$	**Simplify.**
$x^2 - 5x - 8x = 3x - 15$	**Simplify.**
$x^2 - 16x + 15 = 0$	**Write in standard form.**
$(x-1)(x-15) = 0$	**Factor.**
$x = 1$ or $x = 15$	**Zero product property**

▸ The solutions are 1 and 15. Check these in the original equation.

EXTRANEOUS SOLUTIONS When solving a rational equation, you may obtain solutions that are extraneous. Be sure to check for extraneous solutions by substituting back into the original equation.

EXAMPLE 5 Check for extraneous solutions

Solve: $\dfrac{6}{x-3} = \dfrac{8x^2}{x^2-9} - \dfrac{4x}{x+3}$

Solution

Write each denominator in factored form. The LCD is $(x+3)(x-3)$.

$$\frac{6}{x-3} = \frac{8x^2}{(x+3)(x-3)} - \frac{4x}{x+3}$$

$$(x+3)(x-3) \cdot \frac{6}{x-3} = (x+3)(x-3) \cdot \frac{8x^2}{(x+3)(x-3)} - (x+3)(x-3) \cdot \frac{4x}{x+3}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2x^2 + 3x - 9$$

$$0 = (2x-3)(x+3)$$

$$2x - 3 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

REVIEW EXTRANEOUS SOLUTIONS
For help with extraneous solutions, see p. 51.

You can use algebra or a graph to check whether either of the two solutions is extraneous.

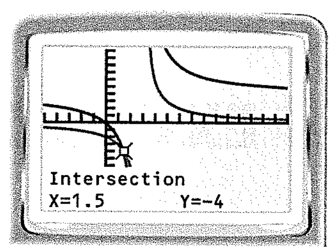
Algebra The solution $\frac{3}{2}$ checks, but the apparent solution -3 is extraneous, because substituting it in the equation results in division by zero, which is undefined.

$$\frac{6}{-3-3} \neq \frac{8(-3)^2}{(-3)^2-9} - \frac{4(-3)}{-3+3}$$

Division by zero is undefined

Graph Graph $y = \dfrac{6}{x-3}$ and $y = \dfrac{8x^2}{x^2-9} - \dfrac{4x}{x+3}$.

The graphs intersect when $x = \frac{3}{2}$, but not when $x = -3$.



▸ The solution is $\frac{3}{2}$.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

Solve the equation by using the LCD. Check for extraneous solutions.

5. $\dfrac{7}{2} + \dfrac{3}{x} = 3$

6. $\dfrac{2}{x} + \dfrac{4}{3} = 2$

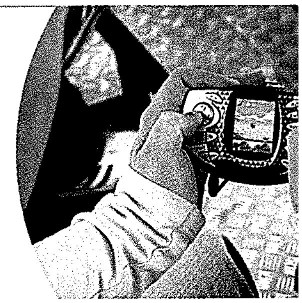
7. $\dfrac{3}{7} + \dfrac{8}{x} = 1$

8. $\dfrac{3}{2} + \dfrac{4}{x-1} = \dfrac{x+1}{x-1}$

9. $\dfrac{3x}{x+1} - \dfrac{5}{2x} = \dfrac{3}{2x}$

10. $\dfrac{5x}{x-2} = 7 + \dfrac{10}{x-2}$

EXAMPLE 6 Solve a rational equation given a function



VIDEO GAME SALES From 1995 through 2003, the annual sales S (in billions of dollars) of entertainment software can be modeled by

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}, \quad 0 \le t \le 8$$

where t is the number of years since 1995. For which year were the total sales of entertainment software about \$5.3 billion?

ANOTHER WAY
For alternative methods for solving the problem in Example 6, turn to page 596 for the **Problem Solving Workshop**.

Solution

$S(t) = \dfrac{848t^2 + 3220}{115t^2 + 1000}$	**Write given function.**
$5.3 = \dfrac{848t^2 + 3220}{115t^2 + 1000}$	**Substitute 5.3 for $S(t)$.**
$5.3(115t^2 + 1000) = 848t^2 + 3220$	**Multiply each side by $115t^2 + 1000$.**
$609.5t^2 + 5300 = 848t^2 + 3220$	**Simplify.**
$5300 = 238.5t^2 + 3220$	**Subtract $609.5t^2$ from each side.**
$2080 = 238.5t^2$	**Subtract 3220 from each side.**
$8.72 \approx t^2$	**Divide each side by 238.5.**
$\pm 2.95 \approx t$	**Take square roots of each side.**

Because -2.95 is not in the domain $(0 \le t \le 8)$, the only solution is 2.95.

▶ So, the total sales of entertainment software were about \$5.3 billion about 3 years after 1995, or in 1998.

✓ **GUIDED PRACTICE** for Example 6

11. **WHAT IF?** Use the information in Example 6 to determine in which year the total sales of entertainment software were about \$4.5 billion.

8.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 5, 15, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 28, 29, 34, and 36

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: When you write $\frac{x}{3} = \frac{x+2}{5}$ as $5x = 3(x + 2)$, you are __?__.

2. ★ **WRITING** A student solved the equation $\frac{5}{x-4} = \frac{x}{x-4}$ and got the solutions 4 and 5. Which, if either, of these is extraneous? *Explain.*

3. **REASONING** *Describe* how you can use a graph to determine if an apparent solution of a rational equation is extraneous.

EXAMPLE 1 on p. 589 for Exs. 4–13

CROSS MULTIPLYING Solve the equation by cross multiplying. Check for extraneous solutions.

4. $\frac{4}{2x} = \frac{5}{x+6}$

5. $\frac{9}{3x} = \frac{4}{x+2}$

6. $\frac{6}{x-1} = \frac{9}{x+1}$

7. $\frac{8}{3x-2} = \frac{2}{x-1}$

8. $\frac{x}{x+1} = \frac{3}{x+1}$

9. $\frac{x-3}{x+5} = \frac{x}{x+2}$

10. $\frac{x}{x^2-2} = \frac{-1}{x}$

11. $\frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4}$

12. $\frac{9}{x^2-6x+9} = \frac{3x}{x^2-3x}$

13. ★ **MULTIPLE CHOICE** What is the solution of $\frac{3}{x+2} = \frac{6}{x-1}$?

Ⓐ -5 Ⓑ -4 Ⓒ -1 Ⓓ 4

EXAMPLES 3, 4, and 5 on pp. 590–591 for Exs. 14–27

LEAST COMMON DENOMINATOR Solve the equation by using the LCD. Check for extraneous solutions.

14. $\frac{4}{x} + x = 5$

15. $\frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$

16. $\frac{5}{x} - 2 = \frac{2}{x+3}$

17. $\frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x}$

18. $\frac{1}{x-2} + 2 = \frac{3x}{x+2}$

19. $\frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2}$

20. $\frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$

21. $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$

22. $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

23. $\frac{10}{x} + 3 = \frac{x+9}{x-4}$

24. $\frac{18}{x^2-3x} - \frac{6}{x-3} = \frac{5}{x}$

25. $\frac{x+3}{x-3} + \frac{x}{x-5} = \frac{x+5}{x-5}$

ERROR ANALYSIS *Describe* and correct the error in the first step of solving the equation.

26.

$$\frac{3}{2x} + \frac{4}{x^2} = 1$$
$$3x^2 + 8x = 1$$

27.

$$\frac{5}{x} + \frac{23}{6} = \frac{45}{x}$$
$$\frac{28}{x+6} = \frac{45}{x}$$

28. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$?

Ⓐ $-3, -\frac{1}{2}$ Ⓑ $-\frac{1}{2}, 3$ Ⓒ $-\frac{1}{2}$ Ⓓ 3

29. ★ **OPEN-ENDED MATH** Give an example of a rational equation that you would solve using cross multiplication. Then give an example of a rational equation that you would solve by multiplying each side by the LCD of the fractions.

CHALLENGE In Exercises 30–32, *a* is a nonzero real number. Tell whether the algebraic statement is *always true, sometimes true,* or *never true. Explain* your answer.

30. For the equation $\frac{1}{x-a} = \frac{x}{x-a}$, $x = a$ is an extraneous solution.

31. The equation $\frac{3}{x-a} = \frac{x}{x-a}$ has exactly one solution.

32. The equation $\frac{1}{x-a} = \frac{2}{x+a} + \frac{2a}{x^2-a^2}$ has no solution.

PRACTICE

EXAMPLE 1
on p. 598
for Exs. 1–6

Use a table to solve the inequality.

1. $\frac{5}{x-2} < 0$
2. $\frac{x-5}{x+3} > 1$
3. $\frac{x^2-3x+2}{x-3} < x$
4. $\frac{10}{x+2} > 0$
5. $\frac{-2x-3}{x-4} > 0$
6. $\frac{x^2-4x+8}{x-1} < x$

EXAMPLE 2
on p. 598
for Exs. 7–12

Use a graph to solve the inequality.

7. $-\frac{4}{x+5} < 0$
8. $\frac{4}{x-3} < 0$
9. $\frac{8}{x^2+1} \geq 4$
10. $\frac{20}{x^2+1} < 2$
11. $\frac{3x+2}{x-1} < -2$
12. $\frac{3x+2}{x-1} > x$

EXAMPLE 3
on p. 599
for Exs. 13–18

Solve the inequality algebraically.

13. $\frac{3}{x+2} > 0$
14. $-\frac{1}{x+5} \leq -2$
15. $\frac{2}{x+2} > \frac{1}{x+3}$
16. $\frac{5}{x-4} < \frac{1}{x+4}$
17. $\frac{5}{x+3} \geq \frac{4}{x+2}$
18. $\frac{2}{x+6} > \frac{-3}{x-3}$

19. **EGG PRODUCTION** From 1994 to 2002, the total number E (in billions) of eggs produced in the United States can be modeled by

$$E = \frac{-3680}{t-50}, \quad 0 \leq t \leq 8$$

where t is the number of years since 1994. For what years was the number of eggs produced greater than 80 billion?

20. **PHONE COSTS** One phone company advertises a flat rate of \$.07 per minute for long-distance calls. Your long-distance plan charges \$5.00 per month plus a rate of \$.05 per minute. How many minutes do you have to talk each month so that your average cost is less than \$.07 per minute?

21. **SATELLITE TV** You subscribe to a satellite television service. The monthly cost for programming is \$43, and there is a one-time installation fee of \$50. The average monthly cost c of the service is given by $c = \frac{43t+50}{t}$ where t is the time (in months) that you have subscribed to the service. For what subscription times is the average monthly cost at most \$47? Solve the problem using a table and using a graph.

22. **FUNDRAISER** Your school is publishing a wildlife calendar to raise money for a local charity. The total cost of using the photos in the calendar is \$710. In addition to this one-time charge, the unit cost of printing each calendar is \$4.50.
    a. The school wants the average cost per calendar to be below \$10. Write a rational inequality relating the average cost per calendar to the desired cost per calendar.
    b. Solve the inequality from part (a) by graphing. How many calendars need to be printed to bring the average cost per calendar below \$10?
    c. Suppose the school wanted to have the average cost per calendar be below \$6. How many calendars would then need to be printed?

Lessons 8.4–8.6

1. **TRAVEL** A car travels 120 miles in the same amount of time that it takes a truck to travel 100 miles. The car travels 10 miles per hour faster than the truck. Use the verbal model to find the speed of the truck.

$$\frac{\text{Distance for car}}{\text{Speed of car}} = \frac{\text{Distance for truck}}{\text{Speed of truck}}$$

A 40 miles/hour

B 50 miles/hour

C 55 miles/hour

D 60 miles/hour

2. **RIVER CURRENT** The speed of a river's current is 3 miles per hour. You travel 2 miles with the current and then return to where you started in a total time of 1.25 hours. What is your approximate speed in still water?

A 3.0 miles/hour

B 3.9 miles/hour

C 5.0 miles/hour

D 5.5 miles/hour

3. **CYCLING** A cyclist travels 50 miles from her home to a state park at a speed of s miles per hour. On the return trip, she increases her speed by 5 miles per hour. Which expression represents the *total* time of the cyclist's round trip?

A $100s + 250$

B $\frac{2s+5}{50}$

C $\frac{250}{s^2 - 5s}$

D $\frac{100s + 250}{s^2 + 5s}$

4. **ALLOYS** Brass is an alloy composed of 55% copper and 45% zinc by weight. You have 25 ounces of copper. About how many ounces of zinc do you need to make brass?

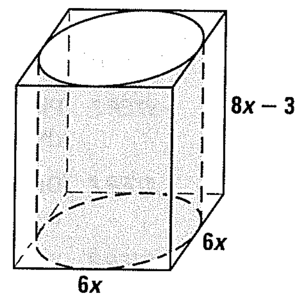
A 20.45 ounces

B 22.73 ounces

C 25.45 ounces

D 30.56 ounces

5. **GEOMETRY** In simplest form, what is the ratio of the volume of the rectangular prism to the volume of the inscribed cylinder?



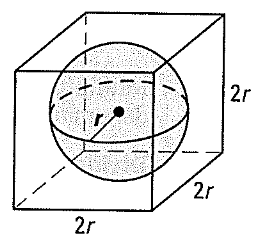
A $\frac{\pi}{4}$

B $\frac{4}{\pi}$

C π

D $\frac{12}{\pi}$

6. **OPEN-ENDED** Consider the following sphere and cube.



A. Find the ratio of the volume of the sphere to the volume of the cube. Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere and the formula $V = s^3$ for the volume of a cube where r is the radius of the sphere and s is the side length of the cube. Write your answer as a decimal rounded to the nearest hundredth.

B. *Explain* how you found your answer.

8 CHAPTER SUMMARY

Animated Algebra
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BIG IDEAS

For Your Notebook

Big Idea 1

Graphing Rational Functions

Use the following steps to graph $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$

where $p(x)$ and $q(x)$ have no common factors other than ± 1.

STEP 1 **Plot** the x-intercepts. The x-intercepts are the real zeros of $p(x)$.

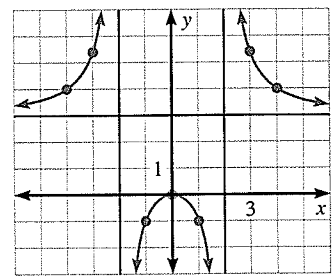
STEP 2 **Draw** the vertical asymptote(s). A vertical asymptote occurs at each real zero of $q(x)$.

STEP 3 **Draw** the horizontal asymptote, if it exists.

If $m < n$, $y = 0$ is a horizontal asymptote.

If $m = n$, $y = \dfrac{a_m}{b_n}$ is a horizontal asymptote.

If $m > n$, there is no horizontal asymptote.



$y = \dfrac{3x^2}{x^2 - 4}$

STEP 4 **Plot** several points on both sides of each vertical asymptote.

Big Idea 2

Performing Operations with Rational Expressions

Operation	Example
Simplify Divide out common factors from the numerator and denominator.	$\dfrac{x^2 + 3x}{x^2 + 8x + 15} = \dfrac{x\cancel{(x+3)}}{(x+5)\cancel{(x+3)}} = \dfrac{x}{x+5}$
Multiply Multiply numerators and denominators. Then simplify.	$\dfrac{x}{15} \cdot \dfrac{3}{x^2 + 7x} = \dfrac{3x}{15x(x+7)} = \dfrac{1}{5(x+7)}$
Divide Multiply the first expression by the reciprocal of the second expression. Then simplify.	$\dfrac{x^2}{3x+1} \div \dfrac{1}{6x+2} = \dfrac{x^2}{3x+1} \cdot \dfrac{2(3x+1)}{1} = 2x^2$
Add or Subtract Write the expressions with like denominators. Then add or subtract the numerators over the common denominator. Lastly, simplify.	$\dfrac{5}{x} + \dfrac{x}{x+2} = \dfrac{5(x+2)}{x(x+2)} + \dfrac{x^2}{x(x+2)} = \dfrac{x^2 + 5x + 10}{x(x+2)}$

Big Idea 3

Solving Rational Equations

Solve $\dfrac{x}{x+1} + \dfrac{2}{x+4} = 1.$

STEP 1 **Find** the LCD. — LCD is $(x + 1)(x + 4)$.

STEP 2 **Multiply** each side of the equation by the LCD. — $x(x + 4) + 2(x + 1) = (x + 1)(x + 4)$

STEP 3 **Solve** the resulting equation.

$$x^2 + 4x + 2x + 2 = x^2 + 5x + 4$$
$$6x + 2 = 5x + 4$$
$$x = 2$$

8 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

- inverse variation, *p. 551*
- constant of variation, *p. 551*
- joint variation, *p. 553*
- rational function, *p. 558*
- simplified form of a rational expression, *p. 573*
- complex fraction, *p. 584*
- cross multiplying, *p. 589*

VOCABULARY EXERCISES

1. Copy and complete: If two variables x and y are related by an equation of the form $y = \frac{a}{x}$ where $a \neq 0$, then x and y show _?_.
2. Suppose z varies jointly with x and y. What can you say about $\frac{z}{xy}$?
3. Copy and complete: A function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$ is called a(n) _?_.
4. Give two examples of a complex fraction.
5. Copy and complete: When you rewrite the equation $\frac{3}{x} = \frac{2}{x-1}$ as $3(x-1) = 2x$, you are _?_.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Model Inverse and Joint Variation

pp. 551–557

EXAMPLE

The variables x and y vary inversely, and $y = 12$ when $x = 3$. Write an equation that relates x and y. Then find y when $x = -4$.

$y = \frac{a}{x}$ **Write general equation for inverse variation.**

$12 = \frac{a}{3}$ **Substitute 12 for y and 3 for x.**

$36 = a$ **Solve for a.**

▶ The inverse variation equation is $y = \frac{36}{x}$. When $x = -4$, $y = \frac{36}{-4} = -9$.

EXERCISES

EXAMPLE 2 on p. 551 for Exs. 6–9

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = -3$.

6. $x = 1, y = 5$
7. $x = -4, y = -6$
8. $x = \frac{5}{2}, y = 18$
9. $x = -12, y = \frac{2}{3}$

8 CHAPTER REVIEW

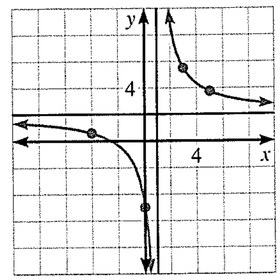
8.2 Graph Simple Rational Functions

pp. 558–563

EXAMPLE

Graph $y = \frac{2x+5}{x-1}$. State the domain and range.

STEP 1 **Draw** the asymptotes. Solve $x - 1 = 0$ for x to find the vertical asymptote $x = 1$. The horizontal asymptote is the line $y = \frac{2}{1} = 2$.



STEP 2 **Plot** points to the left and to the right of the vertical asymptote.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

▶ The domain is all real numbers except 1. The range is all real numbers except 2.

EXERCISES

EXAMPLES 2 and 3 on pp. 559–560 for Exs. 10–12

Graph the function. State the domain and range.

10. $y = \frac{4}{x-3}$ **11.** $y = \frac{1}{x+5} + 2$ **12.** $f(x) = \frac{3x-2}{x-4}$

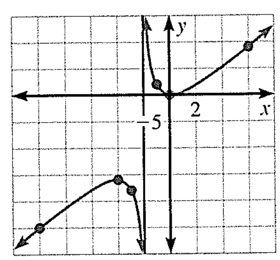
8.3 Graph General Rational Functions

pp. 565–571

EXAMPLE

Graph $y = \frac{2x^2}{x+2}$.

- The numerator has 0 as its only zero, so the graph has an x-intercept at (0, 0).
- The denominator has -2 as its only zero, so the graph has a vertical asymptote at $x = -2$.
- The degree of the numerator (2) is greater than the degree of the denominator (1). So, there is no horizontal asymptote. The graph has the same end behavior as the graph of $y = \frac{2}{1}x^{2-1} = 2x$.



EXERCISES

EXAMPLES 1, 2, and 3 on pp. 565–566 for Exs. 13–18

Graph the function.

13. $y = \frac{5}{x^2+1}$ **14.** $y = \frac{4x^2}{x-1}$ **15.** $h(x) = \frac{6x^2}{x-2}$

16. $y = \frac{-8}{x^2+3}$ **17.** $y = \frac{x^2+6}{x^2-3x-40}$ **18.** $g(x) = \frac{x^2-1}{x+4}$

8.4 Multiply and Divide Rational Expressions

pp. 573–580

EXAMPLE

Divide: $\frac{3x+27}{6x-48} \div \frac{x^2+9x}{x^2-4x-32}$

$$\frac{3x+27}{6x-48} \div \frac{x^2+9x}{x^2-4x-32} = \frac{3x+27}{6x-48} \cdot \frac{x^2-4x-32}{x^2+9x}$$ **Multiply by reciprocal.**

$$= \frac{3(x+9)}{6(x-8)} \cdot \frac{(x+4)(x-8)}{x(x+9)}$$ **Factor.**

$$= \frac{\cancel{3(x+9)}(x+4)\cancel{(x-8)}}{2\cancel{(3)}\cancel{(x-8)}(x)\cancel{(x+9)}}$$ **Divide out common factors.**

$$= \frac{x+4}{2x}$$ **Simplified form**

EXERCISES

EXAMPLES 3, 4, 6, and 7 on pp. 575–577 for Exs. 19–22

Perform the indicated operation. Simplify the result.

19. $\frac{80x^4}{y^3} \cdot \frac{xy}{5x^2}$

20. $\frac{x-3}{2x-8} \cdot \frac{6x^2-96}{x^2-9}$

21. $\frac{16x^2-8x+1}{x^3-7x^2+12x} \div \frac{20x^2-5x}{15x^3}$

22. $\frac{x^2-13x+40}{x^2-2x-15} \div (x^2-5x-24)$

8.5 Add and Subtract Rational Expressions

pp. 582–588

EXAMPLE

Add: $\frac{x}{6x+24} + \frac{x+2}{x^2+9x+20}$

The denominators factor as $6(x+4)$ and $(x+4)(x+5)$, so the LCD is $6(x+4)(x+5)$. Use this result to rewrite each expression with a common denominator, and then add.

$$\frac{x}{6x+24} + \frac{x+2}{x^2+9x+20} = \frac{x}{6(x+4)} + \frac{x+2}{(x+4)(x+5)}$$

$$= \frac{x}{6(x+4)} \cdot \frac{x+5}{x+5} + \frac{x+2}{(x+4)(x+5)} \cdot \frac{6}{6}$$

$$= \frac{x^2+5x}{6(x+4)(x+5)} + \frac{6x+12}{6(x+4)(x+5)}$$

$$= \frac{x^2+11x+12}{6(x+4)(x+5)}$$

EXERCISES

EXAMPLES 3 and 4 on pp. 583–584 for Exs. 23–25

Perform the indicated operation and simplify.

23. $\frac{5}{6(x+3)} + \frac{x+4}{2x}$

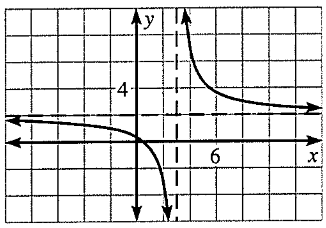
24. $\frac{5x}{x+8} + \frac{4x-9}{x^2+5x-24}$

25. $\frac{x+2}{x^2+4x+3} - \frac{5x}{x^2-9}$

PA Pennsylvania **Test Practice**

MULTIPLE CHOICE

In Exercises 1 and 2, use the given graph of a rational function.



1. What is the range of the function?
   - A All real numbers
   - B All real numbers except 2
   - C All real numbers except 3
   - D All real numbers except 5

2. Which statement is false?
   - A The line $x = 3$ is an asymptote.
   - B The line $y = 2$ is an asymptote.
   - C The function is undefined for $x = 2$.
   - D The value of y is unbounded.

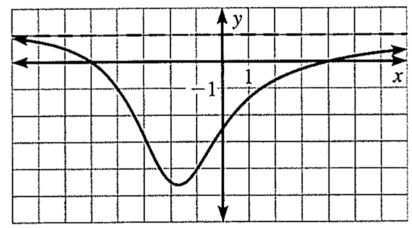
In Exercises 3 and 4, use the given table.

p	−12	3	30	−1.5
q	2	1	−5	−0.5
r	−2	1	−2	1

3. What is the relationship among the variables?
   - A The variable p varies jointly with q and r.
   - B The variable p varies jointly with q and the square of r.
   - C The variable r varies inversely with the sum of p and q.
   - D The variable q varies inversely with the sum of p and r.

4. What is the value of r when $p = 20$ and $q = -4$?
   - A -5
   - B $-\frac{10}{3}$
   - C $-\frac{5}{3}$
   - D $-\frac{1}{5}$

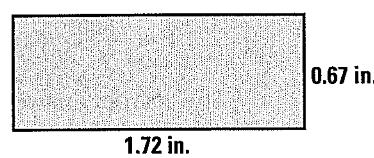
In Exercises 5 and 6, use the given graph of a rational function.



5. What are the x-intercepts of the graph?
   - A -5 and -2.5
   - B -2.5 and 4
   - C -4 and 5
   - D -5 and 4

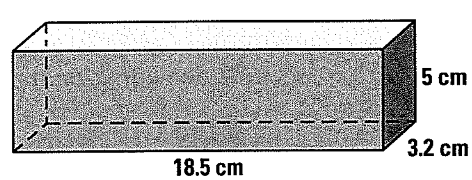
6. What is the horizontal asymptote of the graph?
   - A $x = -2$
   - B $y = 0$
   - C $x = 1$
   - D $y = 1$

7. Consider a rectangle whose dimensions change but whose area remains constant. The rectangle's length ℓ varies inversely with its width w. The diagram below shows the rectangle at one moment in time. Which equation relates ℓ and w?



   - A $w = 0.67\ell$
   - B $\ell w = 1.1524$
   - C $w = 1.72\ell$
   - D $\ell = 1.1524w$

8. What is the approximate radius of a cylinder that has the same volume as the rectangular prism below and has the least surface area possible?



   - A 3.61 cm
   - B 3.84 cm
   - C 6.00 cm
   - D 7.66 cm

TEST PREPARATION

MULTIPLE CHOICE

9. What is the solution to the equation below?

$$\frac{4}{x+1} - \frac{1}{x} = 1$$

A -1

B 0

C $\frac{2}{3}$

D 1

10. What is the domain of the function below?

$$f(x) = \frac{x-3}{2x^2 - 4x + 2}$$

A all real numbers except 3

B all real numbers except 1

C all real numbers except 1 and 3

D all real numbers

11. The variables m and n vary inversely, and $m = -2$ when $n = -8$. What is the value of n when $m = 5$?

A -4

B -3.2

C 3.2

D 4

12. So far, Luis has turned in 15 of 17 homework assignments. However, Luis plans to do every future assignment. What is the *total* number of assignments that Luis must turn in to have a homework percentage of exactly 95%? Find out by solving the equation $\frac{15+x}{17+x} = 0.95$.

A 3 assignments

B 23 assignments

C 38 assignments

D 40 assignments

OPEN-ENDED

13. From 1980 to 2001, the number n (in millions) of males enrolled in high school in the United States can be modeled by

$$n = \frac{0.0441x^2 - 1.08x + 7.25}{0.00565x^2 - 0.142x + 1.00}$$

where x is the number of years since 1980.

A. Make a table of values showing the enrollment at 2 year intervals for the years 1980 to 2000.

B. Graph the model.

C. Use your graph to estimate the year in which 7.7 million males were enrolled in high school.

D. Analyze the graph's trend. Do you think it will it continue indefinitely? *Explain* your answer.

14. A digital video recorder costs \$99.99, and a programming service for the digital video recorder costs \$12.95 per month.

A. Write a model that gives the average cost per month C as a function of the number of months m you have subscribed to the service.

B. Graph the model. Use the graph to estimate the number of months that you need to subscribe before the average cost drops to \$14 per month.

C. What is the equation of the horizontal asymptote? What does the asymptote represent?

9 Quadratic Relations and Conic Sections

PA M11.C.3.1.1

Before

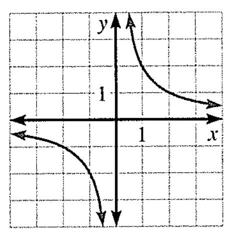
In previous chapters, you learned the following skills, which you'll use in Chapter 9: graphing quadratic functions, completing the square, and solving linear systems.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The graph of a(n) _?_ function is a **parabola**.
2. The graph of the **rational function** $y = \frac{2}{x}$, shown at the right, is a _?_.
3. Two equations of the form $Ax + By = C$ and $Dx + Ey = F$ form a _?_ **system of equations**.



SKILLS CHECK

Graph. Label the vertex and axis of symmetry. ***(Review pp. 236, 245 for 9.2.)***

4. $y = x^2 - 3$
5. $y = -0.25x^2$
6. $y = 3(x + 1)^2$
7. $y = 0.5(x - 2)^2 + 4$

Solve the equation by completing the square. ***(Review p. 284 for 9.6.)***

8. $x^2 - 4x + 7 = 0$
9. $x^2 - 8x - 15 = 0$
10. $3x^2 + 9x - 12 = 0$

Solve the system using any algebraic method. ***(Review p. 160 for 9.7.)***

11. $2x - y = 11$
    $-x - 2y = -3$
12. $x + 5y = -17$
    $-2x - 3y = 13$
13. $-4x + 7y = -14$
    $2x - 6y = 12$

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